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ABOUT THE SPECTRUM OF REGULAR ELEMENTS IN A q-ALGEBRA

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Abstract. We define holomorphic functions taking their values in a quotient bornological space and we use this to show that the spectrum of a regular element of a unital q-algebra is compact and nonempty, also we prove that the resolvent is a holomorphic function which vanishes at infinity.

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1. Introduction and Notations

Waelbroeck was the first who introduced regular elements for b-algebras (see [8-10]). Next, Allan [2] considered elements that he called "analytic" in unital locally convex algebras. Note that, Allan's analytic elements are Waelbroeck's regular elements. Finally, Vasilescu considered regular operators on Fréchet locally convex spaces.

For Waelbroeck, an element a of a unital b-algebra $\mathcal{A}$ is regular if there exists $M > 0$ such that $(a - s.e)$ is invertible if $|s| > M$ and $\{(a - s.e)^{-1} : |s| > M\}$ is bounded, where e is the unity of $\mathcal{A}$. For Allan, an element a of a unital locally convex algebra $\mathcal{B}$ is analytic if there exists $M > 0$ such that $\{\frac{a^n}{M^n} : n \in \mathbb{N}\}$ is bounded. Finally, Vasilescu [7] considered regular operators on quotient Fréchet spaces. These operators are such that the function $s \longmapsto (1 - sa)$ is invertible in the space of holomorphic functions on $\varepsilon\mathbf{D}$ taking their values in quotient Fréchet spaces, for some $\varepsilon > 0$ where $\mathbf{D}$ is the unit disc of $\mathbb{C}$ Applying the "joint corollary" of [12], we see that Vasilescu's regular operators are regular elements in the sense of Waelbroeck.

In this paper, we will define a holomorphic function f on a complex manifold U taking its values in a quotient bornological spaces $E \mid F$ as

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a function induced by a system (V_z, f_{1z}) where each V_z is an open neighborhood of z, each $f_{1z} \in O(V_z, E)$ and for all $z \neq y$, we have $f_{1z_{|V_z \cap V_y}} - f_{1y_{|V_z \cap V_y}} \in O(V_z \cap V_y, F)$. for all z, $f_{1z} \in O(V_z, F)$. The space of these holomorphic functions will be called $O(U, E \mid F)$. We will show that if U is an open subset of $\mathbb{C}$ and $f \in O(U, E \mid F)$, then for each relatively compact subset V of U, there exists a function $f_V \in O(V, E)$ such that $f_{V|_{V \cap V_z}} - f_{1z|_{V \cap V_z}} \in O(V \cap V_z, F)$. Next, if f is a holomorphic function on $\mathbb{C}^*$, there exists a family $(V_z)_z$ of open neighborhoods of z, such that $f_{1z} \in O(V_z, E)$ and $f_{1z|_{V_z \cap V_{z'}}} - f_{1z'|_{V_z \cap V_{z'}}} \in O(V_z \cap V_{z'}, F)$, we will prove that an element $e \in E$ exists such that for all z, we have $f_z - e \in O(V_z, F)$. As application, we will deduce that the spectrum of a regular element a is compact and nonempty, and the resolvent mapping $s \longmapsto (a - s)^{-1}$ is holomorphic and vanishes at infinity.

To prove our results, we need to fix some notations and recall some definitions that will be used in this paper. Let **EV** be the category of vector spaces and linear mappings over the scalar field $\mathbb{R}$ or $\mathbb{C}$, and **Ban** the subcategory of Banach spaces and bounded linear mappings.

1) Let $(E, \|\|_E)$ be a Banach space. A Banach subspace F of E is a vector subspace equipped with a Banach norm $\|\|_F$ such that the inclusion $(F, \|\|_F) \longrightarrow (E, \|\|_E)$ is continuous. A quotient Banach space $E \mid F$ is a vector space E/F, where E is a Banach space and F a Banach subspace.
Given two quotient Banach spaces $E \mid F$ and $E' \mid F'$, a strict morphism $u : E \mid F \longrightarrow E' \mid F'$ is a linear mapping $u : x + F \longmapsto u_1(x) + F'$, where $u_1 : E \longrightarrow E'$ is a bounded linear mapping such that $u_1(F) \subseteq F'$, we shall say that u_1 induces u. Two bounded linear mappings $u_1, u_2 : E \longrightarrow E'$ both inducing a strict morphism, induce the same strict morphism iff $u_1 - u_2$ is a bounded linear mapping $E \longrightarrow F'$. For more information about quotient Banach spaces the reader is refered to [13].

2) In a similar way, we define the category of quotient bornological spaces. Let E be a real or complex vector space, and let B be an absolutely convex set of E. Let E_B be the vector space generated by B i.e. $E_B = \cup_{\lambda > 0} \lambda B$. The Minkowski functional of B is a semi-norm on E_B. It is a norm, if and only if B does not contain any nonzero subspace of E. The set B is completant if its Minkowski functional is a Banach norm.

A bounded structure β on a vector space E is defined by a set of "bounded" subsets of E with the following properties:

(1) Every finite subset of E is bounded.
(2) Every union of two bounded subsets is bounded.
(3) Every subset of a bounded subset is bounded.
(4) A set homothetic to a bounded subset is bounded.
(5) Each bounded subset is contained in a completant bounded subset.

A b-space (E, β) is a vector space E with a boundedness β. A subspace F of a b-space E is bornologically closed if $F \cap E_B$ is closed in E_B for every completant bounded subset B of E.

Given two b-spaces (E, β_E) and (F, β_F), a linear mapping $u : E \longrightarrow F$ is bounded, if it maps bounded subsets of E into bounded subsets of F.
We denote by $\mathbf{b}$ the category of b-spaces and bounded linear mappings. For more information about b-spaces we refer the reader to [4] and [11].

Let (E, β_E) be a b-space. A b-subspace of E is a subspace F with a boundedness β_F such that (F, β_F) is a b-space and $\beta_F \subseteq \beta_E$. A quotient bornological space $E \mid F$ is a vector space E/F, where E is a b-space and F a b-subspace of E.

Let $E \mid F$ and $E' \mid F'$ be two quotient bornological spaces. A strict morphism $u : E \mid F \longrightarrow E' \mid F'$ is induced by a bounded linear mapping $u_1 : E \longrightarrow E'$ whose restriction to F is a bounded linear mapping $F \longrightarrow F'$. Two bounded linear mappings u_1 and $v_1 : E \longrightarrow E'$, both inducing a strict morphism, induce the same strict morphism $E \mid F \longrightarrow E' \mid F'$ iff the linear mapping $u_1 - v_1 : E \longrightarrow F'$ is bounded. A strict morphism u is a class of equivalence of bounded linear mappings, for the equivalence just defined.
The class of quotient bornological spaces and strict morphisms is a category, that we call $\widetilde{\mathbf{q}}$. A pseudo-isomorphism $u : E \mid F \longrightarrow E' \mid F'$ is a strict morphism induced by a bounded linear mapping $u_1 : E \longrightarrow E'$ which is bornologically surjective and such that $u_1^{-1}(F') = F$ as b-spaces e.g. $B \in \beta_F$ if $B \in \beta_E$ and $u_1(B) \in \beta_{F'}$.

The category $\tilde{\mathbf{q}}$ is not abelian. In fact, there are pseudo-isomorphisms which do not have strict inverses. For example, if E is a Banach space and F a closed subspace of E, the pseudo-isomorphism $E \mid F \longrightarrow (E/F) \mid \{0\}$, induced by the quotient mapping $E \longrightarrow E/F$ is not always an isomorphism. Waelboeck [14] constructed an abelian category $\mathbf{q}$ that contains $\widetilde{\mathbf{q}}$ and in which all pseudo-isomorphisms of $\widetilde{\mathbf{q}}$ are isomorphisms.

3) A b-algebra $\mathcal{A}$ is an algebra on $\mathbb{C}$, equipped with a boundedness of b-space $\beta_{\mathcal{A}}$ such that for each completant bounded subsets B_1 and B_2 of $\mathcal{A}$, there exists a completant bounded subset B_3 of $\mathcal{A}$ such that the bilinear mapping $\mathcal{A}_{B_1} \times \mathcal{A}_{B_2} \longrightarrow \mathcal{A}_{B_3}$ is bounded. A left b-ideal of a b-algebra ($\mathcal{A}$, $\beta_{\mathcal{A}}$) is a left ideal I of $\mathcal{A}$, equipped with a boundedness of b-space β_I such that the mapping

$$(\mathcal{A}, \beta) \times (I, \beta_I) \longrightarrow (I, \beta_I), (a, x) \longmapsto a.x$$

is bounded. Similary, we define right b-ideal. If $\mathcal{A}_1$ and $\mathcal{A}_2$ are two b-algebras, a homomorphism $u : \mathcal{A}_1 \longrightarrow \mathcal{A}_2$ is a bounded linear mapping such that

$$u(a.b) = u(a).u(b) \quad \text{for all} \quad a, b \in \mathcal{A}_1.$$

If $\mathcal{A}$ is a b-algebra, a b-subspace I of $\mathcal{A}$ is a (two-sided) b-ideal of $\mathcal{A}$ if it is a left b-ideal and a right b-ideal. For more information about b-algebras and b-ideals we refer the reader to [5] and [12].

2. Main Results

We begin this section by the definition of holomorphic functions taking theirs values in quotient bornological spaces.

Definition 2.1. A holomorphic function f on a complex manifold U taking its values in a quotient bornological space $E \mid F$, is induced by a system (V_z, f_{1z}) where each V_z is an open neighborhood of z, each $f_{1z} \in O(V_z, E)$and for all $z \neq y$, we have $f_{1z_{|V_z \cap V_y}} - f_{1y_{|V_z \cap V_y}} \in O(V_z \cap V_y, F)$. Such a system (V_z, f_{1z}) induces the zero function if for all z, $f_{1z} \in O(V_z, F)$. The space of these holomorphic functions will be denoted by $O(U, E \mid F)$.

In the next Proposition we use a result called by Gunning and Rossi [3] the "generalized Cauchy formula" and by Vasilescu [6], "the Pompeiu formula". We observe that Gunning and Rossi consider scalar-valued functions, Vasilescu considers vector-valued functions, but their proofs are identical.

Proposition 2.2. *Let $E \mid F$ be a quotient bornological space, U be an open subset of $\mathbb{C}$ and $f \in O(U, E \mid F)$. Then for each relatively compact subset V of U, there exists a function $f_V \in O(V, E)$ such that $f_{V|_{V \cap V_z}} - f_{1z|_{V \cap V_z}} \in O(V \cap V_z, F)$.*

Proof. We cover the closure $\overline{V}$ of V by a finite number of open neighborhoods V_z that we call $V_1, \ldots, V_N$. Consider smooth functions $\varphi_1, \ldots, \varphi_N$, where each φ_i has a compact support in V_i, and $\sum_{i=1}^{N} \varphi_i(x) = 1$. We let also $f_i = f_{z_i}$. We see that

$$f_{i|_{V_i \cap V_j}} - f_{j|_{V_i \cap V_j}} \in O(V_i \cap V_j, F)$$

We consider the form $\overline{\partial}(\sum_{j=1}^{N} (\varphi_j f_j))$. This is a form with smooth coefficients in F. Consider any $s \in \overline{V}$, and assume that $s \in \left(\cap_{i=1}^{k} V_i\right) \setminus f_{k+1}^{N}(V_N)$, then on a neighborhood of s, we have

$$\sum_{j=1}^{N} \varphi_j f_j = \sum_{j=1}^{k} \varphi_j f_j$$

We see that

$$\sum_{j=1}^{N} \varphi_i f_j = \varphi_1 (f_1 - f_2) + (\varphi_1 + \varphi_2)(f_2 - f_3) + \cdots + \\ + (\varphi_1 + \cdots + \varphi_{k-1})(f_{N-1} - f_N) + \varphi_N f_N$$

By applying the operator $\overline{\partial}$ to the above relation, we see that

$$\overline{\partial}(\sum_{j=1}^{N}\varphi_j f_j) = \sum_{j=1}^{k}\overline{\partial}\Psi_j\,(f_i - f_j)$$

So the form is differentiable and F-valued (the function Ψ_i is $\sum_{j=1}^{i}\varphi_j$).

Now, let U be an open subset of $\mathbb{C}$ and U' be a relatively compact subset of U. Let g be a closed smooth form on U. Then by the Pompeiu formula, we obtain a smooth F-valued function h on U' such that $\frac{\partial h}{\partial \overline{z}} = g$ on U'.

We have proved that $\sum_{i=1}^{N}\overline{\partial}\varphi_i f_i$ is a smooth F-valued form near to s. The Pompeiu formula [6] gives a F-valued smooth function h on U' such that $\sum_{i=1}^{N}\varphi_i f_i - h$ is holomorphic. The proposition is proved. □

We must next prove the "Liouville theorem". Let $\mathbb{C}^* = \mathbb{C}\cup\{\infty\}$ be the Riemann sphere. Let f be a holomorphic function on $\mathbb{C}^*$, we shall show that f is in a way "constant".

Proposition 2.3. *Let f be a holomorphic function on $\mathbb{C}^*$ taking its values in a quotient bornological space $E \mid F$, induced by a system (V_z, f_{1z}). Then there exists an element $e \in E$ such that for all z, we have $f_{1z} - e \in O\,(V_z, F)$.*

Proof. The proposition 2.2 shows that for all open subset U which is not dense in $\mathbb{C}^*$, there exists a function $f_U \in O\,(U, E)$ such that for all $z \in U$, we have

$$f_{U|_{U\cap V_z}} - f_{1z|_{U\cap V_z}} \in O\,(U\cap V_z, F)\,.$$

We consider two sets

$$U_1 = \{z \in \mathbb{C}^* : |z| < 1+\varepsilon\}$$

and

$$U_2 = \{z \in \mathbb{C}^* : |z| > 1-\varepsilon\}$$

We have two holomorphic functions $f_{U_1} \in O\,(U_1, E)$ and $f_{U_2} \in O\,(U_2, E)$ such that

$$f_{U_{1|U1\cap U_2}} - f_{U_{2|U1\cap U_2}} \in O\,(U_1\cap U_2, F)$$

The Laurent series gives two holomorphic F-valued functions $g_{U_1} \in O\,(U_1, F)$ and $g_{U_2} \in O\,(U_2, F)$ such that

$$f_{U_1} - f_{U_2} = g_{U_1} - g_{U_2}$$

on $U_1 \cap U_2$. Therefore, we obtain

$$f_{U_1} - g_{U_1} = f_{U_2} - g_{U_2}$$

on $U_1\cap U_2$. These two functions $f_{U_1} - g_{U_1}$ and $f_{U_2} - g_{U_2}$ can be united, define a holomorphic E-valued function f on the sphere $\mathbb{C}^*$ such that for all $z \in \mathbb{C}^*$, we have $f - f_{1z} \in O\,(V_z, F)$. The Liouville theorem shows that the function f is constant and the proof is finished. □

We need to introduce the definition of q-algebras. Let $E \mid F$ be a quotient bornological space. Call $\mathbf{q}^1(E \mid F, E \mid F)$ the space of bounded linear mappings $E \longrightarrow E$ which map F into F, and $\mathbf{q}^o(E \mid F, E \mid F)$ the space of bounded linear mappings $E \longrightarrow F$. Let

$$\mathbf{q}(E \mid F, E \mid F) = \mathbf{q}^1(E \mid F, E \mid F) \mid \mathbf{q}^o(E \mid F, E \mid F).$$

and define by induction:

$$\mathbf{q}_1(E \mid F, E \mid F) = \mathbf{q}(E \mid F, E \mid F)$$

and

$$\mathbf{q}_n(E \mid F, \ldots, E \mid F; E \mid F) = \mathbf{q}(E \mid F, \mathbf{q}_{n-1}(E \mid F, \ldots, E \mid F); E \mid F).$$

A *q*-algebra is a quotient bornological space $E \mid F$ on which a multiplication is defined by an element $m \in \sigma q_2(E \mid F, E \mid F; E \mid F)$.

Now, we are in position to define regular elements in a q-algebra $\mathcal{A} \mid \alpha$. In fact, an element $a \in \mathcal{A} \mid \alpha$ is regular if the $\mathcal{A} \mid \alpha$-valued function $s \longmapsto (1 - sa)$ is invertible in the q-algebra $\beta(\varepsilon\mathbf{D}, \mathcal{A} \mid \alpha)$ for some $\varepsilon > 0$, where $\mathbf{D}$ is the unit disc of $\mathbb{C}$ and $\beta(\mathbf{D}, \mathcal{A})$ is the space of mappings $f : \mathbf{D} \to \mathcal{A}$ such that $f(\mathbf{D})$ is bounded in $\mathcal{A}$, equipped with the equibounded boundedness. Note that it follows from [1] that

$$\beta(\varepsilon\mathbf{D}, \mathcal{A} \mid \alpha) = \beta(\varepsilon\mathbf{D}, \mathcal{A}) \mid \beta(\varepsilon\mathbf{D}, \alpha).$$

Thus, we decide that $a \in \mathcal{A} \mid \alpha$ is regular if there exist functions $u \in \beta(\varepsilon\mathbf{D}, \mathcal{A})$, $v \in \beta(\varepsilon\mathbf{D}, \alpha)$ and $v' \in \beta(\varepsilon\mathbf{D}, \alpha)$ such that

$$1 = (1 - sa)\, u(s) + v'(s) = u(s)\,(1 - sa) + v'(s)$$

for some $\varepsilon > 0$.

Now if $a \in \mathcal{A} \mid \alpha$ is a regular element, the resolvent set of a is the set of $s \in \mathbb{C}$ such that $(s.e - a)^{-1}$ exists and is a regular element of $\mathcal{A} \mid \alpha$, where e is the unity of $\mathcal{A}$. The spectrum of a is the complement of its resolvent set.

On the other hand, if E is a b-space, f is a function from a set X into E and $g : X \longrightarrow \mathbb{C}$ is a numerical function, then the notation $f = o_E(g)$ means that the quotient function $\frac{f}{g}$ is bounded in E i.e. there exists a bounded set B of E such that $f(s) \in g(s).B$ for all $s \in X$.

The following result gives some well known properties.

Proposition 2.4. *The spectrum of a regular element $a$of a unital q-algebra $\mathcal{A} \mid \alpha$, is compact, nonempty and the resolvent function $s \longmapsto (a - s.e)^{-1}$ is a holomorphic which vanishes at infinity, where e is the unity of $\mathcal{A}$.*

Proof. First we prove that the spectrum is closed. In fact, let s_0 be an element of the resolvent set of a. The function $t \longmapsto \left(1 - t\,(a - s_0.e)^{-1}\right)$

is invertible in $O(\varepsilon\mathbf{D}, \mathcal{A} \mid \alpha)$. We assume that $(a - s_0.e)$ has a regular inverse, the element is therefore a fortiori invertible. The function $t \longmapsto (a - (s_0 + t).e)$ is invertible in $O(\varepsilon\mathbf{D}, \mathcal{A} \mid \alpha)$. Let s_1 be another complex number such that $|s_1 - s_0| < \varepsilon$, and put $\varepsilon' = \varepsilon - |s_1 - s_0|$. We see that the function $t \longmapsto (a - (s_1 + t).e)$ is invertible in $O(\varepsilon'\mathbf{D}, \mathcal{A} \mid \alpha)$ and $(a - s_1.e)^{-1}$ is a regular element.

Now, we show that the spectrum is bounded. We assume that the function $t \longmapsto (1 - ta)$ is invertible in $O(\varepsilon\mathbf{D}, \mathcal{A} \mid \alpha)$, and we assume that $|s_0| > \varepsilon^{-1}$. We see that $t \longmapsto (a - s_0)$ is invertible and then $t \longmapsto (1 - ta)$ is invertible in $O(\varepsilon\mathbf{D}, \mathcal{A} \mid \alpha)$. For t near to s_0^{-1}, we see that the function $t \longmapsto t$ is invertible, and its inverse $t \longmapsto \frac{1}{t}$ is holomorphic and bounded. Therefore $t \longmapsto \left(a - t^{-1}\right)$ is invertible in a holomorphic way for t near to s_0^{-1}. And $t \longmapsto (a - s_0.e)^{-1}$ is regular.

Finally, the resolvent is holomorphic on the resolvent set. Indeed, near each s of the resolvent set, we have an $\mathcal{A}$-valued holomorphic function $u(z)$ and an α-valued holomorphic function $v(z)$ such that $(a - z)u(z) + v(z) = 1$ (we assume here that $\mathcal{A}$ is commutative, otherwise we must take two functions v, v' such that

$$(a - z)u(z) + v(z) = u(z)(a - z) + v'(z) = 1.$$

This shows that the inverse is holomorphic on the resolvent set. It is also holomorphic at infinity. It has then a series $u(z) = \sum_k a_k z^k$ where $a_k = o_{\mathcal{A}}(M_k)$ and $a_{n+k} - a_n.a_k = o_\alpha\left(M^{n+k}\right)$ with a_1 an element of the equivalence class a.

Apply now the Liouville theorem (e.g. Proposition 2.3), there exists $e \in \mathcal{A}$ such that $u(z) - e \in O(s_0 + \varepsilon\mathbf{D}, \alpha)$, we can take also $e \in \alpha$. Therefore if the spectrum is empty, then a is invertible modulo α and the inverse is 0 what is impossible. This ends the proof. □

References

1. B. Aqzzouz, The ϵ_C-product of a Schwartz b-space by a quotient Banach space and applications. *Appl. Categ. Structures* **10**(2002), No. 6, 603–616.
2. G. R. Allan, On a class of locally convex algebras. *Proc. London Math. Soc. (3).* **17**(1967), 91–114.
3. R. C. Gunning and H. Rossi, Analytic functions of several complex variables. *Prentice-Hall, Inc., Englewood Cliffs, N.J.* 1965.
4. H. Hogbe Nlend, Théorie des bornologies et applications. (French) *Lecture Notes in Mathematics*, Vol. 213. *Springer-Verlag, Berlin-New York*, 1971.
5. H. Hogbe Nlend, Les fondements de la théorie spectrale des algébres bornologiques. (French) *Bol. Soc. Brasil. Mat.* **3**(1972), No. 1, 19–56.
6. F. H. Vasilescu, Analytic functional calculus and spectral decompositions. Translated from the Romanian. *Mathematics and its Applications (East European Series), 1. D. Reidel Publishing Co., Dordrecht; Editura Academiei Republicii Socialiste România, Bucharest*, 1982.

7. F. H. Vasilescu, Spectral theory in quotient Fréchet spaces. *Revue Roumaine de Math. Pures et Appl.* **32**(1986), 561–571.
8. L. Waelbroeck, Le calcul symbolique dans les algébres commutatives. (French) *J. Math. Pures Appl. (9)* **33**(1954), 147–186.
9. L. Waelbroeck, Algébres commutatives: éléments réguliers. (French) *Bull. Soc. Math. Belg.* **9**(1957), 42–49.
10. L. Waelbroeck, Etude spectrale des algébres complétes. (French) *Acad. Roy. Belg. Cl. Sci. Mém. Coll. in-8 (2)* **31**(1960), No. 7, 142.
11. L. Waelbroeck, Topological vector spaces and algebras. *Lecture Notes in Mathematics*, Vol. 230. *Springer-Verlag, Berlin-New York*, 1971.
12. L. Waelbroeck, A preliminary lemma and its application to J. L. T.'s h.f.c. Operators in indefinite Metric spaces, Scattering Theory and Other topics. *Operator Theory. Advance and Applications. Birkhäuser Verlag, Basel-Boston-Stuttgart*, 355-364, 1986.
13. L. Waelbroeck, Quotient Banach spaces. *Spectral theory* (*Warsaw*, 1977), 553–562, *Banach Center Publ.*, 8, *PWN, Warsaw*, 1982.
14. L. Waelbroeck, The category of quotient bornological spaces. *Aspects of mathematics and its applications*, 873–894, *North-Holland Math. Library*, 34, *North-Holland, Amsterdam*, 1986.

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