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THE CONDUCTING LIQUID FLOW BETWEEN POROUS WALLS WITH HEAT TRANSFER

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Abstract. The paper studies a viscous electroconductive liquid flow between porous walls, when external homogeneous magnetic field is perpendicular to the walls. The liquid flow is caused both by pulsating pressure drop and by pulsating motion of porous walls. Physical characteristics of the flow are found.

რეზიუმე. შესწავლილია ელექტროგამტარი ბლანტი არაკუმშვადი სითხის დინება ფოროვან კედლებს შორის, როდესაც კედლების მართობულად მოდებულია გარეგანი ერთგვაროვანი მაგნიტური ველი. სითხის დინება გამოწვეულია წნევის პულსაციური დაცემით და ფოროვანი კედლების პულსაციური მოძრაობით. ნაპოვნია სითხის ფიზიკური მახასიათებლები.

1. Introduction

In the present paper we study an electroconductive viscous incompressible liquid flow pulsating between porous walls with heat transfer when external homogeneous magnetic field is perpendicular to the walls. The liquid flow is caused by pulsating motion of porous walls and by pulsating pressure drop which is given by the formula: $-\dfrac{1}{\rho}\dfrac{\partial P}{\partial z} = Ae^{-i\omega t}$. Temperature change in the porous pipe walls and in the pipe is pulsatory. The heat transfer equation includes dissipation energy $\eta\left(\dfrac{\partial V}{\partial x}\right)^2$, due to friction, and Joule's heat $\nu\left(\dfrac{\partial H}{\partial x}\right)^2$.

Exact solutions of the Stokes-Navier and heat transfer equations for nonstationary motion of electroconductive viscous incompressible liquid are obtained. Physical characteristics of motion and heat transfer are studied making allowance for the action of Hartman, Prandtl and Reynolds numbers and similarity criteria of a pulsating flow. Exact solutions of heat

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transfer equations are obtained in three cases when they include: (a) friction heat $\eta\left(\frac{\partial V}{\partial x}\right)^2$, and Joule's heat $\nu\left(\frac{\partial H}{\partial x}\right)^2$. only friction heat; (c) only Joule's heat.

The problems corresponding to that formulated in the present paper have been studied in [1-10], while the problems dealing with a laminar liquid flow free from heat transfer in the pipe, when in the pipe walls there takes place intensive inflow or outflow, can be found in [6,7,8].

2. The Basic Part

Let us consider the flow of electroconductive viscous incompressible liquid in a plane porous pipe, when external homogeneous magnetic field (H_0) is perpendicular to the liquid motion.

If we direct the oz axis in the fluid flow direction, and the ox axis in perpendicular to walls, then for the desired values we will have: $\vec{V}(u_0^*, 0, v_z(x,t))$, $\vec{H}\,(H_0, 0, H_z(x,t))$, $T(x,t)$.

If the oz-axis is directed along the liquid flow and the ox-axis is perpendicular to the pipe walls, then for the unknown quantities we have $\vec{V}(u_0^*, 0, v_z(x,t))$, $\vec{H}\,(H_0, 0, H_z(x,t))$, $T(x,t)$.

Taking into account the above-said, equations for the motion, induction and heat transfer in dimensionless values take the form

$$\begin{cases} \dfrac{\partial u}{\partial \tau} = f(\tau) + \dfrac{\partial^2 u}{\partial \xi^2} + M^2 \dfrac{\partial h}{\partial \xi} + R\dfrac{\partial u}{\partial \xi}, \\ \dfrac{\partial h}{\partial \tau} = \dfrac{\nu_m}{\nu}\dfrac{\partial^2 h}{\partial \xi^2} + \dfrac{\nu_m}{\nu}\dfrac{\partial u}{\partial \xi} + R\dfrac{\partial h}{\partial \xi}, \\ \dfrac{\partial \theta}{\partial \tau} - R\,\mathrm{Pr}\,\dfrac{\partial \theta}{\partial \xi} = \dfrac{\partial^2 \theta}{\partial \xi} + \left(\dfrac{\partial u}{\partial \xi}\right)^2 + M^2\left(\dfrac{\partial H}{\partial \xi}\right)^2. \end{cases} \tag{1}$$

where

$$\xi = \frac{x}{L},\ \tau = \frac{\nu}{L^2}t,\ U = \frac{V}{V_0^*},\ f(\tau) = -\frac{L^2}{\nu V_0^* \rho}\frac{\partial P}{\partial z},\ h = \frac{H}{H_0 R_m},\ \theta = \frac{k}{\eta {V_0^*}^2}T-$$

are dimensionless quantities, V_0^* and L are, respectively, characteristic velocity and characteristic length, and u_0^* is the characteristic velocity of porosity. $M = H_0 L\sqrt{\frac{\sigma}{\eta}}-$ is Hartman's number, $\alpha = \frac{\omega L^2}{\nu}-$ is a similarity criterion of steady pulsation motion, $P_r = \frac{\eta C_\nu}{k}-$ is Prandtl's number, $R = \frac{u_0^* L}{\nu}$ is Reynolds number characteristic liquid leakage, $R_m = \frac{V_0^* L}{\nu_m}$ is magnetic Reynolds number, and $D = \frac{AL^2}{\nu V_0^*}$ is amplitude of pulsating pressure drop.

ν is coefficient of kinematic viscosity, η is friquency, ω – is the frequency, $C_\nu-$ is specific heat, k is the heat conductivity coefficient.

The initial boundary conditions for equations of motion and induction are of the form:

$$u(\xi,0)=h(\xi,0)=0,\quad u(\pm 1,\tau)=\varphi_{1,2}(\tau),\quad h(\pm 1,\tau)=0, \tag{2}$$

and the initial boundary condition for equations of heat transfer have the form:

$$\begin{cases}\theta(\xi,0)=\theta_1(\xi,\tau)+\theta_2(\xi,\tau)=0,\\ \theta(\pm 1,\tau)=\theta_1(\pm 1,\tau)+\theta_2(\pm 1,\tau)=q_{1,2}^{(1)}(\tau)+q_{1,2}^{(2)}(\tau)=q_{1,2}(\tau),\end{cases} \tag{3}$$

where $\theta(\xi,\tau)=\theta_1(\xi,\tau)+\theta_2(\xi,\tau)$ is a full temperature of the liquid, $\theta_1(\xi,\tau)$ is temperature when the heat transfer equation includes only heat due to the friction, and $\theta_2(\xi,\tau)$ is temperature when this equation includes only Joule's heat.

To solve the boundary value problem (1)–(2), we use the Laplace integral transformation and get

$$S\overline{u}=\overline{f}(S)+\overline{u}''+R\overline{u}'+M^2\overline{h}', \tag{4}$$

$$S\overline{h}=\frac{\nu_m}{\nu}\overline{h}''+\frac{\nu_m}{\nu}\overline{u}'+R\overline{h}', \tag{5}$$

$$\overline{u}(\pm 1,S)=\overline{\varphi}_{1,2}(S),\quad \overline{h}(\pm 1,S)=0, \tag{6}$$

where

$$\overline{u}(\xi,S)=\int_0^\infty u(\xi,\tau)e^{-S\tau}d\tau, \overline{h}(\xi,S)=\int_0^\infty h(\xi,\tau)e^{-S\tau}d\tau,$$

$$\overline{f}(S)=\int_0^\infty f(\tau)e^{-S\tau}d\tau, \overline{\varphi}_{1,2}(S)=\int_0^\infty \varphi_{1,2}(\tau)e^{-S\tau}d\tau.$$

Consider the flow when kinematic coefficient of viscosity (ν) is equal to the kinematic coefficient of magnetic viscosity (ν_m) (i.e. $\nu=\nu_m$). Then the solution of the problem (4)–(6) after transformations takes the form:

$$2\overline{u}-\frac{2\overline{f}}{S}=$$

$$=\left(\overline{\varphi}_1-\frac{\overline{f}}{S}\right)\left[\frac{\exp[(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2+4S}(1+\xi)/2\right)}{sh\sqrt{(R+M)^2+4S}}+\right.$$

$$\left.+\frac{\exp[(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2+4S}(1+\xi)/2\right)}{sh\sqrt{(R-M)^2+4S}}\right]+$$

$$
+\left(\overline{\varphi}_2-\frac{\overline{f}}{S}\right)\left[\frac{\exp[(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2+4S}(1-\xi)/2\right)}{sh\sqrt{(R+M)^2+4S}}+\right.
$$

$$
\left.+\frac{\exp[(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2+4S}(1-\xi)/2\right)}{sh\sqrt{(R-M)^2+4S}}\right], \tag{7}
$$

$$
2M\overline{h}=
$$

$$
=\left(\overline{\varphi}_1-\frac{\overline{f}}{S}\right)\left[\frac{\exp[(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2+4S}(1+\xi)/2\right)}{sh\sqrt{(R+M)^2+4S}}-\right.
$$

$$
\left.-\frac{\exp[(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2+4S}(1+\xi)/2\right)}{sh\sqrt{(R-M)^2+4S}}\right]+
$$

$$
+\left(\overline{\varphi}_2-\frac{\overline{f}}{S}\right)\left[\frac{\exp[(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2+4S}(1-\xi)/2\right)}{sh\sqrt{(R+M)^2+4S}}-\right.
$$

$$
\left.-\frac{\exp[(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2+4S}(1-\xi)/2\right)}{sh\sqrt{(R-M)^2+4S}}\right]. \tag{8}
$$

Consider the liquid flow which is caused by the pulsating motion of pipe walls, $\left(u(\pm1,\tau)=\varphi_{1,2}(\tau)=A_{1,2}e^{-i\alpha\tau}\right)$ and by pulsating pressure drop, $\left(-\dfrac{1}{\rho}\dfrac{\partial P}{\partial z}=f^*(t)=Ae^{-i\alpha\tau}\right)$.

If we take into account the above-said in formulas (7) and (8), then for the velocity and magnetic induction we get in originals the following formulas:

$$
2u(\xi,\tau)+\frac{2D}{i\alpha}e^{-i\alpha\tau}=\frac{e^{-i\alpha\tau}}{sh\sqrt{(R+M)^2-4i\alpha}}\times
$$

$$
\times\left[\frac{\exp[(R+M)(1-\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.
$$

$$
\left.+\frac{\exp[-(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]+
$$

$$
+\frac{e^{-i\alpha\tau}}{sh\sqrt{(R-M)^2-4i\alpha}}\times
$$

$$\times\left[\frac{\exp[(R-M)(1-\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$

$$\left.+\frac{\exp[-(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]+$$

$$+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1}\left\{\frac{e^{S_n\tau}}{S_n(S_n+i\alpha)}\times\right.$$

$$\times\Big[(A_1S_n-D)\exp[(R+M)(1-\xi)/2]\sin\mu_n(1+\xi)/2+$$

$$+(A_2S_n-D)\exp[-(R+M)(1+\xi)/2]\sin\mu_n(1-\xi)/2\Big]+$$

$$+\frac{e^{S_n^*\tau}}{S_n^*(S_n^*+i\alpha)}\Big[(A_1S_n^*-D)\exp[(R-M)(1-\xi)/2]\sin\mu_n(1+\xi)/2+$$

$$\left.+(A_2S_n^*-D)\exp[-(R-M)(1+\xi)/2]\sin\mu_n(1-\xi)/2\Big]\right\},\qquad(9)$$

$$2Mh(\xi,\tau)=\frac{e^{-i\alpha\tau}}{sh\sqrt{(R+M)^2-4i\alpha}}\times$$

$$\times\left[\frac{\exp[(R+M)(1-\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$

$$\left.+\frac{\exp[-(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]-$$

$$-\frac{e^{-i\alpha\tau}}{sh\sqrt{(R-M)^2-4i\alpha}}\times$$

$$\times\left[\frac{\exp[(R-M)(1-\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$

$$\left.+\frac{\exp[-(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]+$$

$$+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1}\left\{\frac{e^{S_n\tau}}{S_n(S_n+i\alpha)}\times\right.$$

$$\times\Big[(A_1S_n-D)\exp[(R+M)(1-\xi)/2]\sin\mu_n(1+\xi)/2+$$
$$+(A_2S_n-D)\exp[-(R+M)(1+\xi)/2]\sin\mu_n(1-\xi)/2 \Big]-$$
$$-\frac{e^{S_n^*\tau}}{S_n^*(S_n^*+i\alpha)}\Big[(A_1S_n^*-D)\exp[(R-M)(1-\xi)/2]\sin\mu_n(1+\xi)/2+$$
$$+(A_2S_n^*-D)\exp[-(R-M)(1+\xi)/2]\sin\mu_n(1-\xi)/2 \Big] \Bigg\}, \qquad (10)$$

where $S_n=-\frac{1}{4}\left[\mu_n^2+(R+M)^2\right]$, $S_n^*=-\frac{1}{4}\left[\mu_n^2+(R-M)^2\right]$, $\mu_n=\pi n$, while A_1, A_2 and D are arbitrary complex numbers.

In the equation of heat transfer (third equation of the first system) we neglect first the Joule's heat and then the heat due to the friction. We obtain, respectively, the following equations:

$$P_r\frac{\partial\theta_1}{\partial\tau}-RP_r\frac{\partial\theta_1}{\partial\xi}-\frac{\partial^2\theta_1}{\partial\xi^2}=\left(\frac{\partial u}{\partial\xi}\right)^2, \qquad (11)$$

$$P_r\frac{\partial\theta_2}{\partial\tau}-RP_r\frac{\partial\theta_2}{\partial\xi}-\frac{\partial^2\theta_2}{\partial\xi^2}=\left(M\frac{\partial h}{\partial\xi}\right)^2. \qquad (12)$$

In a porous pipe, the process of setting a pulsating temperature mode is slower as compared with the change of velocity in a pulsation mode. Therefore we can substitute into equations (11)–(12) the values of velocity and induction which are given by the formulas:

$$2u(\xi,\tau)+\frac{2D}{i\alpha}e^{-i\alpha\tau}=\frac{e^{-i\alpha\tau}}{sh\sqrt{(R+M)^2-4i\alpha}}\times$$
$$\times\left[\frac{\exp[(R+M)(1-\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$
$$\left.+\frac{\exp[-(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]+$$
$$+\frac{e^{-i\alpha\tau}}{sh\sqrt{(R-M)^2-4i\alpha}}\times$$
$$\times\left[\frac{\exp[(R-M)(1-\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$

$$+\left.\frac{\exp[-(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right], \tag{13}$$

$$2Mh(\xi,\tau)=\frac{e^{-i\alpha\tau}}{sh\sqrt{(R+M)^2-4i\alpha}}\times$$
$$\times\left[\frac{\exp[(R+M)(1-\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$
$$+\left.\frac{\exp[-(R+M)(1+\xi)/2]sh\left(\sqrt{(R+M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]-$$
$$-\frac{e^{-i\alpha\tau}}{sh\sqrt{(R-M)^2-4i\alpha}}\times$$
$$\times\left[\frac{\exp[(R-M)(1-\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1+\xi)/2\right)}{i\alpha/(D+i\alpha A_1)}+\right.$$
$$+\left.\frac{\exp[-(R-M)(1+\xi)/2]sh\left(\sqrt{(R-M)^2-4i\alpha}(1-\xi)/2\right)}{i\alpha/(D+i\alpha A_2)}\right]. \tag{14}$$

If in equations (11)–(12) we bear in mind formulas (13)–(14) and apply the Laplace formula of integral transformation, then taking into account the initial boundary conditions (3), the temperature after transformations will take the form

$$\overline{\theta}_{1,2}(\xi,\tau)=$$
$$=\frac{sh\left(\sqrt{P_r^2R^2+4SP_r}(1+\xi)/2\right)}{sh\sqrt{P_r^2R^2+4SP_r}}e^{\frac{RP_r(1-\xi)}{2}}\times$$
$$\times\left[\frac{1}{S+2i\alpha}\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+2P_rR\beta_k-SP_r}+\overline{q}_1^{(1,2))}(S)\right]+$$
$$+\frac{sh\left(\sqrt{P_r^2R^2+4SP_r}(1-\xi)/2\right)}{sh\sqrt{P_r^2R^2+4SP_r}}e^{-\frac{RP_r(1+\xi)}{2}}\times$$
$$\times\left[\frac{1}{S+2i\alpha}\sum_{k=1}^{10}\frac{b_k^2e^{-2\beta_k}}{4\beta_k^2+2P_rR\beta_k-SP_r}+\overline{q}_2^{(1,2))}(S)\right]-$$
$$-\frac{1}{S+2i\alpha}\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+P_rR\beta_k-SP_r}, \tag{15}$$

where

$$\begin{cases} \beta_{1,2} = -\frac{R+M}{2} \pm \dfrac{1}{2}\sqrt{(R+M)^2 - 4i\alpha}, \\ \beta_{3,4} = -\frac{R-M}{2} \pm \dfrac{1}{2}\sqrt{(R-M)^2 - 4i\alpha}, \\ 2\beta_5 = \beta_1 + \beta_2, \ \ 2\beta_6 = \beta_1 + \beta_3, \ \ 2\beta_7 = \beta_1 + \beta_4, \\ 2\beta_8 = \beta_2 + \beta_3, \ \ 2\beta_9 = \beta_2 + \beta_4, \ \ 2\beta_{10} = \beta_3 + \beta_4. \end{cases} \tag{16}$$

$$\begin{cases} b_1 = \dfrac{\beta_1}{sh(\beta_1 - \beta_2)} \left(\dfrac{A_1}{4} e^{-\beta_2} - \dfrac{A_2}{4} e^{\beta_2} - \dfrac{D}{2i\alpha} sh\beta_2 \right), \\ \qquad b_5^2 = 2b_1 b_2, \quad b_6^2 = \pm 2b_1 b_3, \\ b_2 = \dfrac{\beta_2}{sh(\beta_1 - \beta_2)} \left(\dfrac{A_2}{4} e^{\beta_1} - \dfrac{A_1}{4} e^{-\beta_1} + \dfrac{D}{2i\alpha} sh\beta_1 \right), \\ \qquad b_7^2 = \pm 2b_1 b_4 \quad b_9^2 = \pm 2b_2 b_3, \\ b_3 = \dfrac{\beta_3}{sh(\beta_1 - \beta_2)} \left(\dfrac{A_1}{4} e^{-\beta_4} - \dfrac{A_2}{4} e^{\beta_4} - \dfrac{D}{2i\alpha} sh\beta_4 \right), \\ \qquad b_9 = \pm 2b_2 b_4, \quad b_{10}^2 = 2b_3 b_4, \\ b_4 = \dfrac{\beta_4}{sh(\beta_1 - \beta_2)} \left(\dfrac{A_2}{4} e^{\beta_3} - dA_1 4 e^{-\beta_3} - \dfrac{D}{2i\alpha} sh\beta_3 \right). \end{cases} \tag{17}$$

It should be noted that for the values b_6^2, b_7^2, b_8^2 and b_9^2 we use the sign "+" for $\overline{\theta}_1(\xi, S)$ and the sign "-" for $\overline{\theta}_2(\xi, S)$.

Consider now temperature change for steady-state pulsating flow when temperature change at the initial moment equals zero, while on the plane pipe walls it runs according to the pulsation law $\left(q_{1,2}^{(1)}(\tau) = B_{1,2}^{(1)} e^{-2i\alpha\tau}, \ q_{1,2}^{(2)}(\tau) = B_{1,2}^{(2)} e^{-2i\alpha\tau} \right)$.

If we take the above-said into account in formula (15), then for the temperature in originals we get

$$\theta_{1,2}(\xi, \tau) = \Bigg\{ \left[\sum_{k=1}^{10} \frac{b_k^2 e^{2\beta_k}}{4\beta_k^2 + 2P_r R\beta_k + 2i\alpha P_r} + B_1^{(1,2)} \right] \times$$
$$\times \frac{sh\left(\sqrt{P_r^2 R^2 - 8i\alpha P_r}(1+\xi)/2 \right)}{sh\sqrt{P_r^2 R^2 - 8i\alpha P_r}} e^{\frac{RP_r(1-\xi)}{2}} +$$
$$+ \left[\sum_{k=1}^{10} \frac{b_k^2 e^{-2\beta_k}}{4\beta_k^2 + 2P_r R\beta_k + 2i\alpha P_r} + B_2^{(1,2)} \right] \times$$

$$\times\frac{sh\left(\sqrt{P_r^2R^2-8i\alpha P_r}(1-\xi)/2\right)}{sh\sqrt{P_r^2R^2-8i\alpha P_r}}e^{-\frac{RP_r(1+\xi)}{2}}-$$

$$\left.-\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+2P_rR\beta_k+2i\alpha P_r}\right\}e^{-2i\alpha\tau}+$$

$$+\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^{n+1}\mu_ne^{S_n\tau}}{P_r(S_n+2i\alpha)}\left\{\left[B_1^{(1,2)}+\right.\right.$$

$$\left.+\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+2P_rR\beta_k-SP_r}\right]e^{\frac{RP_r(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\left[B_2^{(1,2)}+\right.$$

$$\left.\left.+\sum_{k=1}^{10}\frac{b_k^2e^{-2\beta_k}}{4\beta_k^2+2P_rR\beta_k-SP_r}\right]e^{-\frac{RP_r(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right\}, \qquad (18)$$

where $S_n=-\frac{1}{4P_r}\left(\mu_n^2+R^2P_r\right)$, $\mu_n=\pi n$.

I. Consider now pulsating liquid flow caused by only pulsating motion of walls. Assume that pulsating motion of walls takes place in one and the same phase, with the same amplitude ($A_1=A_2=u_0$), while temperature change in the pipe walls is pulsatory in one and the same phase, with the same amplitude $\left(B_{1,2}^{(1)}=\theta_1^{(1)}=\text{const}, B_{1,2}^{(2)}=\theta_1^{(2)}=\text{const}\right)$, and the pressure drop equals zero ($D=0$).

Taking into account the above-said, from formulas (9), (10) and (18) for velocity, induction and temperature we have

$$\frac{2u^I(\xi,\tau)}{u_0}=\left[\frac{sh\beta_1e^{\beta_2\xi}-sh\beta_2e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}+\frac{sh\beta_3e^{\beta_4\xi}-sh\beta_4e^{\beta_3\xi}}{sh(\beta_3-\beta_4)}\right]e^{-i\alpha\tau}+$$

$$+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n-1}\left\{\frac{e^{S_n\tau}}{S_n+i\alpha}\left(e^{\frac{(R+M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.\right.$$

$$\left.+e^{-\frac{(R+M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)+$$

$$+\frac{e^{S_n^*\tau}}{S_n^*+i\alpha}\left(e^{\frac{(R-M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$

$$\left.\left.+e^{-\frac{(R-M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right\}, \qquad (9^I)$$

$$\frac{2Mh^I(\xi,\tau)}{u_0}=\left[\frac{sh\beta_1e^{\beta_2\xi}-sh\beta_2e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}-\frac{sh\beta_3e^{\beta_4\xi}-sh\beta_4e^{\beta_3\xi}}{sh(\beta_3-\beta_4)}\right]e^{-i\alpha\tau}+$$

$$+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n-1}\left\{\frac{e^{S_n\tau}}{S_n+i\alpha}\left(e^{\frac{(R+M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.\right.$$
$$\left.+e^{-\frac{(R+M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)-\frac{e^{S_n^*\tau}}{S_n^*+i\alpha}\left(e^{\frac{(R-M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$
$$\left.\left.+e^{-\frac{(R-M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right\}, \qquad (10^I)$$

$$\theta_{1,2}^I(\xi,\tau)=\left[\frac{\theta_1^{(1,2)}}{sh\gamma}\left(e^{\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}+e^{-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}\right)+\right.$$
$$+\frac{1}{sh\gamma}\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+2P_rR\beta_k+2i\alpha P_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}+\right.$$
$$\left.\left.+e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}-e^{2\beta_k\xi}sh\gamma\right)\right]e^{-2i\alpha\tau}+$$
$$+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_ne^{S_n\tau}}{P_r(S_n+2i\alpha)}\left[\theta_1^{(1,2)}\left(e^{\frac{RP_r(1-\xi)}{2}}\sin\frac{\gamma(1+\xi)}{2}+\right.\right.$$
$$\left.+e^{-\frac{RP_r(1+\xi)}{2}}\sin\frac{\gamma(1-\xi)}{2}\right)+$$
$$+\frac{1}{2}\sum_{k=1}^{10}\frac{b_k^2}{4\beta_k^2+2P_rR\beta_k-SP_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$
$$\left.\left.+e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right], \qquad (18^I)$$

where

$$\gamma=\sqrt{P_r^2R^2-8i\alpha P_r},\quad b_1=-\frac{u_0}{2}\frac{\beta_1sh\beta_2}{sh(\beta_1-\beta_2)},$$
$$b_2=\frac{u_0}{2}\frac{\beta_2sh\beta_1}{sh(\beta_1-\beta_2)},\quad b_3=-\frac{u_0}{2}\frac{\beta_3sh\beta_4}{sh(\beta_3-\beta_4)},$$
$$b_4=\frac{u_0}{2}\frac{\beta_4sh\beta_3}{sh(\beta_3-\beta_4)}. \qquad (17^I)$$

If we calculate friction force in the liquid and on the pipe walls, we will get, respectively, the following expressions:

$$F^I=\frac{u_0}{2}\left[\frac{\beta_2sh\beta_1e^{\beta_2\xi}-\beta_1sh\beta_2e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}+\frac{\beta_4sh\beta_3e^{\beta_4\xi}-\beta_3sh\beta_4e^{\beta_3\xi}}{sh(\beta_1-\beta_2)}\right]e^{-1\alpha\tau},$$
$$F_1^I=\frac{u_0}{2}\left[\frac{\beta_2sh\beta_1e^{\beta_2}-\beta_1sh\beta_2e^{\beta_1}}{sh(\beta_1-\beta_2)}+\frac{\beta_4sh\beta_3e^{\beta_4}-\beta_3sh\beta_4e^{\beta_3}}{sh(\beta_1-\beta_2)}\right]e^{-1\alpha\tau},$$

$$F_2^I = \frac{u_0}{2}\left[\frac{\beta_2 sh\beta_1 e^{-\beta_2} - \beta_1 sh\beta_2 e^{-\beta_1}}{sh(\beta_1-\beta_2)} + \frac{\beta_4 sh\beta_3 e^{-\beta_4} - \beta_3 sh\beta_4 e^{-\beta_3}}{sh(\beta_1-\beta_2)}\right] e^{-1\alpha\tau},$$

and for the liquid discharge and for the avarage flow velosity we obtain

$$Q^I = \frac{iu_0 e^{-i\alpha\tau}}{\alpha}\left[\frac{\beta_1-\beta_2}{sh(\beta_1-\beta_2)} sh\beta_1 sh\beta_2 + \frac{\beta_3-\beta_4}{sh(\beta_3-\beta_4)} sh\beta_3 sh\beta_4\right],$$

$$U^I = \frac{iu_0 e^{-i\alpha\tau}}{2\alpha}\left[\frac{\beta_1-\beta_2}{sh(\beta_1-\beta_2)} sh\beta_1 sh\beta_2 + \frac{\beta_3-\beta_4}{sh(\beta_3-\beta_4)} sh\beta_3 sh\beta_4\right].$$

When the walls of the plane pipe move in one and the same phase, with the same amplitude according to the pulsation law, then the velocity on the pipe axis fails to reach its maximum (as this is the case in a non-porous pipe), and friction force on the pipe walls has the same direction, but different value.

When Reynolds leakage number ($R = u_0^* L/\nu$) increases, liquid discharge decreases.

II. Let us consider pulsating flow of the liquid which is caused by the pulsating motion of pipe walls, when pulsating motion occurs in one and the same phase, with amplitude having different signs ($A_1 = V_0, A_2 = -V_0$). Temperature change in the pipe walls is also pulsative, in one and the same phase, with amplitude having different signs ($B_1^{(1)} = -B_2^{(1)} = \theta_2^{(1)} = \text{const}$, $B_1^{(2)} = -B_2^{(2)} = \theta_2^{(2)} = \text{const}$). The pressure fall is equal to zero ($D = 0$).

If we take into account the above-said, then from formulas (9), (10) and (18) for the velocity, induction and temperature we obtain

$$\frac{2u^{II}(\xi,\tau)}{V_0} = \left[\frac{ch\beta_2 e^{\beta_1\xi} - ch\beta_1 e^{\beta_2\xi}}{sh(\beta_1-\beta_2)} + \frac{ch\beta_4 e^{\beta_3\xi} - ch\beta_3 e^{\beta_4\xi}}{sh(\beta_3-\beta_4)}\right] e^{-i\alpha\tau} +$$
$$+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n-1}\left\{\frac{e^{S_n\tau}}{S_n+i\alpha}\left(e^{\frac{(R+M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2} - \right.\right.$$
$$\left. -e^{-\frac{(R+M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right) +$$
$$+\frac{e^{S_n^*\tau}}{S_n^*+i\alpha}\left(e^{\frac{(R-M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2} - \right.$$
$$\left.\left. -e^{-\frac{(R-M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right\}, \qquad (9^{II})$$

$$\frac{2Mh^{II}(\xi,\tau)}{V_0} = \left[\frac{ch\beta_2 e^{\beta_1\xi} - ch\beta_1 e^{\beta_2\xi}}{sh(\beta_1-\beta_2)} - \frac{ch\beta_4 e^{\beta_3\xi} - ch\beta_3 e^{\beta_4\xi}}{sh(\beta_3-\beta_4)}\right] e^{-i\alpha\tau} +$$
$$+\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n-1}\left\{\frac{e^{S_n\tau}}{S_n+i\alpha}\left(e^{\frac{(R+M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2} - \right.\right.$$

$$-e^{-\frac{(R+M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\Bigg)-$$

$$-\frac{e^{S_n^*\tau}}{S_n^*+i\alpha}\left(e^{\frac{(R-M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}-\right.$$

$$\left.\left.-e^{-\frac{(R-M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right\},\qquad (10^{II})$$

$$\theta_{1,2}^{II}(\xi,\tau)=\left[\frac{\theta_2^{(1,2)}}{sh\gamma}\left(e^{\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}-\right.\right.$$

$$\left.-e^{-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}\right)+$$

$$+\frac{1}{sh\gamma}\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+2P_rR\beta_k+2i\alpha P_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}-\right.$$

$$\left.\left.-e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}-e^{2\beta_k\xi}sh\gamma\right)\right]e^{-2i\alpha\tau}+$$

$$+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_ne^{S_n\tau}}{P_r(S_n+2i\alpha)}\left[\theta_1^{(1,2)}\left(e^{\frac{RP_r(1-\xi)}{2}}\sin\frac{\gamma(1+\xi)}{2}+\right.\right.$$

$$\left.+e^{-\frac{RP_r(1+\xi)}{2}}\sin\frac{\gamma(1-\xi)}{2}\right)x+$$

$$+\frac{1}{2}\sum_{k=1}^{10}\frac{b_k^2}{4\beta_k^2+2P_rR\beta_k-SP_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$

$$\left.\left.+e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right],\qquad (18^{II})$$

where

$$\gamma=\sqrt{P_r^2R^2-8i\alpha P_r},\quad b_1=\frac{V_0}{2}\frac{\beta_1ch\beta_2}{sh(\beta_1-\beta_2)},\quad b_2=-\frac{V_0}{2}\frac{\beta_2ch\beta_1}{sh(\beta_1-\beta_2)},$$

$$b_3=\frac{V_0}{2}\frac{\beta_3ch\beta_4}{sh(\beta_3-\beta_4)},\quad b_4=-\frac{V_0}{2}\frac{\beta_4ch\beta_3}{sh(\beta_3-\beta_4)}.\qquad (17^{II})$$

Calculating friction force in the liquid and on the pipe walls, we get, respectively, the following expressions:

$$F^{II}=\frac{V_0}{2}\Big[\frac{\beta_1ch\beta_2e^{\beta_1\xi}-\beta_1ch\beta_2e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}+\frac{\beta_3ch\beta_4e^{\beta_3\xi}-\beta_4ch\beta_3e^{\beta_4\xi}}{sh(\beta_1-\beta_2)}\Big]e^{-1\alpha\tau},$$

$$F_1^{II}=\frac{V_0}{2}\Big[\frac{\beta_1ch\beta_2e^{\beta_1}-\beta_2ch\beta_1e^{\beta_2}}{sh(\beta_1-\beta_2)}+\frac{\beta_3ch\beta_4e^{\beta_3}-\beta_4ch\beta_3e^{\beta_4}}{sh(\beta_1-\beta_2)}\Big]e^{-1\alpha\tau},$$

$$F_2^{II}=\frac{V_0}{2}\Big[\frac{\beta_1ch\beta_2e^{-\beta_1}-\beta_2ch\beta_1e^{-\beta_2}}{sh(\beta_1-\beta_2)}+\frac{\beta_3ch\beta_4e^{-\beta_3}-\beta_4ch\beta_3e^{-\beta_4}}{sh(\beta_1-\beta_2)}\Big]e^{-1\alpha\tau},$$

and for the liquid discharge and for average velocity we have

$$Q^{II} = \frac{V_0 e^{-i\alpha\tau}}{i\alpha}\Big[\frac{\beta_2 ch\beta_2 sh\beta_1 - \beta_1 ch\beta_1 sh\beta_2}{sh(\beta_1-\beta_2)} + \frac{\beta_4 ch\beta_4 sh\beta_3 - \beta_3 ch\beta_3 sh\beta_4}{sh(\beta_1-\beta_2)}\Big],$$

$$U^{II} = \frac{V_0 e^{-i\alpha\tau}}{2i\alpha}\Big[\frac{\beta_2 ch\beta_2 sh\beta_1 - \beta_1 ch\beta_1 sh\beta_2}{sh(\beta_1-\beta_2)} + \frac{\beta_4 ch\beta_4 sh\beta_3 - \beta_3 ch\beta_3 sh\beta_4}{sh(\beta_1-\beta_2)}\Big].$$

When the plane pipe walls are in the same phase, with amplitude having different signs, move according to the pulsative law, then the pulsating velocity of the flow on the axis ($\xi = 0$) of a porous pipe does not equal to zero (as this is the case in a non-porous pipe).

In Case I, just as in Case II, when the leakage Reynolds number increases, the liquid discharge decreases, and the friction force increases.

When leakage Reynolds number decreases, temperature due to the friction heat increases, and temperature caused by Joule's heat decreases.

In general, the change of leakage Reynolds number (increase, or decrease) on the porous pipe axis exerts miner influence both on the average velocity of liquid flow and on total temperature.

The process of setting changes in a mode of pulsating velocity, interior induction and temperature takes a long period of time and starts after oscillatory motion of liquid, that is, as $\tau \to \infty$, the summands in formulas (9^I), (9^I), (10^I), (10^{II}), (18^I) and (18^{II}) expresed by infinite sums are equal to zero, and the law of variation of velocity, interior induction and liquid temperature will be pulsatory; it can be calculated by virtue of formulas

$$\frac{2u^I(\xi,\tau)}{u_0} = \Big[\frac{sh\beta_1 e^{\beta_2\xi} - sh\beta_2 e^{\beta_1\xi}}{sh(\beta_1-\beta_2)} + \frac{sh\beta_3 e^{\beta_4\xi} - sh\beta_4 e^{\beta_3\xi}}{sh(\beta_3-\beta_4)}\Big]e^{-i\alpha\tau},$$

$$\frac{2u^{II}(\xi,\tau)}{V_0} = \Big[\frac{ch\beta_2 e^{\beta_1\xi} - ch\beta_1 e^{\beta_2\xi}}{sh(\beta_1-\beta_2)} + \frac{ch\beta_4 e^{\beta_3\xi} - ch\beta_3 e^{\beta_4\xi}}{sh(\beta_3-\beta_4)}\Big]e^{-i\alpha\tau},$$

$$\frac{2Mh^I(\xi,\tau)}{u_0} = \Big[\frac{sh\beta_1 e^{\beta_2\xi} - sh\beta_2 e^{\beta_1\xi}}{sh(\beta_1-\beta_2)} - \frac{sh\beta_3 e^{\beta_4\xi} - sh\beta_4 e^{\beta_3\xi}}{sh(\beta_3-\beta_4)}\Big]e^{-i\alpha\tau},$$

$$\frac{2Mh^{II}(\xi,\tau)}{V_0} = \Big[\frac{ch\beta_2 e^{\beta_1\xi} - ch\beta_1 e^{\beta_2\xi}}{sh(\beta_1-\beta_2)} - \frac{ch\beta_4 e^{\beta_3\xi} - ch\beta_3 e^{\beta_4\xi}}{sh(\beta_3-\beta_4)}\Big]e^{-i\alpha\tau},$$

$$\theta^I_{1,2}(\xi,\tau) = \frac{\theta_1^{(1,2)}}{sh\gamma}\Big(e^{\frac{RP_r(1-\xi)}{2}} sh\frac{\gamma(1+\xi)}{2} + e^{-\frac{RP_r(1+\xi)}{2}} sh\frac{\gamma(1-\xi)}{2} +$$

$$+\frac{1}{sh\gamma}\sum_{k=1}^{10}\frac{b_k^2 e^{2\beta_k}}{4\beta_k^2 + 2P_r R\beta_k + 2i\alpha P_r}\Big(e^{2\beta_k + \frac{RP_r(1-\xi)}{2}} sh\frac{\gamma(1+\xi)}{2} +$$

$$+e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}-e^{2\beta_k\xi}sh\gamma\Big)\Big]e^{-2i\alpha\tau},\theta_{1,2}^{II}(\xi,\tau)=$$

$$=\left[\frac{\theta_2^{(1,2)}}{sh\gamma}\left(e^{\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}-e^{-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}\right)+\right.$$

$$+\frac{1}{sh\gamma}\sum_{k=1}^{10}\frac{b_k^2e^{2\beta_k}}{4\beta_k^2+2P_rR\beta_k+2i\alpha P_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}-\right.$$

$$\left.\left.-e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}-e^{2\beta_k\xi}sh\gamma\right)\right]e^{-2i\alpha\tau}.$$

III. Consider pulsatory flow caused by only pulsating pressure drop ($D\neq 0$). The pipe walls are fixed ($A_1=A_2=0$), and temperature changes on the pipe walls are equal to zero $\left(B_{1,2}^{(1)}=B_{1,2}^{(2)}=0\right)$.

Taking into account the above-said, for velocity, induction and temperature we obtain the following formulas:

$$2u^{III}(\xi,\tau)+\frac{2D}{i\alpha}=$$

$$=\frac{D}{i\alpha}\left[\frac{sh\beta_1e^{\beta_2\xi}-sh\beta_2e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}+\frac{sh\beta_3e^{\beta_4\xi}-sh\beta_4e^{\beta_3\xi}}{sh(\beta_3-\beta_4)}\right]e^{-i\alpha\tau}-$$

$$-\frac{D}{2}\sum_{n=1}^{\infty}(-1)^{n-1}\left\{\frac{e^{S_n\tau}}{S_n(S_n+i\alpha)}\left(e^{\frac{(R+M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.\right.$$

$$\left.+e^{-\frac{(R+M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)+\frac{e^{S_n^*\tau}}{S_n^*(S_n^*+i\alpha)}\left(e^{\frac{(R-M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$

$$\left.\left.+e^{-\frac{(R-M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right\}, \qquad (9^{III})$$

$$2Mh^{III}(\xi,\tau)=$$

$$=\frac{D}{i\alpha}\left[\frac{sh\beta_1e^{\beta_2\xi}-sh\beta_2e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}-\frac{sh\beta_3e^{\beta_4\xi}-sh\beta_4e^{\beta_3\xi}}{sh(\beta_3-\beta_4)}\right]e^{-i\alpha\tau}-$$

$$-\frac{D}{2}\sum_{n=1}^{\infty}(-1)^{n-1}\left\{\frac{e^{S_n\tau}}{S_n(S_n+i\alpha)}\left(e^{\frac{(R+M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.\right.$$

$$\left.+e^{-\frac{(R+M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)-$$

$$-\frac{e^{S_n^*\tau}}{S_n^*(S_n^*+i\alpha)}\left(e^{\frac{(R-M)(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$

$$\left.\left.+e^{-\frac{(R-M)(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right)\right\}, \qquad (10^{III})$$

$$\theta_{1,2}^{III}(\xi,\tau) = \frac{e^{-2i\alpha\tau}}{sh\gamma}\sum_{k=1}^{10}\frac{b_k^2}{4\beta_k^2+2P_rR\beta_k+2i\alpha P_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}sh\frac{\gamma(1+\xi)}{2}+\right.$$
$$\left.+e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}sh\frac{\gamma(1-\xi)}{2}-e^{2\beta_k\xi}sh\gamma\right)+$$
$$+\frac{1}{2}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}\mu_n e^{S_n\tau}}{P_r(S_n+2i\alpha)}\sum_{k=1}^{10}\frac{b_k^2}{4\beta_k^2+2P_rR\beta_k-SP_r}\left(e^{2\beta_k+\frac{RP_r(1-\xi)}{2}}\sin\frac{\mu_n(1+\xi)}{2}+\right.$$
$$\left.+e^{-2\beta_k-\frac{RP_r(1+\xi)}{2}}\sin\frac{\mu_n(1-\xi)}{2}\right), \qquad (18^{III})$$

where

$$\gamma=\sqrt{P_r^2R^2-8i\alpha P_r},\quad b_1=-\frac{D}{2i\alpha}\frac{\beta_1 sh\beta_2}{sh(\beta_1-\beta_2)},\quad b_2=\frac{D}{2i\alpha}\frac{\beta_2 sh\beta_1}{sh(\beta_1-\beta_2)},$$
$$b_3=-\frac{D}{2i\alpha}\frac{\beta_3 sh\beta_4}{sh(\beta_3-\beta_4)},\quad b_4=\frac{D}{2i\alpha}\frac{\beta_4 sh\beta_3}{sh(\beta_3-\beta_4)}. \qquad (17^{III})$$

Calculating friction force in the liquid and on the pipe walls, we obtain, respectively, the expressions:

$$F^{III}=\frac{D}{2i\alpha}\left[\frac{\beta_2 sh\beta_1 e^{\beta_2\xi}-\beta_1 sh\beta_2 e^{\beta_1\xi}}{sh(\beta_1-\beta_2)}+\frac{\beta_4 sh\beta_3 e^{\beta_4\xi}-\beta_3 sh\beta_4 e^{\beta_3\xi}}{sh(\beta_1-\beta_2)}\right]e^{-1\alpha\tau},$$
$$F_1^{III}=\frac{D}{2i\alpha}\left[\frac{\beta_2 sh\beta_1 e^{\beta_2}-\beta_1 sh\beta_2 e^{\beta_1}}{sh(\beta_1-\beta_2)}+\frac{\beta_4 sh\beta_3 e^{\beta_4}-\beta_3 sh\beta_4 e^{\beta_3}}{sh(\beta_1-\beta_2)}\right]e^{-1\alpha\tau},$$
$$F_2^{III}=\frac{u_0}{2i\alpha}\left[\frac{\beta_2 sh\beta_1 e^{-\beta_2}-\beta_1 sh\beta_2 e^{-\beta_1}}{sh(\beta_1-\beta_2)}+\frac{\beta_4 sh\beta_3 e^{-\beta_4}-\beta_3 sh\beta_4 e^{-\beta_3}}{sh(\beta_1-\beta_2)}\right]e^{-1\alpha\tau},$$

and for the liquid discharge and average velocity we have:

$$Q^{III}=\frac{4De^{-i\alpha\tau}}{i\alpha}\left[\frac{\beta_1-\beta_2}{\beta_1\beta_2 sh(\beta_1-\beta_2)}sh\beta_1 sh\beta_2+\right.$$
$$\left.+\frac{\beta_3-\beta_4}{\beta_3\beta_4 sh(\beta_3-\beta_4)}sh\beta_3 sh\beta_4-1\right],$$
$$U^{III}=\frac{De^{-i\alpha\tau}}{i\alpha}\left[2\frac{\beta_1-\beta_2}{i\alpha sh(\beta_1-\beta_2)}sh\beta_1 sh\beta_2+\right.$$
$$\left.+2\frac{\beta_3-\beta_4}{i\alpha sh(\beta_3-\beta_4)}sh\beta_3 sh\beta_4-1\right].$$

If pulsating flow of the liquid is due to the pulsating pressure drop, then the friction force on the pipe axis ($\xi=0$) does not equal to zero (as this is the case in a non-porous pipe), and temperature fails to reach its maximum.

Calculations performed by means of the above formulas show that the action of magnetic field on pulsating motion of the liquid and on the increase of leakage Reynolds number leads to braking of pulsating motion.

The process of setting changes in liquid velocity and interior induction by pulsation mode runs faster than that in temperature under pulsating mode.

Velocity, interior induction, temperature, friction and liquid discharge have periodic properties with respect to time.

When pulsating flow in a porous pipe is due to pulsating motion of pipe walls, then the influence of temperature caused by the friction heat on pulsating changes of liquid temperature is more significant than that caused by the Joule's heat, but if pulsating flow of the liquid is due to the pulsating pressure drop, then the influence of Joule's heat is more significant than that due to friction heat.

3. Conclusion

In general, pulsating flow in a porous pipe caused by pulsating motion of pipe walls and by pulsating pressure drop leads to temperature decrease in electroconductive liquids.

The increase of external magnetic field and increase of leakage Reynolds number leads to the pulsating flow braking.

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