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OSCILLATIONS AND STABILITY OF SHELLS OF REVOLUTION, CLOSE BY THEIR FORM TO CYLINDRICAL ONES, WITH ELASTIC FILLER, UNDER THE ACTION OF NORMAL PRESSURE AND TEMPERATURE

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Abstract. The paper studies natural oscillations of closed shells of revolution which by their form are close to cylindrical ones, with elastic filler, under the action of heat and external pressure. The shell is assumed to be thin and elastic. Temperature is uniformly distributed in a shell body. The filler is simulated by an elastic base. The shells of positive and negative Gaussian curvature are considered. Formulas for determination least frequencies and forms of wave formation depending on temperature, pressure, rigidity of an elastic filler, sign of Gaussian curvature and amplitude of shell deviation from cylindrical form, are given. The problem of shell stability is also investigated.

რეზიუმე. განხილულია საკუთარი რხევები და მდგრადობა ბრუნვითი, ცილინდრულთან მახლობელი, დრეკად შემავსებლიანი ბრუნვითი გარსებისა, რომლებიც იმყოფებიან ნორმალური გარეგანი წნევისა და ტემპერატურის მოქმედების ქვეშ, განხილულია როგორც დადებითი, ასევე უარყოფითი გაუსის სიმრუდის მქონე გარსები, მოყვანილია ფორმულები უმცირესი სიხშირის და კრიტიკული დატვირთვის განსასაზღვრავად.

In the present work we consider natural oscillations and stability of closed shells of revolution which by their form are close to cylindrical ones, with elastic filler, under the action of heat and external pressure. We consider a light filler for which the effect of tangential stresses on the contact surface and inertia forces are neglected. The shell is assumed to be thin and elastic. Temperature is uniformly distributed in the shell body. An elastic filler is simulated by Winkler's base, its extension caused by heating is not taken into account. We investigate both the shells of middle length whose form of

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midsurface generatrix is described by the parabolic function. We consider shells of positive and negative Gaussian curvature. The boundary conditions on the end-faces correspond to a free support admitting a certain radial shift in the initial state.

Formulas and universal curves of dependence of the least frequency and form of wave formation on external pressure, temperature, rigidity of elastic filler, as well as on amplitude of shell deviation from the cylinder, are obtained. It is shown that temperature in the presence of preliminary stress and elastic filler may affect lowest frequencies and forms of wave formation differently, depending on the sign of Gaussian curvature of the shell. The problem on the stability is also investigated, and formulas for determination of critical load are presented.

1. We consider the shell whose middle surface is formed by the rotation of a sufficiently smooth curve around the x-axis of rectangular system of coordinates x, y, z with origin in the middle segment of the axis of revolution. The cross-section radius of shell's middle surface is defined by the equality

$$R = r + \delta_0 F(\xi), \quad \xi = z/r, \tag{1.1}$$

where $F(\xi)$ is a positive function given on the interval $(-\ell/r, \ell/r)$ so that $F(\pm\ell/r) = 0$, $\max F(\xi) = 1$, $|F'(\xi)| \le 1$, $L = 2\ell$ is the shell length; r is radius of the end-face section; δ_0 is small parameter characterizing maximal deviation from cylindrical form. For $\delta_0 > 0$, the midsurface generatrix is convex, while for $\delta_0 < 0$, it is concave. We consider shells of middle length [1] and assume that

$$(\delta_0/r)^2, \ (\delta_0/\ell)^2 \ll 1. \tag{1.2}$$

The midsurface equation represented parametrically has the form

$$x = R(\xi) \cos\varphi, \quad y = R(\xi) \sin\varphi, \quad z = \xi r,$$

where φ is the angular coordinate. Thus we find that coefficients of the first quadratic form for the middle surface are

$$A^2 = r^2 + \delta_0^2 (F')^2, \quad B^2 = R^2(\xi).$$

Relying on the above assumptions, the second term in A^2 may be neglected. Consequently,

$$A \approx r, \quad B = R(\xi). \tag{1.3}$$

The principle curvature radii have the form

$$k_1 = 1/R_1 = -R''/r^2, \quad k_2 = 1/R_2 = 1/R(\xi). \tag{1.4}$$

In the capacity of basic oscillation equations we take those corresponding to the theory of shallow shells [2]. For shells of middle length, the forms of oscillations corresponding to lowest frequencies are slightly varying in longitudinal direction as compared with circumferential one, therefore the relation

$$\frac{\partial^2 f}{\partial \xi^2} \ll \frac{\partial^2 f}{\partial \varphi^2} \quad (f = w, \psi), \tag{1.5}$$

is valid, where w and ψ are the functions of radial displacement and stress, respectively. As a result, the system of equations for the shells under consideration reduces to the following resolving equation (according to the above-adopted assumption, temperature terms are equal to zero [3]):

$$\begin{aligned} &\varepsilon \frac{\partial^8 w}{\partial \varphi^8} + \frac{\partial^4 w}{\partial \xi^4} + \frac{\partial^2}{\partial \xi^2}\left[\left(-\frac{R''}{r}\right)\frac{\partial^2 w}{\partial \varphi^2}\right] + \\ &+ \left(-\frac{R''}{r}\right)\frac{\partial^4 w}{\partial \xi^2\,\partial \varphi^2} + \left(-\frac{R''}{r}\right)^2 \frac{\partial^4 w}{\partial \varphi^4} + \\ &+ \frac{\partial^4}{\partial \varphi^4}\left[\frac{\partial}{\partial \xi}\left(t_1^0 \frac{\partial w}{\partial \xi}\right) + \frac{\partial}{\partial \varphi}\left(t_2^0 \frac{\partial w}{\partial \varphi}\right) + \frac{\partial}{\partial \xi}\left(s^0 \frac{\partial w}{\partial \varphi}\right) + \frac{\partial}{\partial \varphi}\left(s^0 \frac{\partial w}{\partial \xi}\right)\right] + \\ &+ \gamma \frac{\partial^4 w}{\partial \varphi^4} + \frac{\rho r^2}{E}\frac{\partial^2}{\partial t^2}\left(\frac{\partial^4 w}{\partial \varphi^4}\right) = 0, \end{aligned} \tag{1.6}$$

$$\varepsilon = h^2/12\,r^2(1-\nu^2),\ t_i^0 = T_i^0/Eh,\ s^0 = S^0/Eh \quad (i = 1,2),\ \gamma = \beta r^2/Eh,$$

where T_1^0 and T_2^0 are meridional and circumferential compressive forces in the initial state; S^0 is shearing stress in the initial state; F and ν are, respectively, elastic module and Poisson coefficient; h is the shell thickness; ρ is material density of the shell; β is the "bed" coefficient of an elastic filler (characterizing elastic rigidity of a filler); t is time.

2. We investigate a concrete shell whose midsurface generatrix is defined by the parabolic function

$$F(\xi) = 1 - \xi^2 (r/\ell)^2. \tag{2.1}$$

Initial state of the shell is assumed to be momentless. On the basis of the corresponding solution, taking into account the filler reaction and inequality (1.2), we obtain the following approximate expressions:

$$T_1^0 \approx q\delta_0\big[1 - \xi^2 (r/\ell)^2\big], \quad T_2^0 \approx r(q - \beta_0 w_0), \quad S^0 = 0, \tag{2.2}$$

where q is external pressure, w_0, β_0 is the deflection and the "bed" coefficient of the filler in the initial state. A full shift in the initial state is equal to

$$w_0 = w_{0_q} - w_{0_T}, \tag{2.3}$$

where w_{0_q} and w_{0_T} are deflections caused by pressure q and temperature T, respectively. They are expressed through the stresses $\sigma^0_{\varphi_q}$ and $\sigma^0_{\varphi_T}$ by the formulas

$$w_{0_q} = \sigma^0_{\varphi_q}(1-\nu^2)R/E, \quad w_{0_T} = \left[\alpha T - \frac{\sigma_{\varphi_T}(1-v^2)}{E}\right]R, \tag{2.4}$$

where $\sigma^0_{\varphi_q}$ is a circumferential normal stress in the shell due to pressure and $\sigma^0_{\varphi_T}$ is that caused by temperature and by filler constraint; T is temperature and α is coefficient of linear extension of the shell material.

Substituting (2.4) into (2.3) and (2.2), we get

$$T_2^0 \approx \frac{r}{g}(q + \alpha r \beta_0 T), \quad g = 1 + (1-\nu^2)\frac{\beta_0 r}{Eh}, \qquad T_2^0 = \sigma^0_\varphi h, \quad \sigma^0_\varphi = \sigma^0_{\varphi_q} + \sigma^0_{\varphi_T}. \tag{2.5}$$

In addition, taking into account the fact that R is close to r, analogously to the above-said, we assume in (2.5) that $R \approx r$. Bearing in mind that

$$\left[1 - \xi^2 (r/\ell)^2\right] \frac{\partial^2 w}{\partial \xi^2} \ll \frac{\partial^2 w}{\partial \varphi^2},$$

in view of inequalities (1.2) and (2.5), it is not difficult to show that

$$\frac{\partial}{\partial \xi}\left(T_1^0 \frac{\partial w}{\partial \xi}\right) \ll T_2^0 \frac{\partial^2 w}{\partial \varphi^2}. \tag{2.5$'$}$$

Bearing in mind (2.5), (2.5′), we find that equation (1.6) takes the form

$$\varepsilon \frac{\partial^8 w}{\partial \varphi^8} + \frac{\partial^4 w}{\partial \xi^4} + 4\delta \frac{\partial^4 w}{\partial \xi^2 \partial \varphi^2} + \left(4\delta^2 + \frac{\beta r^2}{Eh}\right)\frac{\partial^4 w}{\partial \varphi^4} + \left(\frac{q r^2}{Eh} + \frac{\alpha r^2 T \beta_0}{Eh}\right) g^{-1} \frac{\partial^6 w}{\partial \varphi^6} + \frac{\beta r^2}{E}\frac{\partial}{\partial t^2}\left(\frac{\partial^4 w}{\partial \varphi^4}\right) = 0, \tag{2.6}$$

$$\delta = \delta_0 r/\ell^2.$$

We consider harmonic oscillations. The above boundary conditions and equation (2.6) are satisfied by the solution

$$w = A_{mn}\cos\lambda_m \xi \, \sin n\varphi \, \cos\omega t, \quad \lambda_m = \frac{m\pi r}{2\ell} \qquad (m = 2i+1;\ \ i = 0, 1, 2, \dots). \tag{2.7}$$

Substituting (2.7) into equation (2.6), we obtain the equality to find eigen oscillations

$$\omega^2 = \frac{E}{\rho r^2}\left[\varepsilon n^4 + \lambda_m^4 n^{-4} + 4\delta\lambda_m^2 n^{-2} + 4(\delta^2 + \gamma/4) - (\overline{q} + \gamma_0 \alpha T) g^{-1} n^2\right], \tag{2.8}$$

$$\overline{q} = qr/Eh, \quad \gamma_0 = \beta_0 r^2/Eh, \quad \gamma = \beta r^2/Eh \tag{2.9}$$

allowing one to determine eigenfrequencies.

It is easily seen that for $\delta > 0$, to the least frequency there corresponds the value $m = 1$. It can also be shown that this condition holds for $\delta < 0$, in view of inequality (1.2) and the fact that $\omega^2 > 0$. Therefore we consider first the forms of oscillations under which along the full shell length there arises only one half-wave ($m = 1$), whereas in circumferential direction there arise n waves.

To represent (2.8) for $m = 1$ in dimensionless form, we introduce dimensionless values N, Q and notation

$$N = n^2/n_0^2,\quad Q = \overline{q}/\overline{q}_{0*},\quad \delta_* = \delta\varepsilon_*^{-1/2},\quad \overline{\delta}_k^{\,2} = \delta_*^2 + \delta_*/4,\quad \gamma_* = \gamma\varepsilon_*^{-1},$$
$$\varepsilon_* = (1-\nu^2)^{-1/2}\,\frac{h}{r}\Big(\frac{r}{L}\Big)^2,\quad n_0^2 = \lambda_1\varepsilon^{-1/4},\quad \lambda_1 = \pi r/L, \tag{2.10}$$
$$\overline{q}_{0*} = 0,855(1-\nu^2)^{-1/4}\Big(\frac{h}{r}\Big)^{3/2}\frac{r}{L},$$

$$\omega_0^2 = 2\lambda_1^2\varepsilon^{1/2}E/\rho r^2,\quad K = T/\overline{q}_{0*}\,, \tag{2.10$'$}$$

where ω_0 and $\overline{q}_{0_*}$ are, respectively, the least frequency and critical pressure of a cylindrical midlength shell [1,5]. As a result, (2.8) can be written in dimensionless form

$$\omega^2(N)/\omega_0^2 = 0,5\big(N^2+N^{-2}+2,37\,\delta_*N^{-1}+1,404\,\overline{\delta}_*^{\,2}-1,755\,\overline{Q}N\big), \tag{2.11}$$
$$\overline{Q} = Q + \gamma_0\alpha K. \tag{2.11$'$}$$

Consider now the case for $\overline{Q} = 0$. Then

$$\omega^2(N)/\omega_0^2 = 0,5\big(N^2 + N^{-2} + 2,37\,\delta_*N^{-1} + 1,404\,\overline{\delta}_*^{\,2}\big). \tag{2.12}$$

It is easy to show that the least frequency in this case is defined from the condition $\omega^2(N)' = 0$. Thus we get

$$N^4 + dN + e = 0,\quad d = -1,185\,\delta_*,\quad e = -1. \tag{2.13}$$

The roots of equation (2.13) coincide with those of the two quadratic equations

$$N^2 + \frac{a_{1,2}}{2}\,N + \left(y_1 - \frac{d}{a_{1,2}}\right) = 0,\quad a_{1,2} = \pm\sqrt{8\,y_1}.$$

Consequently, the roots of equation (2.13) have the form

$$N_{1,2} = -\sqrt{\frac{y_1}{2}} \pm \sqrt{\frac{d}{\sqrt{8\,y_1}} - \frac{y_1}{2}}\,,\quad N_{3,4} = \sqrt{\frac{y_1}{2}} \pm \sqrt{-\frac{d}{\sqrt{8\,y_1}} - \frac{y_1}{2}}\,, \tag{2.14}$$

where y_1 is any root of the cubic equation $y^3 + 3py + 2q = 0$, $p = 1/3$, $q = -1,185^2\,\delta_*^2/16$. Since the discriminant of that equation is $D = q^2+p^3 > 0$, therefore the equation has only one real root

$$y_1 = u + u_2,\quad u_{1,2} = \sqrt[3]{-q \pm \sqrt{q^2+p^3}}\,.$$

Thus we obtain

$$y_1 = \frac{1}{\sqrt{3}}\Big[\Big(\sqrt{1+0,208\,\delta_*^4}+0,456\,\delta_*^2\Big)^{1/3} - \\ - \Big(\sqrt{1+0,208\,\delta_*^4}-0,456\,\delta_*^2\Big)^{1/3}\Big]. \tag{2.15}$$

If we take

$$0,208\,\delta_*^4 \ll 1 \tag{2.16}$$

expand the expression appearing in (2.15) into series and neglecting values of second order smallness, we obtain $y_1 \approx 0,1755\,\delta_*^2$. Substituting values y_1 and d into (2.14) and taking into account that of our interest are only positive values of N (since $n^2 > 0$), we find that for $d > 0$ ($\delta < 0$), the root N_1 is positive, whereas for $d < 0$ ($\delta > 0$), positive will be the root N_3. Consequently, we have

$$\begin{aligned} N_1 &= \sqrt{1-0,0876\,\delta_*^2}+0,2692\,|\delta_*|, \\ N_3 &= \sqrt{1-0,0876\,\delta_*^2}-0,2692\,|\delta_*|, \end{aligned} \tag{2.17}$$

or taking into account (2.10), we get

$$\begin{aligned} n_1^2 &= \Big(\sqrt{1-0,0876\,\delta_*^2}+0,2962|\delta_*|\Big)\lambda_1\varepsilon^{-1/4}, \\ n_2^2 &= \Big(\sqrt{1-0,0876\,\delta_*^2}-0,2962|\delta_*|\Big)\lambda_1\varepsilon^{-1/4}, \end{aligned} \tag{2.17$'$}$$

where n_1 is related to $\delta > 0$, and n_2 to $\delta < 0$. In particular, for $\delta_* = 0$, we have the well-known formula for cylindrical shell of middle length $n_0^2 = \lambda_1\varepsilon^{-1/4}$. For $|\delta_*| \gtrsim 0,5$, we have to come from the complete expression (2.15).

Defining in such a way the value of N_0 (for fixed δ_*) and substituting if into (2.12), we obtain the least frequency values for an unloaded shell $\omega(N_0)$.

Figure 1 shows the dependence of N_0 (curves 1 and $1'$) and $\omega(N_0)/\omega_0(1)$ (curves 2 and $2'$) on the parameter δ_*, when $q = T = 0$ for $\gamma_* = 0$, $\gamma = 13,816$. Curves 1 and 2 correspond to the case $\gamma_* = 0$, and curves $1'$ and $2'$ correspond to the case, where $\gamma_* = 3,816$. It is not difficult to see that curve 1 coincides with curve $1'$.

For $\omega = 0$, from formula (2.11) we obtain the expresion allowing one to determine critical load

$$1,755\,\overline{Q} = N + N^{-3} + 2,37\,\delta_* N^{-2} + 1,404\,\overline{\delta}_*^{\,2} N^{-1}. \tag{2.18}$$

The least value of $\overline{Q}$ depending on N is realized for $\overline{Q}'_N = 0$. Hence we obtain

$$N^4 + cN^2 + dN + e = 0, \quad c = -1,404\,\overline{\delta}_*^{\,2}, \quad d = -4,74\,\delta_*, \quad e = -3. \tag{2.19}$$

The roots of that equation coincide with those of two quadratic equations

$$N^2 + \frac{A_{1,2}}{2}\,N + \Big(y_1 - \frac{d}{A_{1,2}}\Big) = 0, \quad A_{1,2} = \pm\sqrt{8\alpha}\,,$$

$$N_{1,2} = -\sqrt{\frac{\alpha}{2}} \pm \sqrt{\frac{d}{\sqrt{8\alpha}} - \frac{\alpha_1}{2}}\,, \quad N_{3,4} = \sqrt{\frac{\alpha}{2}} \pm \sqrt{-\frac{d}{\sqrt{8\alpha}} - \frac{\alpha_1}{2}}\,, \tag{2.20}$$

$$\alpha = y_1 - c/2, \quad \alpha_1 = y_1 + c/2, \tag{2.21}$$

where y_1 is any real root of the cubic equation

$$y^3 - \frac{c}{2}\,y^2 - ey + \Big(\frac{ce}{2} - \frac{d^2}{8}\Big) = 0, \tag{2.22}$$

or

$$z^3 + 3pz + 2q = 0 \quad (z = y - c/6), \tag{2.23}$$

$$p = 1 - \big(1,404\,\overline{\delta}_*^{\,2}\big)^2/36\,, \quad q = -\frac{1}{2}\,1,404\Big(\delta_*^2 - \frac{\gamma_*}{4}\Big) + \frac{(1,404\,\overline{\delta}_*^{\,2})^2}{216}\,. \tag{2.24}$$

If we assume that

$$\big(1,404\,\overline{\delta}_*^{\,2}\big)^2/36 \ll 1, \tag{2.25}$$

then expressions (2.24) take the form $p = 1$, $q = -0,702\,\overline{\delta}_*^{\,2}$. Since the discriminant of equation (2.23) is $D > 0$, we have one real root

$$z = \big(-q + \sqrt{q^2 + p^3}\big)^{1/3} + \big(-q - \sqrt{q^2 + p^3}\big)^{1/3}. \tag{2.26}$$

Using condition (2.25) and expanding expressions appearing in (2.26), neglecting the values of second order smallness, we obtain $z \approx 1,404(\delta_*^2 - \frac{\gamma_*}{4})/3$. Then on the basis of (2.19), (2.21) and (2.23), we get

$$\begin{aligned} \alpha &= z - c/3 = 2\cdot 1,404\,\delta_*^2/3\,,\\ \alpha_1 &= z + 2c/3 = -1,404\Big(\delta_*^2 + \frac{3}{4}\,\gamma_*\Big)/3\,. \end{aligned} \tag{2.27}$$

Taking into account that y_1 is the root of equation (2.22), we have

$$\frac{d^2}{8(y_1 - \frac{c}{2})} = y_1^2 - e.$$

Thus we get

$$\frac{|d|}{\sqrt{8\alpha}} = \sqrt{y_1^2 - e} > y_1. \tag{2.28}$$

We represent y_1 in the form

$$y_1 = \frac{1}{2}\Big(y_1 + \frac{c}{2}\Big) + \frac{1}{2}\Big(y_1 - \frac{c}{2}\Big) = \frac{\alpha_1}{2} + \frac{\alpha}{2}\,.$$

Then according to inequality (2.28), we obtain

$$\frac{|d|}{\sqrt{8\alpha}} - \frac{\alpha_1}{2} > \frac{\alpha}{2}\,. \tag{2.29}$$

Since $N = n^2/n_0^2$, of our interest are only positive roots of equation (2.19). Bearing in mind inequality (2.29), we find that for $\delta > 0$ $(d > 0)$, only the root N_1 is positive, whereas for $\delta > 0$ $(d < 0)$, positive is the root N_3. Substituting the values d, α, α_1, according to equalities (2.19) and (2.27), into (2.20), we arrive at (2.30)

$$\begin{aligned} N_* &= \sqrt{\sqrt{3} + 0,234\Big(\delta_*^2 + \frac{3}{4}\gamma_*\Big)} - 0,684\,|\delta_*| \quad (\delta_* < 0),\\ N_* &= \sqrt{\sqrt{3} + 0,234\Big(\delta_*^2 + \frac{3}{4}\gamma_*\Big)} + 0,684\,|\delta_*| \quad (\delta_* > 0). \end{aligned} \tag{2.30}$$

As a result, we obtain

$$\begin{aligned} n_{1,2}^2 = &\Bigg(\sqrt{\sqrt{3} + 0,2703\,\varepsilon^{-1/2}\Big[\Big(\frac{\delta_0}{\ell}\Big)^2 + \frac{\gamma}{4}\Big(\frac{\ell}{r}\Big)^2\Big]} \pm \\ &\pm 0,735\,\frac{|\delta_0|}{\ell}\,\varepsilon^{-1/4}\Bigg)\lambda_1\varepsilon^{1/4} \end{aligned} \tag{2.31}$$

the index " 1 " corresponds to $\delta_0 > 0$, and the index " 2 " to $\delta_0 < 0$. In particular, for $\delta_0 = \gamma = 0$, we obtain the well-known formula for critical number of waves of cylindrical shells of middle length [1]. Substituting the values of (2.30) into (2.18), we obtain critical value of $\overline{Q}_*$.

Charts in Figure 2 presented in dimensionless form show dependence of N_* and $\overline{Q}_*$ on δ_* for $\gamma_* = 0$ and $\gamma_* = 3,816$. To N_* there correspond curves 1 and $1'$, while for $\overline{Q}_*$ the curves 2 and $2'$.

Note that expression (2.18) for determination of a critical load can be simplified on the basis of equation (2.19). Equation (2.19) yields

$$2,37\,\delta_* N^{-2} + 1,404\,\overline{\delta}_*^{\,2} N^{-1} = -(2,37\,\delta_* N^{-2} + 3N^{-3} - N). \tag{2.32}$$

Substituting equation (2.32) into (2.18), we obtain

$$\overline{Q}_* = 1,15(N_* - N_*^{-3} - 1,185\,\delta_* N_*^{-2}). \tag{2.33}$$

Consider now expression (2.11), when $\overline{Q} \neq 0$. From the condition of frequency smallness which is defined by virtue of (2.11), we obtain the following dependence between $\overline{Q}$ and N

$$\overline{Q} = 1,15(N - N^{-3} - 1,185\,\delta_* N^{-2}). \tag{2.34}$$

It is not difficult to notice that this equality results in the relation (2.33). On the basis of (2.34), for $\overline{Q} = 0$, we obtain equation (2.13), whose root $N = N_0$ corresponds, as is mentioned above, to the least frequency of an unloaded shell $\omega(N_0)$, while for $\overline{Q} = \overline{Q}_*$, we get equation (2.33) whose root $N = N_*$ corresponds to the critical load and to $\omega = 0$. Thus, as $\overline{Q}$ varies in the interval

$$0 \le \overline{Q} \le \overline{Q}_* \tag{2.35}$$

the least frequency $\omega(N,\overline{Q})$ varies in the interval $\omega(N_0,\overline{Q}=0)\geq\omega(N,\overline{Q})$ ≥ 0.

Relying on the reasoning analogous to [5], we can show that when $\overline{Q}$ varies in the interval (2.35), the value of N, realizing the least frequency $\omega(N,\overline{Q})$, is in the interval

$$N_0 \leq N \leq N_*. \tag{2.36}$$

In Figures 1 and 2 we can see values of N_0 and N_* depending on δ_* for $\gamma_*=0$ and $\gamma_*=3,816$. They are presented by curves 1 and $1'$. In particular, for $\delta_*=0$ and $\gamma_*=0$, inequalities (2.35) and (2.36) take the form [5]: $0\leq\overline{Q}\leq 1$, $1\leq N\leq 1,315$.

Next, on the basis of (2.34), it is easy to construct the curves $N(\overline{Q})$ which realize minimal frequency for different values δ_*,γ_*. Towards this end, we fix δ_*,γ_*, and taking values of N from the interval (2.36), we define the value corresponding to $\overline{Q}$ by formula (2.34).

In Figure 3, we see the curves of $N(\overline{Q})$ for $\delta_*=0;\,0,8;\,-0,8$ and $\gamma_*=0;\,3,816$ (plotted, respectively, by solid and dotted curves). On the basis of these curves and by expression (2.11), we can find N and a corresponding minimal frequency ω for the given δ_*, γ_*, $\overline{Q}$.

Charts in Figure 4 show variations of dimensionless least frequency depending on dimensionless prestressed $\overline{Q}$ for the above given values δ_*,γ_*; in addition, the relation $\omega/\omega_0(1,0)$ ($\omega_0(1,0)$ is laid off along the Y-axis ($\omega_0(1,0)$ is the least frequency, free from the action of a cylindrical shell and defined by equality ($2.10'$)), and $\overline{Q}$ is laid off along the X-axis (to $\overline{Q}$ there corresponds expression (2.11'), where $\overline{q}_{0*}$ characterizes critical pressure of cylindrical shell and is defined by equality (2.10)); for $\gamma_*=0$, the curves are denoted by $0_0,1_0,2_0$, and for $\gamma_*=3,816$ by $0_1,1_1,2_1$.

The above formulas and charts show to what extent vary the least frequency and the corresponding form of wave formation depending both on the form of a shell and on external effects.

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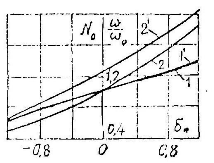
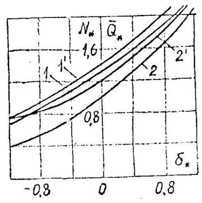
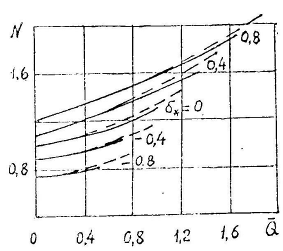


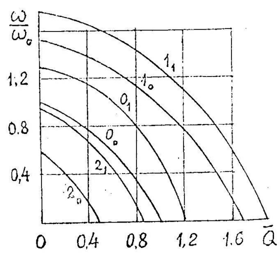
Фиг. 1



Фиг. 2



Фиг. 3



Фиг. 4

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