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# ON THE APPLICATION OF THE METHOD OF A SMALL PARAMETER IN THE THEORY OF NON-SHALLOW I. N. VEKUA'S SHELLS

T. MEUNARGIA

Abstract. In the present paper we suggest the method of a small parameter for the solution of some basic boundary value problems of non-shallow shells applying the methods developed by N. I. Muskhelishvili and his pupils, by means of the theory of functions of a complex variable and integral equations.

**რეზიუმე.** წინამდებარე სტატიაში შემოთავაზებულია მცირე პარამეტრის მეთოდი არადამრეცი გარსების ზოგიერთი ძირითადი სასაზღვრო ამოცანის ამოსახსნელად კომპლექსური ცვლადის ფუქციათა თეორიისა და ინტეგრალურ განტოლებათა აპარატის გამოყენებით, რომელიც გადმოცემულია ნ. მუსხელიშვილისა და მისი მოწაფეების შრომებში [3].

## 1. The Coordinate System Connected Normally with the Surface. Shallow and Non-Shallow Shells

Let $\Omega$ denote a shell and a domain of the space occupied by this shell. Inside the shell we consider a smooth surface $S$ with respect to which the shell $\Omega$ lies symmetrically. The surface $S$ is called a midsurface of the shell $\Omega$. To construct the theory of shells we use the more convenient coordinate system which is normally connected with the midsurface $S$. This means that the radius-vector $R$ of any point of the domain $\Omega$ can be represented in the form [1]

$$\mathbf{R}(\mathrm{x}^1,\mathrm{x}^2,\mathrm{x}^3)=\mathbf{r}(\mathrm{x}^1,\mathrm{x}^2)+\mathrm{x}^3\mathbf{n}(\mathrm{x}^1,\mathrm{x}^2),$$

where $\mathbf{r}$ and $\mathbf{n}$ are the radius-vector and the basis vector of the normal of the midsurface $S(x_3=0)$, respectively. $(x^1,x^2)$ are the Gaussian parameters of the surface $S$, and $x^3$ (or $x_3$) is the thickness coordinate, where

$$-h\le x_3\le h,$$

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$2h$ is the shell thickness (generally speaking, variable).

Covariant and contravariant basis vectors $\mathbf{R_i}$ and $\mathbf{R^i}$ of the surface $\widehat{S}(x_3 =$ const) and the corresponding basis vectors $\mathbf{r}_\mathrm{j}$ and $\mathbf{r}^\mathrm{j}$ of the midsurface $\widehat{S}(x_3 = \text{const})$ are connected by the following relations [1]:

$$\mathbf{R}_\mathrm{i} = \mathrm{A}_{\mathrm{i}.}^{\cdot\mathrm{j}}\mathbf{r}_\mathrm{j} = \mathrm{A}_{\mathrm{ij}}\mathbf{r}^\mathrm{j}, \ \ \mathbf{R}^\mathrm{i} = \mathrm{A}^{\mathrm{i}\cdot}_{\cdot\mathrm{j}}\mathbf{r}^\mathrm{j} = \mathrm{A}^{\mathrm{ij}}\mathbf{r}_\mathrm{j}, \ \ (\mathrm{i},\mathrm{j} = 1,2,3),$$

where

$$A_{i.}^{\cdot j} = \begin{cases} a_\alpha^\beta - x_3 b_\alpha^\beta, \ \ i=\alpha, \ j=\beta, \ (\alpha,\beta=1,2) \\ \delta_i^3, \ \ j=3, \end{cases}$$

$$A^{i\cdot}_{\cdot j} = \begin{cases} \dfrac{(1-2Hx_3)a_\beta^\alpha + x_3 b_\beta^\alpha}{1-2Hx_3+Kx_3^2}, \ \ i=\alpha, \ j=\beta, \ (\alpha,\beta=1,2) \\ \delta_3^i, \ \ j=3, \end{cases} \tag{1.1}$$

$$\mathbf{r}^i = \begin{cases} \mathbf{r}^\alpha, \ \ i=\alpha, \\ \mathbf{n}, \ \ i=3. \end{cases} \tag{1.2}$$

Here $(a_{\alpha\beta}, \ a^{\alpha\beta}, \ a_\beta^\alpha)$ and $(b_{\alpha\beta}, \ b^{\alpha\beta}, \ b_\beta^\alpha)$ are the components (co, contra, mixed) of the metric tensor and curvarure tensor of the midsurface $S$. By $H$ and $K$ we denote a middle and Gaussian curvature of the surface $S$, where

$$2H = b_1^1 + b_2^2, \ \ K = b_1^1 b_2^2 - b_1^2 b_2^1.$$

The main quadratic forms of the midsurface $S$ have the form

$$I = ds^2 = a_{\alpha\beta}dx^\alpha dx^\beta, \tag{1.3}$$

$$II = k_s ds^2 = b_{\alpha\beta}dx^\alpha dx^\beta, \tag{1.4}$$

where $k_s$ is the normal curvature of the surface $S$, and

$$a_{\alpha\beta} = \mathbf{r}_\alpha \mathbf{r}_\beta, \ \ b_{\alpha\beta} = -\mathbf{n}_\alpha \mathbf{r}_\beta, \ \ \partial_\alpha = \frac{\partial}{\partial x^\alpha}.$$

Here and in the sequel, under a repeated indices we mean summation; note that the Greek indices range over 1, 2, while Latin indices range over 1, 2, 3.

It is important to note that under the thin and shallow I. N. Vekua's shells we mean three-dimensional shell-like bodies, satisfying the following geometric conditions:

$$a_\alpha^\beta - x_3 b_\alpha^\beta \cong a_\beta^\alpha, \ \ (\alpha,\beta=1,2). \tag{$*$}$$

These conditions are always fulfilled if the interval $[-h,h]$ is sufficiently small (thin shells), or if $b_\alpha^\beta$ are small values (shallow shells).

For thin and shallow shells the relation of the type $(*)$

$$\mathbf{R}_\alpha \cong \mathbf{r}_\alpha, \ \ \mathbf{R}^\alpha \cong \mathbf{r}^\alpha, \ \ A^{\alpha\cdot}_{\cdot\beta} \cong a_\beta^\alpha, \ \ 1-2Hx_3+Kx_3 \cong 1$$

are valid. In other words, for thin and shallow shells, the interior geometry of the shell does not change in thickness and coincides with that of the midsurface $S$. Therefore thin and shallow shells are also called the shells with notvarying geometry in thickness. I.N. Vekua has constructed a refined theory for thin and shallow shells [1].

Further we omit the assumption of the type ($*$) requiring only for the conditions

$$\left|b_\alpha^\beta h\right| \leq q < 1, \quad (\alpha, \beta = 1,2), \tag{(**)}$$

be fulfilled, which denote the shell-like three-dimensional elastic body.

By analogy with the previous case, such kind of shells will be called non-shallow and non-thin shells, or shells with changeable in thickness geometry [5,6].

To construct the theory of non-shallow shells, it is necessary to obtain formulas for a family of surfaces $\widehat{S}(x_3 = const)$, analogous to (1.3)-(1.4) of the midsurface $S(x_3 = 0)$ which have the form [1]

$$I = d\widehat{s}^2 = g_{\alpha\beta} dx^\alpha dx^\beta, \tag{1.5}$$

$$II = k_{\widehat{s}} d\widehat{s}^2 = \widehat{b}_{\alpha\beta} dx^\alpha dx^\beta, \tag{1.6}$$

where

$$g_{\alpha\beta} = \mathbf{R}_\alpha \mathbf{R}_\beta = a_{\alpha\beta} - 2x_3 b_{\alpha\beta} + x_3^2(2H b_{\alpha\beta} - K_{\alpha\beta}),$$
$$\widehat{b}_{\alpha\beta} = (1 - 2Hx_3) b_{\alpha\beta} + x_3 K a_{\alpha\beta},$$

and $k_{\widehat{s}}$ the normal curvature of the surface $\widehat{S}$.

It is not now difficult to get the expressions for the unit tangent vector $\widehat{\mathbf{s}}$ and for the tangential normal of the surface $\widehat{\mathbf{l}}$ directed to $\widehat{s}$ [5]:

$$\widehat{\mathbf{s}} = \frac{d\mathbf{R}}{d\widehat{s}} = [(1 - x_3 k_s)\mathbf{s} + x_3 \tau_s \mathbf{l}] \frac{ds}{d\widehat{s}},$$
$$\widehat{\mathbf{l}} = \widehat{\mathbf{s}} \times \mathbf{n} = [(1 - x_3 k_s)\mathbf{l} - x_3 \tau_s \mathbf{s}] \frac{ds}{d\widehat{s}},$$
$$d\widehat{s} = \sqrt{1 - 2x_3 k_s + x_3^2(k_s^2 + \tau_s^2)}\, ds.$$
$$(\widehat{\mathbf{l}} \times \widehat{\mathbf{s}} = \mathbf{n}),$$

where $d\widehat{s}$ and $ds$ are the linear elements of the surfaces $\widehat{S}$ and $S$, and $\tau_s$ is the geodesic torsion of the surface $S$.

Note that in deducing these formulas we use the well-known expressions of Rodrige's vector

$$\frac{d\mathbf{n}}{ds} = -k_s \mathbf{s} + \tau_s \mathbf{l}, \quad (\mathbf{l} \times \mathbf{s} = \mathbf{n}),$$

where $\mathbf{s}$ and $\mathbf{l}$ are the unit vectors of the tangent and tangential normal on $S$.

The formula [5]

$$\widehat{\mathbf{l}}\mathbf{R}_\alpha = (1 - 2Hx_3 + Kx_3^2)(\mathbf{l}\mathbf{r}_\alpha) \frac{ds}{d\widehat{s}}, \quad (\alpha = 1,2), \tag{1.7}$$

necessary in writing the refuced basic boundary value problem in stresses, is also valid.

## 2. A System of Equations of Equilibrium, and Hook's Law for Non-Shallow Shells

We write the equation of equilibrium of an elastic medium $\Omega$ in a vector form which is convenient for the reduction to the two-dimensional equations:

$$\frac{1}{\sqrt{a}}\frac{\partial\sqrt{a}\left(\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^\alpha\right)}{\partial x^\alpha}+\frac{\partial\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^3}{\partial x^3}+\frac{g}{a}\mathbf{\Phi}=0, \tag{2.1}$$

where $g$ and $a$ are discriminants of metric quadratic forms of the tree-dimensional domain $\Omega$ and midsurface $S$; $\mathbf{\Phi}$ is the body force per unit of volume, $\boldsymbol{\sigma}^i$ will be called, following to I. N. Vekua, the contravariant components (pseudovector) of the stress vector $\boldsymbol{\sigma}_{(\overset{*}{\boldsymbol{l}})}$ acting on the area with the normal $\overset{*}{\boldsymbol{l}}$ and representable as the Cauchy formula as follows:

$$\boldsymbol{\sigma}_{(\overset{*}{\boldsymbol{l}})}=\boldsymbol{\sigma}^i\overset{*}{\boldsymbol{l}}_i,\quad \left(\overset{*}{\boldsymbol{l}}_i=\overset{*}{\boldsymbol{l}}\mathbf{R}_i,\quad i=1,2,3\right).$$

Using the relation (1.7), for the stress vector acting on the area with the normal $\widehat{\mathbf{l}}$, we obtain

$$\boldsymbol{\sigma}_{(\widehat{\mathbf{l}})}=\boldsymbol{\sigma}^\alpha(\widehat{\mathbf{l}}\mathbf{R}_\alpha)=\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^\alpha(\mathbf{l}\mathbf{r}_\alpha)\frac{ds}{d\widehat{s}} \tag{2.2}$$

where

$$\sqrt{\frac{g}{a}}=1-2Hx_3+Kx_3^2.$$

For the face surfaces $\overset{(\pm)}{S}(x_3=\pm h)$ with the normal $(\pm\mathbf{n})$ we have

$$\boldsymbol{\sigma}_{(\pm\mathbf{n})}=\pm\boldsymbol{\sigma}^3(x^1,x^2,\pm h). \tag{2.2$_1$}$$

Using the relations (1.1) and (1.2), for the Hook's law we obtain the following vector notation [6]:

$$\boldsymbol{\sigma}^i=A^i_{i_1}A^j_{j_1}C^{i_1j_1}\partial_j\mathbf{U},\quad (i,j=1,2,3), \tag{2.3}$$

where $\mathbf{U}$ is the displacement vector, $C^{i_1j_1}$ are dyadic operators, and

$$C^{i_1j_1}=\lambda(\mathbf{r}^{i_1}\otimes\mathbf{r}^{j_1})+\mu(\mathbf{r}^{j_1}\otimes\mathbf{r}^{i_1})+\mu a^{i_1j_1}E,\quad (a^{i_1j_1}=\mathbf{r}^{i_1}\cdot\mathbf{r}^{j_1}) \tag{2.3$_1$}$$

here $\otimes$ is the symbol of dyadic vector products, $E$ is the unit dyad,

$$E=\mathbf{r}^\alpha\otimes\mathbf{r}_\alpha+\mathbf{n}\otimes\mathbf{n},$$

$\lambda$ and $\mu$ are the elastic Lamé constants.

In an expanded form the relation (2.3) can be represented as

$$\boldsymbol{\sigma}^i=A^i_{i_1}A^j_{j_1}\left[\lambda(\mathbf{r}^{j_1}\partial_j\mathbf{U})\mathbf{r}^{i_1}+\mu(\mathbf{r}^{i_1}\partial_j\mathbf{U})\mathbf{r}^{j_1}+\mu a^{i_1j_1}\partial_j\mathbf{U}\right]. \tag{2.3$_2$}$$

Substituting now (2.3) into (2.1), we obtain the equation of equilibrium with respect to the displacement vector $\mathbf{U}$:

$$\nabla_\alpha\left(\sqrt{\frac{g}{a}}A^\alpha_{\alpha_1}A^j_{j_1}C^{\alpha_1 j_1}\partial_j\mathbf{U}\right)+\frac{\partial}{\partial x_3}\left(\sqrt{\frac{g}{a}}A^3_3A^j_{j_1}C^{3j_1}\partial_j\mathbf{U}\right)+\sqrt{\frac{g}{a}}\boldsymbol{\Phi}=0, \quad (2.4)$$

where $\nabla_\alpha$ is the symbol of a covariant derivative on the surface $S$.

The equilibrium equation (2.1) and the Hook's law in a tensor notation takes the form

$$\begin{cases}\nabla_\alpha\left(\sqrt{\frac{g}{a}}\sigma^{\alpha\beta}\right)-b^\beta_\alpha\left(\sqrt{\frac{g}{a}}\sigma^{\alpha3}\right)+\frac{\partial\left(\sqrt{\frac{g}{a}}\sigma^{3\beta}\right)}{\partial x_3}+\sqrt{\frac{g}{a}}\phi^\beta=0, \ (\alpha,\beta=1,2),\\ \nabla_\alpha\left(\sqrt{\frac{g}{a}}\sigma^{\alpha3}\right)-b^\beta_\alpha\left(\sqrt{\frac{g}{a}}\sigma^{\alpha\cdot}_{\cdot\beta}\right)+\frac{\partial\left(\sqrt{\frac{g}{a}}\sigma^3_3\right)}{\partial x_3}+\sqrt{\frac{g}{a}}\phi^3=0,\end{cases} \quad (2.5)$$

where

$$\begin{cases}\sigma^{\alpha\beta}=\boldsymbol{\sigma}^\alpha\mathbf{r}^\beta=A^\alpha_{\alpha_1}A^j_{j_1}\left(C^{\alpha_1j_1}\partial_j\mathbf{U}\right)\mathbf{r}^\beta,\\ \sigma^{\alpha3}=\boldsymbol{\sigma}^\alpha\mathbf{n}=A^\alpha_{\alpha_1}A^j_{j_1}\left(C^{\alpha_1j_1}\partial_j\mathbf{U}\right)\mathbf{n},\\ \sigma^{3\beta}=\boldsymbol{\sigma}^3\mathbf{r}^\beta=A^3_3A^j_{j_1}\left(C^{3j_1}\partial_j\mathbf{U}\right)\mathbf{r}^\beta,\\ \sigma^{33}=\boldsymbol{\sigma}^3\mathbf{n}=A^3_3A^j_{j_1}\left(C^{3j_1}\partial_j\mathbf{U}\right)\mathbf{n}.\end{cases} \quad (2.6)$$

It is not difficult to represent them in expanded form, for example, for $\sigma^{\alpha\beta}$ and $\sigma^{\alpha3}$ we have

$$\begin{cases}\sigma^{\alpha\beta}=A^\alpha_{\alpha_1}\left\{A^\gamma_{\gamma_1}\left[\lambda(\mathbf{r}^{\gamma_1}\partial_\gamma\mathbf{U})a^{\alpha_1\beta}+\mu(\mathbf{r}^{\alpha_1}\partial_\gamma\mathbf{U})a^{\gamma_1\beta}+\right.\right.\\ \qquad\left.\left.+\ \mu a^{\alpha_1\gamma_1}(\mathbf{r}^\beta\partial_\gamma\mathbf{U})\right]+A^3_3\lambda(\mathbf{n}\partial_3\mathbf{U})a^{\alpha,\beta}\right\},\\ \sigma^{\alpha3}=\mu A^\alpha_{\alpha_1}\left\{A^\gamma_{\gamma_1}(\mathbf{n}\partial_\gamma\mathbf{U})a^{\alpha_1\gamma_1}+A^3_3(\mathbf{r}^{\alpha_1}\partial_3\mathbf{U})\right\},\\ (A^i_3=a^3_i=a^{3i}=\delta^{3i}).\end{cases} \quad (2.7)$$

Note that the derivative of the vector $\mathbf{U}$ with respect to $x^\alpha$ can be represented as

$$\begin{aligned}\partial_\alpha\mathbf{U}=\nabla_\alpha\mathbf{U}&=(\nabla_\alpha U_\beta-b_{\alpha\beta}U_3)\mathbf{r}^\beta+(\nabla_\alpha U_3-b_{\alpha\beta}U^\beta)\mathbf{n}\\ &=(\nabla_\alpha U^\beta-b^\beta_\alpha U_3)\mathbf{r}_\beta+(\nabla_\alpha U_3+b^\beta_\alpha U_\beta)\mathbf{n}\end{aligned}$$

i.e.,

$$\begin{aligned}&\mathbf{r}_\beta\partial_\alpha\mathbf{U}=\nabla_\alpha U_\beta-b_{\alpha\beta}U_3, \ \ \mathbf{r}^\beta\partial_\alpha\mathbf{U}=\nabla_\alpha U^\beta-b^\beta_\alpha U_3,\\ &\mathbf{n}\partial_\alpha\mathbf{U}=\nabla_\alpha U_3+b_{\alpha\beta}U^\beta=\nabla_\alpha U_3+b^\beta_\alpha U_\beta, \ \ (\nabla_\alpha U_3=\partial_\alpha U_3).\end{aligned} \quad (2.8)$$

To the system of equations (2.1) and (2.3), or (2.5) and (2.6) we have to add the boundary conditions on the face $\overset{(\pm)}{S}$ $(x_3=\pm h)$ and on the side $\Sigma$ surfaces of the shell $\Omega$. If the shell is closed, then $\Sigma$ is absent. It is assumed that $\Sigma$ are the ruled surfaces and their generators are the normals to $S$. As usual, in the theory of shells it is assumed that stresses are given on the face surfaces $\overset{(\pm)}{S}$ $(x_3=\pm h)$, i.e.

$$\overset{(+)}{\boldsymbol{\sigma}}{}^3=\boldsymbol{\sigma}^3(x^1,\ x^2\ h), \quad \overset{(-)}{\boldsymbol{\sigma}}{}^3=\boldsymbol{\sigma}^3(x^1,\ x^2\ -h)$$

is the given vector field. Suppose that the stresses or displacements are given on the side surfaces $\Sigma$, or the stresses are known on one part $\Sigma'$ and the displacements $\Sigma''$ on the remaining part $\Sigma' \cup \Sigma'' = \Sigma$, $\Sigma' \cap \Sigma'' = \oslash$.

Let $\widehat{\mathbf{l}}$ be a unit vector of the normal of the boundary area $d\Sigma = d\widehat{s}dx_3$, where $d\widehat{s}$ is the linear element of the boundary curve $\widehat{\Gamma}$ of the surface $\widehat{S}(x_3 = const)$.

Then the stress vector $\boldsymbol{\sigma}_{(\widehat{l})}$ is expressed by the formula(2.2),

$$\boldsymbol{\sigma}_{(\widehat{l})} = \boldsymbol{\sigma}^{\alpha}(\widehat{\mathbf{l}}\mathbf{R}_{\alpha}) = \sqrt{\frac{g}{a}}\boldsymbol{\sigma}^{\alpha}(\mathbf{l}\mathbf{r}_{\alpha})\frac{ds}{d\widehat{s}} = \sqrt{\frac{g}{a}}\boldsymbol{\sigma}_{(\mathbf{l})}\frac{ds}{d\widehat{s}}, \tag{2.9}$$

where $\mathbf{l} = l_{\alpha}\mathbf{r}^{\alpha}$ is the tangential normal of the boundary area $dS = dsdx_3$, and $ds$ is the linear element of the boundart curve $\Gamma$ of the midsuface $S$.

For the system (2.5) and (2.6) we consider the following basic boundary value problems.

**Problem I.** Find a solution of the system (2.5) and (2,6), consistent with the physical condition of the type

$$\boldsymbol{\sigma}_{(\widehat{l})} = \sigma_{(\widehat{l}\widehat{l})}\widehat{\mathbf{l}} + \sigma_{(\widehat{l}\widehat{s})}\widehat{\mathbf{s}} + \sigma_{(\widehat{l}n)}\mathbf{n} = \widehat{\mathbf{f}}_{(\widehat{l})} \text{ on } \widehat{\Gamma} \tag{2.10}$$

where $\widehat{\mathbf{f}}_{(\widehat{l})}$ is the given vector function on the contour $\widehat{\Gamma}$. By $\sigma_{(\widehat{l}\widehat{l})}$, $\sigma_{(\widehat{l}\widehat{s})}$, $\sigma_{(\widehat{l}n)}$ we denote respectively the normal, longitudinal and transversal tangential stresses acting on the area with the normal $\widehat{\mathbf{l}}$.

This condition can likewise be written as

$$\sigma_{(\widehat{l}\widehat{l})} = \widehat{f}_{(\widehat{l})}, \ \ \sigma_{(\widehat{l}\widehat{s})} = \widehat{f}_{(\widehat{s})}, \ \ \sigma_{(\widehat{l}n)} = \widehat{f}_{(n)} \text{ on } \widehat{\Gamma}, \tag{2.10$_1$}$$

where $\widehat{f}_{(\widehat{l})}$, $\widehat{f}_{(\widehat{s})}$, $\widehat{f}_{(n)}$ are the given functions of the points $\widehat{\Gamma}$.

By formula (2.9), the condition (2.10) can be represented by means of the unit vectors $\mathbf{l}$, $\mathbf{s}$ and $\mathbf{n}$ ($\mathbf{l} \times \mathbf{s} = \mathbf{n}$) as follows:

$$\boldsymbol{\sigma}_{(l)} = \sigma_{(ll)}\mathbf{l} + \sigma_{(ls)}\mathbf{s} + \sigma_{(ln)}\mathbf{n} = \mathbf{f}_{(l)} \text{ on } \widehat{\Gamma}$$

or

$$\sigma_{(ll)} = f_{(l)}, \ \ \sigma_{(ls)} = f_{(s)}, \ \ \sigma_{(ln)} = f_{(n)} \text{ on } \widehat{\Gamma},$$

where

$$\begin{aligned}
f_{(l)} &= \sqrt{\frac{a}{g}}\left[(1 - x_3k_s)\widehat{f}_{(\widehat{l})} + x_3\tau_s\widehat{f}_{(\widehat{s})}\right], \\
f_{(s)} &= \sqrt{\frac{a}{g}}\left[(1 - x_3k_s)\widehat{f}_{(\widehat{s})} - x_3\tau_s\widehat{f}_{(\widehat{l})}\right], \\
f_{(n)} &= \sqrt{\frac{a}{g}}\widehat{f}_n\frac{d\widehat{s}}{ds},
\end{aligned} \tag{2.10$_2$}$$

are the given functions of the points on the contour $\widehat{\Gamma}$.

**Problem II.** Find a solution of the system (2.5) and (2.6) consistent with the kinematic boundary condition of the type

$$\mathbf{U} = U_{(\widehat{l})}\widehat{\mathbf{l}} + U_{(\widehat{s})}\widehat{\mathbf{s}} + U_3\mathbf{n} = \widehat{g} \text{ on } \widehat{\Gamma}, \tag{2.11}$$

where $\widehat{g}$ is the given vector function on $\widehat{\Gamma}$, and by $U_{(\widehat{l})}$, $U_{(\widehat{s})}$, $U_3$ are denoted respectively the normal, tangential and transversal displacements of the vector $\mathbf{U}$.

The condition (2.11) can likewise be written as

$$U_{(\widehat{l})} = \widehat{g}_{(\widehat{l})}, \ \ U_{(\widehat{s})} = \widehat{g}_{(\widehat{s})}, \ \ U_3 = \widehat{g}_3 \text{ on } \widehat{\Gamma}, \tag{$2.11_1$}$$

where $U_{(\widehat{l})}$, $U_{(\widehat{s})}$, $U_3$ are the given functions of the points on the contour $\widehat{\Gamma}$.

By virtue of the identity

$$\mathbf{U} = U_{(\widehat{l})}\widehat{\mathbf{l}} + U_{(\widehat{s})}\widehat{\mathbf{s}} + U_3\mathbf{n} = U_{(l)}\mathbf{l} + U_{(s)}\mathbf{s} + U_3\mathbf{n},$$

for $(2.11_1)$ we obtain the equivalent conditions of the type

$$U_{(l)} = g_{(l)}, \ \ U_{(s)} = g_{(s)}, \ \ U_3 = g_3, \tag{$2.11_2$}$$

where $g_{(l)}$, $g_{(s)}$, $g_3$ are the given functions on $\widehat{\Gamma}$, and

$$g_{(l)} = \Big[(1 - x_3k_s)\widehat{g}_{(\widehat{l})} + x_3\tau_s\widehat{g}_{(\widehat{s})}\Big]\frac{ds}{d\widehat{s}},$$
$$g_{(s)} = \Big[(1 - x_3k_s)\widehat{g}_{(\widehat{s})} - x_3\tau_s\widehat{g}_{(\widehat{l})}\Big]\frac{ds}{d\widehat{s}}, \ \ g_3 = \widehat{g}_3,$$

**Problem III.** Find a solution of the system (2.5) and (2.6) satisfying on one part of the contour $\widehat{\Gamma}$ the physical boundary conditions of the type (2.10) and the kinematic conditions of the type (2.11) on the remaining part of the boundary.

It should be noted that in the present paper we do not consider the theorems on the existence and uniqueness of these problems, we indicate only the way for obtaining a formal solution of the basic boundary value problems by the method of a small parameter.

## 3. Some Special Coordinate Systems on the Midsurface of a Shell

3.1. **The Coordinate System in Lines of Curvature.** For this system [4] we have

$$\mathbf{R}_\alpha = (1 - k_\alpha x_3)\mathbf{r}_\alpha, \ \ \mathbf{R}^\alpha = \frac{\mathbf{r}^\alpha}{1 - k_\alpha x_3}, \ \ (\alpha = 1, 2),$$

where $k_1$ and $k_2$ is the principal curvature of the midsurface $S$, and

$$\sqrt{\frac{g}{a}} = (1-k_1x_3)(1-k_2x_3), \quad 2H = k_1+k_2, \quad K = k_1k_2$$

The basic quadratic forms are of the type

$$\begin{aligned} &I = ds^2 = a_{11}(dx^1)^2 + a_{22}(dx^2)^2, \\ &II = k_s ds^2 = a_{11}k_1(dx^1)^2 + a_{22}k_2(dx^2)^2, \end{aligned}$$

i.e. $a_{12}=0$, $b_1^1 = k_1$, $b_2^2 = k_2$, $b_2^1 = b_1^2 = 0$.

The system of equations of equilibrium in the lines of curvature can be written as follows:

$$\begin{cases} \nabla_\alpha\left(\sqrt{\frac{g}{a}}\sigma^{\alpha 1}\right) - k_1\left(\sqrt{\frac{g}{a}}\sigma^{13}\right) + \dfrac{\partial\left(\sqrt{\frac{g}{a}}\sigma^{31}\right)}{\partial x_3} + \sqrt{\frac{g}{a}}\Phi^1 = 0, \\ \nabla_\alpha\left(\sqrt{\frac{g}{a}}\sigma^{\alpha 2}\right) - k_2\left(\sqrt{\frac{g}{a}}\sigma^{23}\right) + \dfrac{\partial\left(\sqrt{\frac{g}{a}}\sigma^{31}\right)}{\partial x_3} + \sqrt{\frac{g}{a}}\Phi^2 = 0, \end{cases} \tag{3.1}$$

where

$$\begin{aligned} \sigma^{\alpha\beta} =& \frac{1}{1-k_\alpha x_3}\Big[\lambda\Big(\frac{\mathbf{r}^\gamma\partial_\gamma\mathbf{U}}{1-k_\gamma x_3} + \mathbf{n}\partial_3\mathbf{U}\Big)a^{\alpha\beta} + \\ &+ \frac{(\mathbf{r}^\alpha\partial_\gamma\mathbf{U})a^{\beta\gamma} + (\mathbf{r}^\beta\partial_\gamma\mathbf{U})a^{\alpha\gamma}}{1-k_\gamma x_3}\Big], \\ \sigma^{\alpha 3} =& \frac{\mu}{1-k_\alpha x_3}\Big[\mathbf{r}^\alpha\partial_3\mathbf{U} + \frac{(\mathbf{n}\partial_\gamma\mathbf{U})a^{\alpha\gamma}}{1-k_\gamma x_3}\Big], \\ \sigma^{3\alpha} =& \mu\Big[\mathbf{r}^\alpha\partial_3\mathbf{U} + \frac{(\mathbf{n}\partial_\gamma\mathbf{U})a^{\alpha\gamma}}{1-k_\gamma x_3}\Big], \\ \sigma^{33} =& \lambda\frac{\mathbf{r}^\gamma\partial_\gamma\mathbf{U}}{1-k_\gamma x_3} + (\lambda+2\mu)(\mathbf{n}\partial_3\mathbf{U}) \end{aligned} \tag{3.2}$$

The summation with respect to $\alpha$ is not allowed.

For $A_{i.}^{\cdot j}$ and $A_{\cdot j}^{i.}$ we have

$$\begin{aligned} &A_{1.}^{\cdot 1} = 1-k_1x_3, \quad A_{2.}^{\cdot 2} = 1-k_2x_3, \quad A_{1.}^{\cdot 2} = A_{2.}^{\cdot 1} = 0, \\ &A_{\cdot 1}^{1.} = \tfrac{1}{1-k_1x_3}, \quad A_{\cdot 2}^{2.} = \tfrac{1}{1-k_2x_3}, \quad A_{\cdot 2}^{1.} = A_{\cdot 1}^{2.} = 0. \end{aligned} \tag{3.3}$$

Moreover, the relations (2.8) take the form

$$\begin{aligned} &\mathbf{r}_1\nabla_1\mathbf{U} = \nabla_1U_1 - a_{11}k_1U_3, \quad \mathbf{r}_2\nabla_2\mathbf{U} = \nabla_2U_2 - a_{22}k_2U_3, \\ &\mathbf{r}_1\nabla_2\mathbf{U} = \nabla_2U_1, \quad \mathbf{r}_2\nabla_1\mathbf{U} = \nabla_1U_2, \\ &\mathbf{r}^1\nabla_1\mathbf{U} = \nabla_1U^1 - k_1U_3, \quad \mathbf{r}^2\nabla_2\mathbf{U} = \nabla_2U^2 - k_2U_3, \\ &\mathbf{r}^1\nabla_2\mathbf{U} = \nabla_2U^1, \quad \mathbf{r}^2\nabla_1\mathbf{U} = \nabla_1U^2, \\ &\mathbf{n}\nabla_1\mathbf{U} = \nabla_1U_3 + k_1U_1, \quad \mathbf{n}\nabla_2\mathbf{U} = \nabla_2U_3 + k_2U_2, \\ &(\nabla_\alpha U_3 = \partial_\alpha U_3, \quad \nabla_\alpha\mathbf{U} = \partial_\alpha\mathbf{U}). \end{aligned}$$

Next, the external form for the expressions $\boldsymbol{\sigma}_{(l)}$ and $\mathbf{U}$ remains invariable.

**Isometric System of Coordinates.** The isometric system of coordinates on the surface $S$ is of special interest, because in this system we can obtain basic equations of the theory of shells in a complex form which in turn allows one to construct for a rather wide class of problems complex representations of general solutions by means of analytic functions of one variable $z = x^1 + ix^2$. This circumstance makes it possible to apply the methods, developed by N. I. Muskhelishvili and his pupils, by means of the theory of functions of a complex variable and integral equations.

The main quadratic forms in this system of coordinates are of the type

$$I = ds^2 = \Lambda(x^1, x^2)\left[(dx^1)^2 + (dx^2)^2\right] = \Lambda(z, \bar{z})dzd\bar{z}, \ \ (\Lambda > 0),$$
$$II = k_s ds^2 = b_{\alpha\beta}dx^\alpha dx^\beta = \frac{1}{2}\Lambda\left[\overline{Q}dz^2 + 2Hdzd\bar{z} + Qd\bar{z}^2\right],$$

where

$$Q = \frac{1}{2}(b_1^1 - b_2^2 + 2ib_2^1), \ \ H = \frac{1}{2}(b_1^1 + b_2^2), \ \ z = x^1 + ix^2.$$

Introducing the well-known differential operators

$$\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x^1} - i\frac{\partial}{\partial x^2}\right), \ \ \frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x^1} + i\frac{\partial}{\partial x^2}\right),$$

and the notation

$$\tau^{i\cdot}_{\cdot j} = \sqrt{\frac{g}{a}}\sigma^{i\cdot}_{\cdot j}, \ \ \mathbf{X} = \sqrt{\frac{g}{a}}\mathbf{\Phi}, \tag{3.4}$$

from systems (2,5) and (2.6) we obtain the following complex writing both for the system of equations of equilibrium and for the Hook's law:

$$\begin{cases} \frac{1}{\Lambda}\frac{\partial}{\partial z}\left[\Lambda(\tau_1^1 - \tau_2^2 + i\tau_2^1 + i\tau_1^2)\right] + \frac{\partial}{\partial \bar{z}}(\tau_1^1 + \tau_2^2 + i\tau_2^1 - i\tau_1^2) - \\ -\Lambda(H\tau^+ + Q\bar{\tau}^+) + \frac{\partial\tau_{(+)}}{\partial x_3} + X_+ = 0, \\ \frac{1}{\Lambda}\left(\frac{\partial\Lambda\tau^+}{\partial z} + \frac{\partial\Lambda\bar{\tau}^+}{\partial\bar{z}}\right) + H(\tau_1^1 + \tau_2^2) + \\ +Re\left[\bar{Q}(\tau_1^1 - \tau_2^2 + i\tau_2^1 + i\tau_1^2)\right] + \frac{\partial\tau_3^3}{\partial x_3} + X_3 = 0, \end{cases} \tag{3.5}$$
$$(X^+ = X^1 + iX^2),$$

where

$$\tau^1_{.1} - \tau^2_{.2} + i\tau^1_{.2} + i\tau^2_{.1} = 4\mu\mathbf{r}^+\partial_{\bar{z}}\mathbf{U} + 2\sqrt{\frac{\mathrm{a}}{\mathrm{g}}}\left\{\mathrm{x}_3\mathrm{Q}(1 - \mathrm{H}\mathrm{x}_3)\left[(\lambda+\mu)\Theta + 2\mu\mathbf{r}^+\partial_{\mathrm{z}}\mathbf{U}\right] + \right.$$
$$\left. + x_3^2 Q\left[(\lambda+\mu)\overline{\mathbf{r}^+}\partial_z\mathbf{U} + (\lambda+3\mu)\overline{\mathrm{Q}}\mathbf{r}^+\partial_{\bar{\mathrm{z}}}\mathbf{U}\right]\right\} + 2\lambda x_3 Q\partial_3 U_3,$$

$$\tau^1_{.1} + \tau^2_{.2} + i\tau^1_{.2} - i\tau^2_{.1} = 2(\lambda+\mu)\Theta + 2\sqrt{\frac{a}{g}}\left\{x_3(1 - Hx_3)\left[(\lambda+\mu)Q\overline{\mathbf{r}^+}\partial_z\mathbf{U} + \right.\right.$$
$$\left.\left. + (\lambda+3\mu)\overline{Q}\mathbf{r}^+\partial_{\bar{\mathrm{z}}}\mathbf{U}\right] + x_3^2 Q\overline{Q}\left[(\lambda+\mu)\Theta + 2\mu\mathbf{r}^+\partial_{\mathrm{z}}\mathbf{U}\right]\right\} + 2\lambda(1 - Hx_3)\partial_3 U_3,$$

$$\tau^+ = \sqrt{\frac{g}{a}}(\sigma^{13}+ \tag{3.6}$$

$$+\sigma^{23}) = \frac{2\mu}{\Lambda}\Big\{(\mathbf{n}\partial_{\bar z}\mathbf{U})+2\sqrt{\frac{\mathrm{a}}{\mathrm{g}}}\left[\mathrm{x}_3^2\mathrm{Q}\overline{\mathrm{Q}}\mathbf{n}\partial_{\bar z}\mathbf{U}+\mathrm{x}_3\mathrm{Q}(1-\mathrm{H}\mathrm{x}_3)(\mathbf{n}\partial_z\mathbf{U})\right]$$

$$+(1-Hx_3)\partial_3 U_+ + x_3 Q\partial_3\overline{U}_+\Big\},$$

$$\tau_{(+)} = \sqrt{\frac{g}{a}}(\sigma_1^3+\sigma_2^3) = 2\mu\left[(1-Hx_3)\mathbf{n}\partial_{\bar z}\mathbf{U}+\mathrm{x}_3\mathrm{Q}(\mathbf{n}\partial_z\mathbf{U})+\frac{1}{2}\sqrt{\frac{\mathrm{g}}{\mathrm{a}}}\partial_3\mathrm{U}_+\right],$$

$$\tau_3^3 = \sqrt{\frac{g}{a}}\sigma^{33} = \lambda\left[(1-Hx_3)\Theta+x_3(Q\mathbf{r}^+\partial_z\mathbf{U}+\overline{\mathrm{Q}}\mathbf{r}^+\partial_{\bar z}\mathbf{U})\right]+(\lambda+2\mu)\sqrt{\frac{g}{a}}\partial_3 U_3,$$

Here,

$$\Theta = \mathbf{r}^+\partial_z\mathbf{U}+\bar{\mathbf{r}}^+\partial_{\bar z}\mathbf{U} = \frac{1}{\Lambda}\left(\frac{\partial\mathrm{U}_+}{\partial\mathrm{z}}+\frac{\partial\overline{\mathrm{U}_+}}{\partial\bar{\mathrm{z}}}\right)-2\mathrm{Hu}_3,$$

$$\mathbf{r}^+\partial_z\mathbf{U} = \frac{1}{\Lambda}\frac{\partial\mathrm{U}_+}{\partial\mathrm{z}}-\mathrm{HU}_3,\quad \mathbf{r}^+\partial_{\bar z}\mathbf{U} = \frac{1}{\Lambda}\frac{\partial\mathrm{U}^+}{\partial\bar{\mathrm{z}}}-\mathrm{QU}_3,$$

$$\mathbf{n}\partial_z\mathbf{U}=\partial_z U_3+\frac{1}{2}(\overline{Q}U_+ + H\overline{U}_+),\quad \mathbf{n}\partial_{\bar z}\mathbf{U}=\partial_{\bar z}U_3+\frac{1}{2}(Q\overline{U}_+ + HU_+), \tag{3.6$_1$}$$

$$\mathbf{r}_+ = \mathbf{r}_1+i\mathbf{r}_2,\quad \mathbf{r}^+ = \mathbf{r}^1+i\mathbf{r}^2,\quad U_+ = U_1+iU_2,\quad U^+ = U^1+iU^2,$$

$$\sqrt{a} = \Lambda,\quad \sqrt{g} = \Lambda(1-2Hx_3+K_3^2),\quad \left(\partial_z = \frac{\partial}{\partial z}\right),$$

$$\tau_1^1-\tau_2^2+i(\tau_2^1+\tau_1^2) = \sqrt{\frac{g}{a}}[\sigma_1^1-\sigma_2^2+i(\sigma_2^1+\sigma_1^2)],\ldots,\quad \tau^+ = \sqrt{\frac{g}{a}}(\sigma^{13}+i\sigma^{23}).$$

The displacement vector $\mathbf{U}$, representable in the form

$$\mathbf{U} = \mathrm{U}^\alpha\mathbf{r}_\alpha+\mathrm{U}^3\mathbf{n} = \mathrm{U}_\alpha\mathbf{r}^\alpha+\mathrm{U}_3\mathbf{n} = \mathrm{u}_{(l)}\mathbf{l}+\mathrm{u}_{(s)}\mathbf{s}+\mathrm{u}_3\mathbf{n},\quad (\mathrm{u}_3 = \mathrm{u}^3),$$

can be rewritten as follows:

$$\begin{aligned}\mathbf{U} &= \frac{1}{2}\left(\mathrm{U}^+\bar{\mathbf{r}}_+ + \overline{\mathrm{U}}^+\mathbf{r}_+\right)+\mathrm{U}_3\mathbf{n} = \frac{1}{2}\left(\mathrm{U}_+\bar{\mathbf{r}}^+ + \overline{\mathrm{U}}_+\mathbf{r}^+\right)+\mathrm{U}_3\mathbf{n} = \\ &= -\frac{i}{2}\left[(U_{(l)}+iU_{(s)})\frac{dz}{ds}\bar{\mathbf{r}}_+ - (U_{(l)}-iU_{(s)})\frac{d\bar z}{ds}\mathbf{r}_+\right]+U_3\mathbf{n},\end{aligned} \tag{3.7}$$

where $U_+ = \mathbf{U}\cdot\mathbf{r}_+$, $U^+ = \mathbf{U}\cdot\mathbf{r}^+$, $U_{(\mathbf{l})} = \mathbf{U}\cdot\mathbf{l}$, $U_{(s)} = \mathbf{U}\cdot\mathbf{s}$..

Let us consider the complex writing of formula (2.9) of the type

$$\boldsymbol{\sigma}_{(l)}\frac{d\widehat{s}}{ds} = \sqrt{\frac{g}{a}}\left\{Im\left[(\sigma_{(ll)}+i\sigma_{(ls)})\frac{dz}{ds}\bar{\mathbf{r}}_+\right]+\sigma_{(ln)}\mathbf{n}\right\},$$

where

$$\begin{cases}\sigma_{(ll)}+i\sigma_{(ls)}=\dfrac{1}{2}\left[\sigma^{1\cdot}_{\cdot 1}+\sigma^{2\cdot}_{\cdot 2}+i\sigma^{1\cdot}_{\cdot 2}-i\sigma^{2\cdot}_{\cdot 1}-(\sigma^{1\cdot}_{\cdot 1}-\sigma^{2\cdot}_{\cdot 2}+i\sigma^{1\cdot}_{\cdot 2}+i\sigma^{2\cdot}_{\cdot 1})\dfrac{d\bar z}{dz}\right], \\ \sigma_{(ln)}=-Im\left(\Lambda\sigma^+\dfrac{d\bar z}{ds}\right),\end{cases} \tag{3.8}$$

Below, in solving the boundary value problems in displacements (3.7) and stresses (3.8) the use will be made of the complex representations.

For the spherical shell of radius $R$ we have

$$H=-\frac{1}{R},\quad K=\frac{1}{R^2},\quad Q=0,\quad \Lambda=\frac{4R^2}{(1+z\bar{z})^2}.\quad z=tg\frac{\theta}{2}e^{i\varphi},$$

where $\theta$ and $\varphi$ are the geographic coordinates on the sphere.

The system of equations of equilibrium and the Hook's law for the spherical shell take the form

$$\begin{cases}\frac{1}{\Lambda}\frac{\partial}{\partial z}[\Lambda(\tau_1^1-\tau_2^2+i\tau_2^1+i\tau_1^2)]+\frac{\partial}{\partial\bar{z}}(\tau_1^1+\tau_2^2+i\tau_2^1-i\tau_1^2)+\frac{\Lambda}{R}\tau^+ +\frac{\partial\tau_{(+)}}{\partial x_3}+X_+=0,\\ \frac{1}{\Lambda}\left(\frac{\partial\Lambda\tau^+}{\partial z}+\frac{\partial\Lambda\bar{\tau}^+}{\partial\bar{z}}\right)-\frac{1}{R}(\tau_1^1+\tau_2^2)+\frac{\partial\tau_3^3}{\partial x^3}+X_3=0,\end{cases}$$

where

$$\begin{cases}\tau_1^1-\tau_2^2+i(\tau_2^1+\tau_1^2)=4\mu\mathbf{r}^+\partial_{\bar{z}}\mathbf{U},\\ \tau_1^1+\tau_2^2+i(\tau_2^1-\tau_1^2)=2(\lambda+\mu)\Theta+2\lambda\left(1+\frac{x_3}{R}\right)\partial_3U_3,\\ \tau^+=\frac{2\mu}{\Lambda}\left[\mathbf{n}\partial_{\bar{z}}\mathbf{U}+\frac{1}{2}\left(1+\frac{x_3}{R}\right)\partial_3U_+\right],\\ \tau_{(+)}=2\mu\left[\left(1+\frac{x_3}{R}\right)\mathbf{n}\partial_{\bar{z}}U_++\left(1+\frac{x_3}{R}\right)^2\partial_3U_+\right],\\ \tau_3^3=\left[\lambda\left(1+\frac{x_3}{R}\right)\Theta+(\lambda+2\mu)\left(1+\frac{x_3}{R}\right)^2\partial_3U_3\right],\end{cases}$$

Here

$$\begin{aligned}&\Theta=\mathbf{r}^\alpha\partial_\alpha\mathbf{U}=\mathbf{r}^+\partial_{\mathrm{z}}\mathbf{U}+\overline{\mathbf{r}}^+\partial_{\bar{\mathrm{z}}}\mathbf{U}=\frac{1}{\Lambda}\left(\frac{\partial\mathrm{U}_+}{\partial\mathrm{z}}+\frac{\partial\overline{\mathrm{U}_+}}{\partial\bar{\mathrm{z}}}\right)+\frac{2}{\mathrm{R}}\mathrm{U}_3,\\ &\mathbf{r}^+\partial_{\mathrm{z}}\mathbf{U}=\frac{1}{\Lambda}\frac{\partial\mathrm{U}_+}{\partial\mathrm{z}}+\frac{1}{\mathrm{R}}\mathrm{U}_3,\quad \overline{\mathbf{r}}^+\partial_{\bar{\mathrm{z}}}\mathbf{U}=\partial_{\bar{\mathrm{z}}}\mathrm{U}^+,\\ &\mathbf{n}\partial_z\mathbf{U}=\partial_{\bar{z}}U_3-\frac{1}{2R}U_+,\quad \sqrt{g}=\Lambda\left(1+\frac{x_3}{R}\right)^2,\\ &\tau_j^i=\left(1+\tfrac{x_3}{R}\right)^2\sigma_j^i.\end{aligned}$$

For the circular cylindrical shell of radius $R$ we have

$$\begin{aligned}&H=Q=-\frac{1}{2R}=b_1^1,\quad b_2^2=b_2^1=b_1^2=0,\quad K=0,\quad \Lambda=1\\ &z=x_1+ix_2,\quad \sqrt{g}=1+\tfrac{x_3}{R},\quad a=1,\end{aligned}$$

where $x_1=R\varphi$, and $\varphi$ is the polar angle, $x_2$ is the coordinate along the generatrix.

Now we write out the system of equations and the Hook's law which are obtained from (3.5) and (3.6):

$$\begin{cases} \dfrac{\partial}{\partial z}(\tau_1^1-\tau_2^2+i\tau_1^2+i\tau_2^1)+\dfrac{\partial}{\partial \bar z}(\tau_1^1+\tau_2^2+i\tau_2^1-i\tau_1^2)+\\ \quad +\dfrac{1}{2R}(\tau^+ +\bar\tau_+)+\dfrac{\partial \tau_{(+)}}{\partial x_3}+X_+=0,\\ \dfrac{\partial \tau^+}{\partial z}+\dfrac{\partial \bar\tau^+}{\partial \bar z}-\dfrac{1}{2R}\left[\tau_1^1+\tau_2^2+Re(\tau_1^1-\tau_2^2+i\tau_2^1+i\tau_1^2)\right]+\dfrac{\partial \tau_3^3}{\partial x^3}+X_3=0, \end{cases}$$

where

$$\begin{cases} \tau_1^1-\tau_2^2+i\tau_2^1+i\tau_1^2=4\mu \mathbf{r}^+\partial_{\bar z}\mathbf{U}+\\ \quad +\dfrac{x_3}{R+x_3}\Big\{-\Big(1+\dfrac{x_3}{2R}\Big)\left[(\lambda+\mu)\Theta+\mu\mathbf{r}^+\partial_z\mathbf{U}\right]+\\ \quad +\dfrac{x_3}{2R}\left[(\lambda+\mu)\overline{\mathbf{r}}^+\partial_z\mathbf{U}+(\lambda+3\mu)\mathbf{r}^+\partial_{\bar z}\mathbf{U}\right]\Big\}-\lambda\dfrac{x_3}{R}\lambda\partial_3U_3,\\ \tau_1^1+\tau_2^2+i\tau_2^1-i\tau_1^2=2(\lambda+\mu)\Theta+\dfrac{x_3}{R+x_3}\Big\{-\Big(1+\dfrac{x_3}{2R}\Big)\left[(\lambda+\mu)\mathbf{r}^+\partial_z\mathbf{U}+\right.\\ \quad \left.+(\lambda+3\mu)\mathbf{r}^+\partial_{\bar z}\mathbf{U}\right]+\dfrac{x_3}{2R}\left[(\lambda+\mu)\Theta+2\mu\mathbf{r}^+\partial_z\mathbf{U}\right]\Big\}+2\lambda\Big(1+\dfrac{x_3}{2R}\Big)\partial_3U_3,\\ \tau^+=\mu\Big\{2\mathbf{n}\partial_{\bar z}\mathbf{U}+\dfrac{2x_3}{R+x_3}\Big[\dfrac{x_3}{2R}\mathbf{n}\partial_{\bar z}\mathbf{U}-\Big(1+\dfrac{x_3}{R}\Big)\mathbf{n}\partial_z\mathbf{U}\Big]+\\ +\left(1+\frac{x_3}{2R}\right)\partial_3U_+-\frac{x_3}{2R}\partial_3\bar U_+\Big\},\\ \tau_{(+)}=\mu\Big[\Big(1+\dfrac{x_3}{2R}\Big)\mathbf{n}\partial_{\bar z}\mathbf{U}-\dfrac{x_3}{2R}\mathbf{n}\partial_z\mathbf{U}+\Big(1+\dfrac{x_3}{R}\Big)\partial_3U_+\Big],\\ \tau_3^3=\lambda\Big[\Big(1+\dfrac{x_3}{2R}\Big)\Theta-\dfrac{x_3}{2R}[\overline{\mathbf{r}}^+\partial\mathbf{U}+\mathbf{r}^+\partial_{\bar z}\mathbf{U}]\Big]+(\lambda+2\mu)\Big(1+\dfrac{x_3}{R}\Big)\partial_3U_3, \end{cases}$$

Note that

$$\begin{aligned} &\Theta=\partial_zU_++\partial_{\bar z}\overline{U}_++\frac{1}{R}U_3,\ \ \mathbf{r}^+\partial_z\mathbf{U}=\partial \mathrm{U}_++\frac{1}{2\mathrm{R}}\mathrm{U}_3,\\ &\mathbf{r}^+\partial_{\bar z}\mathbf{U}=\partial_{\bar z}\mathrm{U}^++\tfrac{1}{2\mathrm{R}}\mathrm{U}_3,\ \ \mathbf{n}\partial_z\mathbf{U}=\partial_{\bar z}\mathrm{U}_3-\tfrac{1}{4\mathrm{R}}(\mathrm{U}_3+\overline{\mathrm{U}}_3),\\ &\tau_j^i=\left(1+\tfrac{x_3}{R}\right)^2\sigma_j^i,\ \ (U_+=U^+). \end{aligned}$$

## 4. I. N. Vekua's Method of Reduction

There are many different methods of passage (reduction) from three-dimensional problems of elasticity to two-dimensional problems of the theory of shells (Kirchgoff-Love, E. Reissner, A. Green, A. I. Lur'e, V. Z. Vlasov, W. Koiter, R. Naghdi, A. L. Goldenveiser, I. I. Vorovich, I. N. Vekua, etc.).

In the present paper we realize reduction of three-dimensional problems of the theory of elasticity to the two-dimensional ones by the method suggested by I. N. Vekua the essence of which consists, without going into details, in the following [1]. Since the system of Legendre polynomials $\left\{P_m\left(\frac{x_3}{h}\right)\right\}$ is complete in the interval $[-h,\ h]$,for equation (2.1) we obtain the equivalent infinite system of two-dimensional equations

$$\int_{-h}^{h}\left[\frac{1}{\sqrt{a}}\frac{\partial\sqrt{a}\left(\sqrt{\frac{g}{a}}\sigma^{\alpha}\right)}{\partial x^{\alpha}}+\frac{\partial\sqrt{\frac{g}{a}}\sigma^{3}}{\partial x_3}+\sqrt{\frac{g}{a}}\boldsymbol{\Phi}\right]P_m\left(\frac{x_3}{h}\right)dx_3=0,\quad (m=0,1,\dots)$$

or in a finite form

$$\nabla_{\alpha}\overset{(m)}{\boldsymbol{\sigma}}{}^{\alpha}-\frac{2m+1}{h}\left(\overset{(m-1)}{\boldsymbol{\sigma}}{}^{\alpha}+\overset{(m-3)}{\boldsymbol{\sigma}}{}^{\alpha}+\cdots\right)+\overset{(m)}{\boldsymbol{F}}=0, \tag{4.1}$$

where

$$\left(\overset{(m)}{\boldsymbol{\sigma}}{}^{i},\ \overset{(m)}{\boldsymbol{\Phi}}\right)=\frac{2m+1}{2h}\int_{-h}^{h}\left(\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^{i},\ \sqrt{\frac{g}{a}}\boldsymbol{\Phi}\right)P_m\left(\frac{x_3}{h}\right)dx_3,\quad (i=1,2,3)$$

$$\overset{(m)}{\boldsymbol{F}}=\overset{(m)}{\boldsymbol{\Phi}}+\frac{2m+1}{2h}\left[\sqrt{\frac{g_+}{a}}\overset{(+)}{\boldsymbol{\sigma}}{}^{3}-(-1)^m\sqrt{\frac{g_-}{a}}\overset{(-)}{\boldsymbol{\sigma}}{}^{3}\right],$$

$$\sqrt{\frac{g_{\pm}}{a}}=1\mp 2H+Kh^2.$$

Thus we have obtained the infinite system of two dimensional equation of the theory of shells for which the boundary conditions on the face surface ($x_3=\pm h$) are satisfied, i.e. $\overset{(\pm)}{\boldsymbol{\sigma}}_3=\boldsymbol{\sigma}^3(x^1,\ x^2,\pm)$ is the preassigned vector field.

For the Hook's law we have [6]

$$\begin{cases}\overset{(m)\alpha}{\boldsymbol{\sigma}}=\sum\limits_{s=0}^{\infty}\left[\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1}\left(\overset{(m,s)}{C}{}^{\alpha_1\gamma_1}\partial_{\gamma}\overset{(s)}{\boldsymbol{U}}\right)+\frac{1}{h}\overset{(m,s)}{I}{}^{\alpha 3}_{\alpha_1 3}\overset{(m,s)}{C}{}^{\alpha_1 3}\overset{(s)}{\boldsymbol{U}}{}'\right],\\ \overset{(m)}{\boldsymbol{\sigma}}_3=\sum\limits_{s=0}^{\infty}\left[\overset{(m,s)}{I}{}^{3\gamma}_{3\gamma_1}\overset{(m,s)}{C}{}^{3\gamma_1}\partial_{\gamma}\overset{(s)}{\boldsymbol{U}}+\frac{1}{h}\overset{(m,s)}{I}{}^{33}_{33}\overset{(m,s)}{C}{}^{33}\overset{(s)}{\boldsymbol{U}}{}'\right],\end{cases} \tag{4.2}$$

where

$$\overset{(s)}{\boldsymbol{U}}{}'=\frac{2s+1}{2h}\int_{-h}^{h}\mathbf{U}P_m\left(\frac{x_3}{h}\right)dx_3,\quad \overset{(s)}{\boldsymbol{U}}{}'=(2s+1)\left(\overset{(s+1)}{\boldsymbol{U}}+\overset{(s+3)}{\boldsymbol{U}}+\cdots\right), \tag{4.3}$$

$$\overset{(m,s)}{I}{}^{ij}_{i_1j_1}=\frac{2s+1}{2h}\int_{-h}^{h}\sqrt{\frac{g}{a}}A^{i\cdot}_{\cdot i_1}A^{j\cdot}_{\cdot j_1}P_m\left(\frac{x_3}{h}\right)P_s\left(\frac{x_3}{h}\right)dx_3, \tag{4.4}$$

$$(i,\ i_1,\ j,j_1=1,2,3).$$

By the relations (2.10) and (2.11) we have

$$\frac{2m+1}{2h}\int_{-h}^{h}\boldsymbol{\sigma}_{(\hat{l})}\frac{d\widehat{s}}{ds}P_m\left(\frac{x_3}{h}\right)dx_3=\overset{(m)}{\boldsymbol{\sigma}}{}^{\alpha}l_{\alpha}=$$

$$= \overset{(m)}{\sigma}_{(ll)}\mathbf{l} + \overset{(m)}{\sigma}_{(ls)}\mathbf{s} + \overset{(m)}{\sigma}_{(ln)}\mathbf{n} = \overset{(m)}{\boldsymbol{\sigma}}_{(l)}. \tag{4.5}$$

$$\overset{(m)}{\boldsymbol{U}} = \overset{(m)}{U}_\alpha \mathbf{r}^\alpha + \overset{(m)}{U}_3\mathbf{n} = \overset{(m)}{U}{}^\alpha \mathbf{r}^\alpha + \overset{(m)}{U}_3\mathbf{n} =$$

$$= \overset{(m)}{U}_{(l)}\mathbf{l} + \overset{(m)}{U}_{(s)}\mathbf{s} + \overset{(m)}{U}_3\mathbf{n}, \quad (m = 0, 1, 2, \ldots). \tag{4.6}$$

Thus we have constructed an infinite system of two-dimensional equations of non-shallow shells (4.1), (4.2) which is consistent with the boundary conditions on the face surfaces $x_3 = \pm h$, where the stresses $\overset{(+)}{\boldsymbol{\sigma}}{}^3$ and $\overset{(-)}{\boldsymbol{\sigma}}{}^3$ are, as usual, assumed to be given. As for the boundary conditions on the side surfaces, they can be satisfied by means of the relations (4.5) and (4.6). This system is more preferable, because it involves two independent variables, Gaussian parameters $x^1$, and $x^2$ on the surface $S$. But decrease of a number of independent variables by unity is achieved by increasing a number of equations ad infinitum, but this is surely connected with great difficulties. The passage to the finite Vekua's system can be realized by various methods one of which consists in considering of a finite segment in the Fourier-Legendre series, i.e.

$$\left(\sqrt{\frac{g}{a}}\boldsymbol{\sigma}^i,\ \mathbf{U},\ \sqrt{\frac{g}{a}}\boldsymbol{\Phi}\right) = \sum_{m=0}^{N}\left(\sqrt{\frac{g}{a}}\overset{(m)}{\boldsymbol{\sigma}}{}^i,\ \overset{(m)}{\boldsymbol{U}},\ \overset{(m)}{\boldsymbol{\Phi}}\right) P_m\left(\frac{x_3}{h}\right), \tag{4.7}$$

where $N$ is a fixed nonnegative number. In other words, in the previous equations it is assumed that

$$\overset{(k)}{\boldsymbol{U}} = 0, \quad \overset{(k)}{\boldsymbol{\sigma}}{}^i = 0, \quad k > N.$$

In what follows, approximation of such a type will be called approximations of order $N$.

The second difficulty (not less important) consists in that integrals of the type (4.4) should be calculated explicitly:

$$\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1} = \frac{2s+1}{2h}\int_{-h}^{h} \frac{\left[a^\alpha_{\alpha_1} + x_3(b^\alpha_{\alpha_1} - 2Ha^\alpha_{\alpha_1})\right]\left[a^\gamma_{\gamma_1} + x_3(b^\gamma_{\gamma_1} - 2Ha^\gamma_{\gamma_1})\right]}{1 - 2Hx_3 + Kx_3^2} \times$$
$$\times P_m\left(\frac{x_3}{h}\right) P_s\left(\frac{x_3}{h}\right) dx_3,$$

$$\overset{(m,s)}{I}{}^{\alpha 3}_{\alpha_1 3} = \frac{2s+1}{2h}\int_{-h}^{h}\left[a^\alpha_{\alpha_1} + x_3(b^\alpha_{\alpha_1} - 2Ha^\alpha_{\alpha_1})\right] P_m\left(\frac{x_3}{h}\right) P_s\left(\frac{x_3}{h}\right) dx_3,$$

$$\overset{(m,s)}{I}{}^{33}_{33} = \frac{2s+1}{2h}\int_{-h}^{h}(1 - 2Hx_3 + Kx_3^2) P_m\left(\frac{x_3}{h}\right) P_s\left(\frac{x_3}{h}\right) dx_3.$$

By F. Neumann's [5] and J. Adams [6] formulas

$$\int_{-1}^{1} \frac{P_m(y)dy}{x-y} = 2Q_m(x), \quad |x| > 1,$$

and

$$P_m(x)P_s(x) = \sum_{r=0}^{min(m,s)} \alpha_{msr} P_{m+s-2r}(x),$$

respectively, where

$$\alpha_{msr} = \frac{A_{m-r}A_rA_{s-r}}{A_{m+s-r}} \frac{2m+2s-4r+1}{2m+2s-2r+1}, \quad A_m = \frac{1\cdot 3\cdots(2m-1)}{m!},$$

it is not difficult to obtain expressions of these integrals explicitly [6]:

$$\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1} = \begin{cases} \frac{(b^\alpha_{\alpha_1} - 2Ha^\alpha_{\alpha_1})(b^\gamma_{\gamma_1} - 2Ha^\gamma_{\gamma_1})}{K}\delta^{ms} + \\ + \frac{2m+1}{2h\sqrt{E}}\left[B^\alpha_{\alpha_1}(y)B^\gamma_{\gamma_1}(y)\begin{pmatrix} P_m(y)Q_s(y), & m \le s \\ P_s(y)Q_m(y), & m > s \end{pmatrix}\right]_{y_1}^{y_2}, \\ (E = H^2 - K \ne 0, \quad K \ne 0) \\ a^{\alpha\gamma}_{\alpha_1\gamma_1}\delta^{ms}, \quad E = H^2 - K = 0, \quad K \ne 0, \end{cases} \tag{4.8}$$

$$\overset{(m,s)}{I}{}^{\alpha 3}_{\alpha_1 3} = a^\alpha_{\alpha_1}\delta^{ms} +$$

$$+ h(b^\alpha_{\alpha_1} - 2Ha^\alpha_{\alpha_1})\left(\frac{m}{2m-1}\delta^{s+1}_m + \frac{m+1}{2m+3}\delta^{s-1}_m\right) = \overset{(m,s)}{I}{}^{3\alpha}_{3\alpha_1}, \tag{4.8$_1$}$$

$$\overset{(m,s)}{I}{}^{33}_{33} = \delta^{ms} - 2Hh\left(\frac{m}{2m-1}\delta^{s+1}_m + \frac{m+1}{2m+3}\delta^{s-1}_m\right) +$$

$$+ K\left[\frac{m(m-1)}{(2m-1)(2m-3)}\delta^{s+2}_m + \frac{1}{2m+1}\left(\frac{(m+1)^2}{2m+3} + \right.\right. \tag{4.8$_2$}$$

$$\left.\left. + \frac{m^2}{2m-1}\right)\delta^{ms} + \frac{(m+1)(m+2)}{(2m+3)(2m+5)}\delta^{s-2}_m\right]h^2.$$

where $\delta^{ms} = \delta^m_s$ is the Kronecker symbol, $E = H^2 - K$ is the Euler difference, $Q_s$ is the Legender function of second order, $B^\alpha_{\alpha_1}(y) = (a^\alpha_{\alpha_1} + hy(b^\alpha_{\alpha_1} - 2Ha^\alpha_{\alpha_1})$. The essence of the square brackets consists in the following:

$$[f(y)]_{y_1}^{y_2} = f(y_2) - f(y_1), \quad y_{1,2} = [(H \mp \sqrt{E})h]^{-1}.$$

These integrals take more simple form in the coordinate system in lines of curvature. Taking into account that in this case

$$A^\alpha_{\alpha_1} = \begin{cases} 0, & \alpha \ne \alpha_1, \\ \frac{1}{1-k_\alpha x_3}, & \alpha = \alpha_1, \end{cases}$$

we have

$$\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1}=\frac{2m+1}{2h}\int\limits_{-h}^{h}\sqrt{\frac{g}{a}}A^{\alpha}_{\alpha_1}A^{\gamma}_{\gamma_1}P_mP_s dx_3=$$

$$=\begin{cases}0, \ \ \alpha\neq\alpha_1, \ \ \gamma\neq\gamma_1,\\ \delta^{ms}, \ \ \alpha=\alpha_1\neq\gamma=\gamma_1,\\ \dfrac{k_{3-\alpha}}{k_\alpha}\delta^{ms}+\dfrac{2m+1}{hk_\alpha}\dfrac{k_\alpha-k_{3-\alpha}}{k_\alpha}\left[P_m\left(\dfrac{1}{hk_\alpha}\right)Q_s\left(\dfrac{1}{hk_\alpha}\right)\right],\\ \qquad \alpha=\alpha_1=\gamma=\gamma_1, \ \ k_\alpha\neq 0.\end{cases}$$

If the Gaussian curvature is equal to zero, i.e. $K=k_1k_2=0$ (plates, cylindrical and conical shells) then the above integrals take the form

$$\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1}=\begin{cases}0, \ \ \alpha\neq\alpha_1, \ \ \gamma\neq\gamma_1,\\ \delta^{ms}, \ \ \alpha=\alpha_1\neq\gamma=\gamma_1,\\ \frac{2m+1}{hk_\alpha}\frac{k_\alpha-k_{3-\alpha}}{k_\alpha}\left[P_m\left(\frac{1}{hk_\alpha}\right)Q_s\left(\frac{1}{hk_\alpha}\right)\right],\\ \qquad \alpha=\alpha_1=\gamma=\gamma_1, \ \ k_\alpha\neq 0, \ \ k_{3-\alpha}=0.\end{cases}$$

An explicit expression for the product $P_m(y)Q_s(y)$ has the form [6]

$$P_m(y)Q_s(y)=\sum_{r=0}^{\left[\frac{m}{2}\right]}\sum_{p=0}^{\infty}\overset{(m,s)}{M}_{rp}\frac{1}{y^{s-m+2(r+p)+1}},$$

where

$$\overset{(m,s)}{M}_{rp}=2^{s-m}\frac{(-1)^r}{r!}\frac{(2m-2r)!}{(m-r)!(m-2r)!}\frac{(s+p)!(s+2p)!}{p!(2s+2p+1)!} \tag{4.8$_3$}$$

Therefore the integrals of the type $\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1}$ can be likewise represented as

$$\overset{(m,s)}{I}{}^{\alpha\gamma}_{\alpha_1\gamma_1}=\frac{2m+1}{2h\sqrt{E}}\sum_{r=0}^{\left[\frac{m}{2}\right]}\sum_{p=0}^{\infty}\overset{(m,s)}{M}_{rp}\times$$
$$\times\Bigg[\left(a^{\alpha}_{\alpha_1}+\frac{(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1})}{H+\sqrt{E}}\right)\left(a^{\gamma}_{\gamma_1}+\frac{(b^{\gamma}_{\gamma_1}-2Ha^{\gamma}_{\gamma_1})}{H+\sqrt{E}}\right)\times$$
$$\times((H+\sqrt{E})h)^{s-m+2(r+p)+1}- \tag{4.8$_4$}$$
$$-\left(a^{\alpha}_{\alpha_1}+\frac{(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1})}{H-\sqrt{E}}\right)\left(a^{\gamma}_{\gamma_1}+\frac{(b^{\gamma}_{\gamma_1}-2Ha^{\gamma}_{\gamma_1})}{H-\sqrt{E}}\right)\times$$
$$\times((H-\sqrt{E})h)^{s-m+2(r+p)+1}\Bigg]+\frac{(b^{\alpha}_{\alpha_1}-2Ha^{\alpha}_{\alpha_1})(b^{\gamma}_{\gamma_1}-2Ha^{\gamma}_{\gamma_1})}{K}\delta_{ms}.$$

Thus for the non-shallow elastic shells we have obtained a finite system of two-dimensional equations (approximation of order $N$) which by virtue of (4.1) and (4.2) looks in a tensor form as

$$\begin{cases} \nabla_\alpha \overset{(m)}{\sigma}{}^{\alpha\beta} - b_\alpha^\beta - \frac{2m+1}{h}\Big(\overset{(m-1)}{\sigma}{}^{3\beta} + \overset{(m-3)}{\sigma}{}^{3\beta} + \cdots\Big) + \overset{(m)}{F}{}^\beta = 0, \quad (\beta = 1,2) \\ \nabla_\alpha \overset{(m)}{\sigma}{}^{\alpha 3} + b_\alpha^\beta - \frac{2m+1}{h}\Big(\overset{(m-1)}{\sigma}{}^{33} + \overset{(m-3)}{\sigma}{}^{33} + \cdots\Big) + \overset{(m)}{F}{}^3 = 0, \end{cases} \tag{4.9}$$

$$(m = 0, 1, \ldots, N)$$

where

$$\begin{aligned} \overset{(m)}{\sigma}{}^{\alpha\beta} &= \sum_{s=0}^{N}\Bigg[\overset{(m,s)}{I}{}_{\alpha_1\gamma_1}^{\alpha\gamma}\Big(\overset{(m,s)}{C}{}^{\alpha_1\gamma_1}\nabla_\gamma \overset{(s)}{\boldsymbol{U}}\Big)\mathbf{r}^\beta + \frac{1}{h}\overset{(m,s)}{I}{}_{\alpha_1 3}^{\alpha 3}\Big(\overset{(m,s)}{C}{}^{\alpha_1 3}\overset{(s)}{\boldsymbol{U}}{}'\Big)\mathbf{r}^\beta\Bigg], \\ \overset{(m)}{\sigma}{}^{\alpha 3} &= \sum_{s=0}^{N}\Bigg[\overset{(m,s)}{I}{}_{\alpha_1\gamma_1}^{\alpha\gamma}\Big(\overset{(m,s)}{C}{}^{\alpha_1\gamma_1}\nabla_\gamma \overset{(s)}{\boldsymbol{U}}\Big)\mathbf{n} + \frac{1}{h}\overset{(m,s)}{I}{}_{\alpha_1 3}^{\alpha 3}\Big(\overset{(m,s)}{C}{}^{\alpha_1 3}\overset{(s)}{\boldsymbol{U}}{}'\Big)\mathbf{n}\Bigg], \\ \overset{(m)}{\sigma}{}^{3\beta} &= \sum_{s=0}^{N}\Bigg[\overset{(m,s)}{I}{}_{3\gamma_1}^{3\gamma}\Big(\overset{(m,s)}{C}{}^{3\gamma_1}\nabla_\gamma \overset{(s)}{\boldsymbol{U}}\Big)\mathbf{r}^\beta + \frac{1}{h}\overset{(m,s)}{I}{}_{33}^{33}\Big(\overset{(m,s)}{C}{}^{33}\overset{(s)}{\boldsymbol{U}}{}'\Big)\mathbf{r}^\beta\Bigg], \\ \overset{(m)}{\sigma}{}^{33} &= \sum_{s=0}^{N}\Bigg[\overset{(m,s)}{I}{}_{3\gamma_1}^{3\gamma}\Big(\overset{(m,s)}{C}{}^{3\gamma_1}\nabla_\gamma \overset{(s)}{\boldsymbol{U}}\Big)\mathbf{n} + \frac{1}{h}\overset{(m,s)}{I}{}_{33}^{33}\Big(\overset{(m,s)}{C}{}^{33}\overset{(s)}{\boldsymbol{U}}{}'\Big)\mathbf{n}\Bigg], \end{aligned} \tag{4.10}$$

Here,

$$\begin{gathered} \overset{(m)}{F}{}^i = \overset{(m)}{\Phi}{}^i + \frac{2m+1}{2h}\left[\sqrt{\frac{g_+}{a}}\,\overset{(+)}{\sigma}{}^{3i} - (-1)^m\sqrt{\frac{g_-}{a}}\,\overset{(-)}{\sigma}{}^{3i}\right], \\ \nabla_\alpha \overset{(s)}{\boldsymbol{U}} = \partial_\alpha \overset{(s)}{\boldsymbol{U}} = \Big(\nabla_\alpha \overset{(s)}{U}{}^\beta - b_\alpha^\beta \nabla_\alpha \overset{(s)}{U}_3\Big)\mathbf{r}_\beta + \Big(\nabla_\alpha \overset{(s)}{U}_3 - b_\alpha^\beta \nabla_\alpha \overset{(s)}{U}_\beta\Big)\mathbf{n}, \\ \overset{(s)}{\boldsymbol{U}}{}' = (2s+1)\Big(\overset{(s+1)}{\boldsymbol{U}} + \overset{(s+3)}{\boldsymbol{U}} + \cdots\Big), \quad (s = 0, 1, 2, \ldots, N). \end{gathered} \tag{4.11}$$

The boundary conditions which should be added to the system of equations (4.9), (4.10) can be obtained from the relations (4.5) and (4.6) as follows.

Let $D$ be the domain belonging to the plane $Ox^1x^2$ onto which we topologically map the midsurface $S$ with the boundary contour $\Gamma$. Let $L$ be the boundary of the domain $D$. It is assumed that between the points of $\Gamma$ and $L$ there exists the one-to-one continuous correspondence. Then the boundary conditions in stresses take the form

$$\begin{gathered} \overset{(m)}{\sigma}{}_{(ll)} = \overset{(m)}{\sigma}{}^{\alpha\beta} l_\alpha l_\beta = \overset{(m)}{f}{}_{(ll)}, \quad \overset{(m)}{\sigma}{}_{(ls)} = \overset{(m)}{\sigma}{}^{\alpha\beta} l_\alpha s_\beta = \overset{(m)}{f}{}_{(ls)}, \\ \overset{(m)}{\sigma}{}_{(ln)} = \overset{(m)}{\sigma}{}^{\alpha 3} l_\alpha = \overset{(m)}{f}{}_3, \quad (L) \end{gathered} \tag{4.12}$$

where $\overset{(m)}{f}{}_{(ll)}$, $\overset{(m)}{f}{}_{(ls)}$, $\overset{(m)}{f}{}_3$ are the given functions of points of the curve $L$.

Similarly, for the displacement we have

$$\overset{(m)}{U}_{(l)}=\overset{(m)}{U}{}^{\alpha}l_{\alpha}=\overset{(m)}{g}_{(l)},\quad \overset{(m)}{U}_{(s)}=\overset{(m)}{U}{}^{\alpha}s_{\alpha}=\overset{(m)}{g}_{(s)},\quad \overset{(m)}{U}_{3}=\overset{(m)}{g}_{3},\ (L) \quad (4.13)$$

where $\overset{(m)}{g}_{(l)}$, $\overset{(m)}{g}_{(s)}$, $\overset{(m)}{g}_{3}$ are the given functions on $L$.

## 5. Introduction of a Small Parameter

Three-dimensional shell-like bodies are characterized by the inequalities of the type

$$|hb^{\alpha}_{\beta}|\leq q<1,\ \ (\alpha,\ \beta=1,2).$$

Therefore they can be represented as follows:

$$|\varepsilon hb^{\alpha}_{\beta}R|\leq q<1,\ \ (\alpha,\ \beta=1).$$

where $\varepsilon$ is a small parameter which is expressed in the form

$$\varepsilon=\frac{h}{R}.$$

Here $h$ is semi-thickness of the shell, $R$ is an arbitrary characteristic radius of curvature of the midsurface $S$.

Having introduced a small parameter, we represent the system of equations (4.9), (4.10) of approximation of order $N$ in a complex form:

$$\begin{gathered}
\frac{h}{\Lambda}\frac{\partial}{\partial z}\Big(\overset{(m)}{\sigma}_{11}-\overset{(m)}{\sigma}_{22}+i\overset{(m)}{\sigma}_{12}+i\overset{(m)}{\sigma}_{21}\Big)+\\
+h\frac{\partial}{\partial\bar z}\Big(\overset{(m)}{\sigma}{}^{1}_{1}+\overset{(m)}{\sigma}{}^{2}_{2}+i\overset{(m)}{\sigma}{}^{1}_{2}-i\overset{(m)}{\sigma}{}^{2}_{1}\Big)-\\
-\varepsilon(H\overset{(m)}{\sigma}_{+}+Q\overline{\overset{(m)}{\sigma}_{+}})R-(2m+1)\Big(\overset{(m-1)}{\sigma}_{(+)}+\overset{(m-3)}{\sigma}_{(+)}+\cdots\Big)+h\overset{(m)}{F}_{+}=0,\\
\frac{h}{\Lambda}\Big(\frac{\partial\overset{(m)}{\sigma}_{+}}{\partial z}+\frac{\partial\overline{\overset{(m)}{\sigma}_{+}}}{\partial\bar z}\Big)+\\
+\varepsilon\Big[H\overset{(m)}{\sigma}{}^{\alpha}_{\alpha}+Re\Big(\overline{Q}\Big(\overset{(m)}{\sigma}{}^{1}_{1}-\overset{(m)}{\sigma}{}^{2}_{2}+i\overset{(m)}{\sigma}{}^{2}_{1}+i\overset{(m)}{\sigma}{}^{1}_{2}\Big)\Big)\Big]R-\\
-(2m+1)\Big(\overset{(m-1)}{\sigma}{}^{3}_{3}+\overset{(m-3)}{\sigma}{}^{3}_{3}+\cdots\Big)+h\overset{(m)}{F}_{3}=0,\\
\Big(m=0,1,\dots,N,\ \ \overset{(m)}{F}_{+}=\overset{(m)}{F}_{1}+i\overset{(m)}{F}_{2}\Big)
\end{gathered}\tag{5.1}$$

where

$$\begin{gathered}
h\Big(\overset{(m)}{\sigma}_{11}-\overset{(m)}{\sigma}_{22}+i\overset{(m)}{\sigma}_{12}+i\overset{(m)}{\sigma}_{21}\Big)=4\mu\Lambda\Big(h\frac{\partial\overset{(m)}{U}_{+}}{\partial\bar z}-\varepsilon QR\overset{(m)}{U}_{3}\Big)+\\
+2\Lambda\sum_{s=0}^{\infty}\Big\{\Big(\overset{(m,s)}{I}_{1}-H\overset{(m,s)}{I}_{2}\Big)Q\Big[(\lambda+\mu)(h\overset{(s)}{\Theta}-2H\varepsilon\overset{(s)}{U}_{3}R)+
\end{gathered}$$

$$+2\mu\Big(\frac{h}{\Lambda}\frac{\partial \overset{(s)}{U}_+}{\partial z}-\varepsilon H\overset{(s)}{U}_3 R\Big)\Big]+\overset{(m,s)}{I_2}Q\Big[(\lambda+\mu)Q\Big(h\frac{\partial\overline{\overset{(s)}{U}_+}}{\partial z}-\varepsilon\overline{Q}\overset{(s)}{U}_3R\Big)+$$

$$+(\lambda+3\mu)\overline{Q}\Big(h\frac{\partial\overset{(s)}{U}{}^+}{\partial\bar z}-\varepsilon Q\overset{(s)}{U}_3\Big)\Big]+2\lambda\overset{(m,s)}{I_3}Q\overset{(s)}{U}_3{}'\Big\},\tag{5.2}$$

$$h\Big(\overset{(m)}{\sigma}{}_1^1+\overset{(m)}{\sigma}{}_2^2+i\overset{(m)}{\sigma}{}_1^2-i\overset{(m)}{\sigma}{}_2^1\Big)=2(\lambda+\mu)(h\overset{(m)}{\Theta}-2H\varepsilon\overset{(m)}{U}_3R)+$$

$$+2\sum_{s=0}^{\infty}\Big\{\overset{(m,s)}{I}{}_1-H\overset{(m,s)}{I}{}_2\Big[(\lambda+3\mu)\overline{Q}\Big(h\frac{\partial\overset{(s)}{U}{}^+}{\partial\bar z}-\varepsilon Q\overset{(s)}{U}_3R\Big)+$$

$$+(\lambda+\mu)Q\Big(h\frac{\partial\overline{\overset{(s)}{U}{}^+}}{\partial z}-\varepsilon\overline{Q}\overset{(s)}{U}_3R\Big)\Big]+$$

$$+\overset{(m,s)}{I}{}_2Q\overline{Q}\Big[(\lambda+\mu)(h\overset{(s)}{\Theta}-2H\varepsilon\overset{(s)}{U}_3R)+$$

$$+2\mu\Big(\frac{h}{\Lambda}\frac{\partial\overset{(s)}{U}{}^+}{\partial\bar z}-\varepsilon H\overset{(s)}{U}_3R\Big)\Big]+2\lambda(\delta_{ms}-H\overset{(m,s)}{I}{}_3)\overset{(s)}{U}_3{}'\Big\},$$

$$h\overset{(m)}{\sigma}{}_+=2\mu\Big(h\frac{\partial\overset{(m)}{U}_3}{\partial\bar z}+\varepsilon\frac{H\overset{(m)}{U}_++Q\overline{\overset{(m)}{U}_+}}{2}R\Big)+$$

$$+\mu\sum_{s=0}^{\infty}\Big\{\overset{(m,s)}{I}{}_2Q\overline{Q}\Big[2h\frac{\partial\overset{(s)}{U}_3}{\partial\bar z}+\varepsilon(H\overset{(s)}{U}_++Q\overline{\overset{(s)}{U}_+})R\Big]+$$

$$+(\overset{(m,s)}{I}{}_1-H\overset{(m,s)}{I}{}_2)Q\Big[2h\frac{\partial\overset{(s)}{U}_3+}{\partial z}+\varepsilon(\overline{Q}\overset{(s)}{U}_++H\overline{\overset{(s)}{U}_+})R\Big]+$$

$$+\overset{(m)}{U}_+{}'\delta^{ms}-(H\overset{(s)}{U}_+{}'+Q\overline{\overset{(s)}{U}_+{}'})\overset{(m,s)}{I}{}_3\Big\},$$

$$h\overset{(m)}{\sigma}{}_{(+)}=2\mu\Big(h\frac{\partial\overset{(m)}{U}_3}{\partial\bar z}+\varepsilon\frac{H\overset{(m)}{U}_++Q\overline{\overset{(m)}{U}_+}}{2}R\Big)+$$

$$+\mu\sum_{s=0}^{\infty}\Big\{\overset{(m,s)}{I}{}_3\Big[Q\Big(h\frac{\partial\overset{(s)}{U}_3}{\partial\bar z}+\varepsilon\frac{\overline{Q}\overset{(s)}{U}_++H\overline{\overset{(s)}{U}_+}}{2}R\Big)-$$

$$-H\Big(h\frac{\partial\overset{(s)}{U}_3}{\partial\bar z}+\varepsilon\frac{H\overset{(s)}{U}_++Q\overline{\overset{(s)}{U}_+}}{2}R\Big)\Big]+\overset{(m,s)}{I}{}_4\overset{(s)}{U}_+{}'\Big\},$$

$$h\overset{(m)}{\sigma}{}_3^3=\lambda(h\overset{(m)}{\Theta}-2H\varepsilon\overset{(m)}{U}_3R)+\sum_{s=0}^{\infty}\Big\{\lambda\Big[Q\Big(h\frac{\partial\overline{\overset{(s)}{U}{}^+}}{\partial z}-\varepsilon\overline{Q}R\overset{(s)}{U}_3\Big)+$$

$$+\overline{Q}\Big(h\frac{\partial \overset{(s)}{U}_+}{\partial\bar z}-\varepsilon QR\overset{(s)}{U_3}\Big)-H(h\overset{(s)}{\Theta}-2H\varepsilon\overset{(s)}{U_3}R)\Big]\overset{(m,s)}{I}_3+$$

$$+(\lambda+2\mu)\overset{(m,s)}{I}_4\overset{(s)}{U_3}{}'\Big\},\quad (m=0,1,\dots,N).$$

Here we have introduced the notation

$$\overset{(n)}{\Theta}=\frac{1}{\Lambda}\left(\frac{\partial\overset{(n)}{U}_+}{\partial z}+\overline{\frac{\partial\overset{(n)}{U}_+}{\partial\bar z}}\right),\quad \overset{(s)}{U}{}'_i=(2s+1)(\overset{(s+1)}{U}_i+\overset{(s+3)}{U}_i+\cdots),$$

$$\begin{aligned}
\overset{(m,s)}{I}_1&=\frac{2m+1}{2h}\int_{-h}^{h}\frac{x_3P_m\left(\frac{x_3}{h}\right)P_s\left(\frac{x_3}{h}\right)dx_3}{1-2Hx_3+Kx_3^2},\\
\overset{(m,s)}{I}_2&=\frac{2m+1}{2h}\int_{-h}^{h}\frac{x_3^2P_m\left(\frac{x_3}{h}\right)P_s\left(\frac{x_3}{h}\right)dx_3}{1-2Hx_3+Kx_3^2},\\
\overset{(m,s)}{I}_3&=\frac{2m+1}{2h}\int_{-h}^{h}x_3P_m\left(\frac{x_3}{h}\right)P_s\left(\frac{x_3}{h}\right)dx_3,\\
\overset{(m,s)}{I}_4&=\frac{2m+1}{2h}\int_{-h}^{h}(1-2Hx_3+Kx_3^2)P_mP_sdx_3.
\end{aligned}\tag{5.3}$$

The above integrals can be calculated explicitly (see [4.8$_3$]), and their expressions are represented as

$$\overset{(m,s)}{I}_1=\frac{2m+1}{2\sqrt{E}}\sum_{r=0}^{\left[\frac{m}{2}\right]}\sum_{p=0}^{\infty}\overset{(m,s)}{M}_{rp}\varepsilon^{s-m+2(r+p)}\times$$

$$\times\Big[((H+\sqrt{E})h)^{s-m+2(r+p)}-((H-\sqrt{E})h)^{s-m+2(r+p)}\Big],$$

$$\overset{(m,s)}{I}_2=\frac{2m+1}{2\sqrt{E}}h\sum_{r=0}^{\left[\frac{m}{2}\right]}\sum_{p=0}^{\infty}\overset{(m,s)}{M}_{rp}\varepsilon^{s-m+2(r+p)-1}\times$$

$$\times\Big[((H+\sqrt{E})h)^{s-m+2(r+p)-1}-((H-\sqrt{E})h)^{s-m+2(r+p)-1}\Big]+\frac{1}{K}\delta^{ms},$$

$$\overset{(m,s)}{I}_3=h\left(\frac{m}{2m-1}\delta_m^{s+1}+\frac{m+1}{2m+3}\delta_m^{s-1}\right),$$

$$\overset{(m,s)}{I}_4=\delta^{ms}-2HR\varepsilon\left(\frac{m}{2m-1}\delta_m^{s+1}+\frac{m+1}{2m+3}\delta_m^{s-1}\right)+$$

$$+\varepsilon^2 KR^2\left[\frac{m(m-1)}{(2m-1)(2m-3)}\delta_m^{s+2}+\right.$$
$$\left.+\frac{1}{2m+1}\left(\frac{(m+1)^2}{2m+3}+\frac{m^2}{2m-1}\right)\delta^{ms}+\frac{(m+1)(m+2)}{(2m+3)(2m+5)}\delta_m^{s-2}\right].$$

Note that we can obtain the system of equations (5.1), (5.2) directly from the three-dimensional relations (3.5) and (3.6) using I. N. Vekua' s method of reduction and notation (3.4).

The boundary conditions written in a complex form can also be obtained directly from the relations (3.7) and (3.8). For the first boundary value problem (in stresses) we have

$$\begin{cases} \overset{(m)}{\sigma}_{(ll)}+i\overset{(m)}{\sigma}_{(ls)}=\dfrac{1}{2}\Big[\overset{(m)}{\sigma}{}_1^1+\overset{(m)}{\sigma}{}_2^2+i\overset{(m)}{\sigma}{}_2^1-i\overset{(m)}{\sigma}{}_1^2- \\ \qquad -(\overset{(m)}{\sigma}{}_a^1-\overset{(m)}{\sigma}{}_2^2+i\overset{(m)}{\sigma}{}_2^1+i\overset{(m)}{\sigma}{}_1^2)\dfrac{d\bar z}{dz}\Big]=\overset{(m)}{f}_{(ll)}+i\overset{(m)}{f}_{(ls)}, \\ \overset{(m)}{\sigma}_{(ln)}=-Im\left(\overset{(m)}{\sigma}_+\dfrac{d\bar z}{ds}\right)=\overset{(m)}{f}_3,\quad \text{on } L \end{cases}$$

where $\overset{(m)}{f}_{(ll)}$, $\overset{(m)}{f}_{(ls)}$, $\overset{(m)}{f}_3$ are the given functions of points of the contour $L$.

For the second boundary value problem (in displacements) on the contour $\Gamma$ of the domain $D$ we have

$$\begin{cases} \overset{(m)}{U}_{(l)}+i\overset{(m)}{U}_{(s)}=i\overset{(m)}{U}_+\dfrac{d\bar z}{ds}=\overset{(m)}{g}_{(l)}+i\overset{(m)}{g}_{(s)}, \\ \overset{(m)}{U}_3=\overset{(m)}{g}_3,\ \ (L),\ \ (U_+=\mathbf{U}\cdot\mathbf{r}_1+i\mathbf{U}\cdot\mathbf{r}_2=U_1+iU_2) \end{cases}$$

where $\overset{(m)}{g}_{(l)}$, $\overset{(m)}{g}_{(s)}$, $\overset{(m)}{g}_3$ $(m=0,1,2,\ldots,N)$ are the given functions on $L$.

It is obvious that the larger is the number $N$, the more exact approximations we obtain, in general. But it is clear that with the growth of $N$, practical difficulties of solution of the corresponding system of equations considerably increase. However, in many cases it is practically sufficient to restrict ourselves to the approximations of order $N=0$, or $N=1$. Therefore we will consider these cases separately below.

## 6. Approximation of Order $N=0$

Approximation of order $N=0$ corresponds to the case in which the picture of stressed and strained states of the shell are practically negligibly vary along the midsurface normal. This case represents actually the membrane theory supplemented in such a way that the corresponding system of equations is consistent with three physical boundary conditions of the problem.

Introduce the notation

$$\overset{(0)}{\boldsymbol{U}} = \mathbf{U}, \quad \overset{(0)}{\sigma}_i = T_i, \quad \overset{(0)}{F}_i = X_i, \quad (i = 1, 2, 3),$$

and

$$\overset{(0,0)}{I_j} = I_j, \quad (j = 1, 2, 3, 4),$$

Now the systems (5.1) and (5.2) take the form

$$\begin{cases} \dfrac{h}{\Lambda}\dfrac{\partial}{\partial z}(T_{11} - T_{22} + iT_{12} + iT_{21}) + h\dfrac{\partial}{\partial \bar{z}}(T_1^1 + T_2^2 + iT_2^1 - iT_1^2) \\ \qquad -\varepsilon(HT_+ + Q\bar{T}_+)R + hX_+ = 0, \\ \dfrac{h}{\Lambda}\left(\dfrac{\partial T_+}{\partial z} + \dfrac{\partial \bar{T}_+}{\partial \bar{z}}\right) + \varepsilon[HT_\alpha^\alpha + \\ \qquad +Re(\overline{Q}(T_1^1 - T_2^2 + iT_2^1 + iT_1^2))] + hX_3 = 0, \end{cases} \tag{6.1}$$

where

$$h(T_{11} - T_{22} + iT_{12} + iT_{21}) = 4\mu\Lambda\left(h\frac{\partial u^+}{\partial \bar{z}} - \varepsilon QRu_3\right) +$$
$$+2\Lambda\left\{(I_1 - HI_2)Q\left[(\lambda+\mu)(h\Theta - 2H\varepsilon u_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial u_+}{\partial z} - \varepsilon H u_3 R\right)\right] +\right.$$
$$\left. + I_2 Q\left[(\lambda+\mu)Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon\overline{Q}u_3 R\right) + (\lambda+3\mu)\overline{Q}\left(h\frac{\partial u^+}{\partial \bar{z}} - \varepsilon Q u_3 R\right)\right]\right\},$$
$$h(T_1^1 + T_2^2 + iT_1^2 - iT_2^1) = 2(\lambda+\mu)(h\Theta - 2H\varepsilon u_3 R) +$$
$$+2\left\{(I_1 - HI_2)\left[(\lambda+3\mu)\overline{Q}\left(h\frac{\partial u^+}{\partial \bar{z}} - \varepsilon Q u_3 R\right) + \right.\right.$$
$$\left. + (\lambda+\mu)Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon\overline{Q}u_3 R\right)\right] +$$
$$\left. + I_2 Q\overline{Q}\left[(\lambda+\mu)(h\Theta - 2H\varepsilon u_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial u^+}{\partial \bar{z}} - \varepsilon H u_3 R\right)\right]\right\}, \tag{6.2}$$
$$hT_+ = 2\mu\left\{\left(h\frac{\partial u_3}{\partial \bar{z}} + \varepsilon\frac{Hu_+ + Q\overline{u_+}}{2}R\right) + I_2 Q\overline{Q}\left(h\frac{\partial u_3}{\partial \bar{z}} + \varepsilon\frac{Hu_+ + Q\overline{u_+}}{2}R\right) + \right.$$
$$\left. + (I_1 - HI_2)Q\left(h\frac{\partial u_3}{\partial z} + \varepsilon\frac{\overline{Q}u_+ + H\overline{u_+}}{2}R\right)\right\},$$
$$hT_{(+)} = \mu\left[h\frac{\partial u_3}{\partial \bar{z}} + \varepsilon\frac{Hu_+ + Q\overline{u_+}}{2}R\right],$$
$$hT_{33} = \lambda(h\Theta - 2H\varepsilon u_3 R),$$
$$\Theta = \frac{1}{\Lambda}\left(\frac{\partial u_+}{\partial z} + \frac{\partial \bar{u}_+}{\partial \bar{z}}\right).$$

It follows from formulas (5.3) that

$$
\begin{aligned}
&I_1 - HI_2 = h\sum_{p=1}^{\infty}\frac{(\varepsilon R)^{2p-1}}{2p+1}\sum_{q=0}^{p-1}C_{2p-1}^{2q}H^{2p-2q-1}E^q,\\
&I_2 = h^2\sum_{p=1}^{\infty}\frac{(\varepsilon R)^{2p-1}}{2p+1}\sum_{q=0}^{p-1}C_{2p-1}^{2q+1}H^{2p-2q-2}E^q,\\
&I_3 = 0,\;\; I_4 = 1+\frac{1}{3}\varepsilon^2 KR^2, \quad (E = Q\overline{Q} = H^2 - K)
\end{aligned}
$$

To find components of the displacement vector and stress tensor we take advantage of following series expansions with respect to the small parameter $\varepsilon^n$ [2]:

$$
(U_i,\; T_i,\; X_i) = \sum_{n=0}^{\infty}\Big(\overset{(n)}{U}_i,\; \overset{(n)}{T}_i,\; \overset{(n)}{X}_i\Big)\varepsilon^n, \quad (i=1,2,3).
$$

Substituting the above expansions into the relations (6.1) and (6.2) and then equalizing the coefficients of expansion for $\varepsilon^n$, we obtain the following system of equations:

$$
\begin{cases}
\dfrac{h}{\Lambda}\dfrac{\partial}{\partial z}\Big(\overset{(n)}{T}_{11} - \overset{(n)}{T}_{22} + i\,\overset{(n)}{T}_{12} + i\,\overset{(n)}{T}_{21}\Big)+\\
\qquad + h\frac{\partial}{\partial \bar z}(\overset{(n)}{T}{}^{1}_{1} + \overset{(n)}{T}{}^{2}_{2} + i\,\overset{(n)}{T}{}^{1}_{2} - i\,\overset{(n)}{T}{}^{2}_{1})\\
\qquad = -h\,\overset{(n)}{X}_{+} + (H\overset{(n-1)}{T}_{+} + Q\,\overline{\overset{(n-1)}{T}_{+}})R,\\
\dfrac{h}{\Lambda}\Big(\dfrac{\partial\,\overset{(n)}{T}_{+}}{\partial z} + \dfrac{\partial\,\overline{\overset{(n)}{T}_{+}}}{\partial \bar z}\Big) = -h\,\overset{(n)}{X}_3 - [H\overset{(n-1)}{T}{}^{\alpha}_{\alpha}\\
\qquad + Re(\overline{Q}(\overset{(n-1)}{T}{}^{1}_{1} - \overset{(n-1)}{T}{}^{2}_{2} + i\,\overset{(n-1)}{T}{}^{1}_{2} + i\,\overset{(n-1)}{T}{}^{2}_{1}))]R,
\end{cases}
\tag{6.3}
$$

where

$$
h(\overset{(n)}{T}_{11} - \overset{(n)}{T}_{22} +^{n-i}\overset{(n)}{T}_{12} + i\,\overset{(n)}{T}_{21}) = 4\mu\Lambda\Big(h\frac{\partial\,\overset{(n)}{U}{}^{+}}{\partial \bar z} - QR\overset{(n-1)}{U}{}_{3}\Big)+
$$

$$
+2\Lambda\sum_{p=1}^{[\frac{n}{2}]}\sum_{q=0}^{p-1}\frac{R^{2p}}{2p+1}\Big\{C_{2p-1}^{2q}H^{2p-2q-1}E^qQ\Big[(\lambda+\mu)(h\overset{(n-2p)}{\Theta} - 2H\overset{(n-2p-1)}{U_3 R}) +
$$

$$
+2\mu\Big(\frac{h}{\Lambda}\frac{\partial\,\overset{(n-2p)}{U_+}}{\partial z} - H\overset{(n-2p-1)}{U_3 R}\Big)\Big] + C_{2p-1}^{2q+1}H^{2p-2q-2}E^q\times
$$

$$
\times\Big[(\lambda+\mu)Q^2\Big(h\frac{\partial\,\overline{\overset{(n-2p)}{U}{}^{+}}}{\partial z} - \overline{Q}\overset{(n-2p-1)}{U_3 R}\Big)+
$$

$$+(\lambda+3\mu)Q\overline{Q}\Big(h\frac{\partial \overset{(n-2p)}{U}{}_{+}}{\partial\bar z}-Q\overset{(n-2p-1)}{U_3}R\Big)\Big]\Big\},$$

$$h\Big(\overset{(n)}{T}{}_1^{\,1}+\overset{(n)}{T}{}_2^{\,2}+i\overset{(n)}{T}{}_1^{\,2}-i\overset{(n)}{T}{}_2^{\,1}\Big)=2(\lambda+\mu)(h\overset{(n)}{\Theta}-2H\overset{(n-1)}{U}{}_3R)+ \tag{6.4}$$

$$+2\sum_{p=1}^{[\frac{n}{2}]}\sum_{q=0}^{p-1}\frac{R^{2p}}{2p+1}\Big\{C_{2p-1}^{2q}H^{2p-2q-1}E^q\Big[(\lambda+3\mu)\overline{Q}\Big(h\frac{\partial \overset{(n-2p)}{U}{}_{+}}{\partial\bar z}-Q\overset{(n-2p-1)}{U_3}R\Big)+$$

$$+(\lambda+\mu)Q\Big(h\frac{\partial\overline{\overset{(n-2p)}{U}{}_{+}}}{\partial z}-\overline{Q}\overset{(n-2p-1)}{U}{}_3R\Big)\Big]+C_{2p-1}^{2q+1}H^{2p-2q-2}E^qQ\overline{Q}\times$$

$$\times\Big[(\lambda+\mu)(h\overset{(n-2p)}{\Theta}-2H\overset{(n-2p-1)}{U}{}_3R)+2\mu\Big(\frac{h}{\Lambda}\frac{\partial \overset{(n-2p)}{U}{}_{+}}{\partial z}-H\overset{(n-2p-1)}{U_3}R\Big)\Big]\Big\},$$

$$h\overset{(n)}{T}{}_{+}=2\mu\Big\{h\frac{\partial \overset{(n)}{U_3}}{\partial\bar z}-\frac{H\overset{(n-1)}{U_+}+Q\overline{\overset{(n-1)}{U}{}_{+}}}{2}R+$$

$$+\sum_{p=1}^{[\frac{n}{2}]}\sum_{q=0}^{p-1}\frac{R^{2p}}{2p+1}\Big[C_{2p-1}^{2q}H^{2p-2q-1}E^qQ\times$$

$$\times\Big(h\frac{\partial\overset{(n-2p)}{U_3}}{\partial z}+\frac{\overline{Q}\overset{(n-2p-1)}{U}{}_{+}+H\overline{\overset{(n-2p-1)}{U}{}_{+}}}{2}R\Big)+$$

$$+C_{2p-1}^{2q+1}H^{2p-2q-1}E^qQ\overline{Q}\Big(h\frac{\partial\overset{(n-2p)}{U_3}}{d\bar z}+\frac{H\overset{(n-2p-1)}{U_+}+Q\overline{\overset{(n-2p-1)}{U_+}}}{2}R\Big)\Big]\Big\},$$

$$h\overset{(n)}{T}{}_{(+)}=\mu\Big[h\frac{\partial\overset{(n)}{U_3}}{\partial\bar z}+\frac{H\overset{(n-1)}{U_+}+Q\overline{\overset{(n-1)}{U_+}}}{2}R\Big],$$

$$h\overset{(n)}{T}{}_{33}=\lambda(h\overset{(n)}{\Theta}-2H\overset{(n-1)}{U_3}R).$$

Now we write the system of equations in terms of components of the displacement vector

$$\overset{(m)}{U}=\frac12\left(\overset{(m)}{U}{}_{+}\mathbf{r}^{+}+\overset{(m)}{U}{}_3\mathbf{n}\right)=\frac12\left(\overset{(m)}{U}{}^{+}\mathbf{r}_{+}+\overset{(m)}{U}{}_3\mathbf{n}\right),$$

We have

$$4\mu h^2\frac{\partial}{\partial\bar z}\left(\frac{1}{\Lambda}\frac{\partial\overset{(n)}{U_+}}{\partial\bar z}\right)+2(\lambda+\mu)h^2\frac{\partial\overset{(n)}{\Theta}}{\partial\bar z}=\overset{(n)}{\chi}_{+}\left(\overset{(0)}{U_i},\dots,\overset{(n-1)}{U_i}\right), \tag{6.5$_1$}$$

$$\mu h^2\nabla^2\overset{(n)}{U_3}=\overset{(n)}{\chi}_3\left(\overset{(0)}{U_i},\dots,\overset{(n-1)}{U_i}\right),\quad\left(\nabla^2=\frac{4}{\Lambda}\frac{\partial^2}{\partial z\partial\bar z}\right), \tag{6.5$_2$}$$

$$\overset{(n)}{\Theta} = \frac{1}{\Lambda}\left(\frac{\partial \overset{(n)}{U}_+}{\partial z} + \frac{\partial \overline{\overset{(n)}{U}_+}}{\partial \bar{z}}\right), \quad (i = 1, 2, 3)$$

where $\overset{(n)}{\chi}_+$ and $\overset{(n)}{\chi}_3$ are expressed by $\overset{(0)}{U}_+, \overset{(0)}{U}_3, \dots, \overset{(n-1)}{U}_+, \overset{(n-1)}{U}_3$ and assuming that they are already found. When deducing the system (6.5) we used the formula [1]

$$\frac{4h^2}{\Lambda}\frac{\partial}{\partial z}\Lambda\frac{\partial}{\partial \bar{z}}U^+ = 4h^2\frac{\partial}{\partial \bar{z}}\left(\frac{1}{\Lambda}\frac{\partial \overset{(n)}{U}_+}{\partial z}\right) + 2K\varepsilon^2 U_+, \qquad (6.5_3)$$

where $K$ is the Gaussian curvature of the midsurface of the shell.

The above writing of the type $(6.5_1)$, $(6.5_2)$ brings the suggested version of the theory of shells ($N = 0$) closer to the equations of the classical plane problem of elasticity, and for $\Lambda = 1$ (plates, cylindrical and conical shells) coincide with them. This circumstance plays an important role in practical realization of calculations of a shell on the basis of the obtained equations, since in passing from the $n$-th step to the $(n+1)$ we always have to solve the "plane" problem and the Poisson equation on the midsurface $S$ of the shell $\Omega$. We can see that in passing from the given step to the subsequent one only the right-hand sides of equations undergo variations. The problem of converges of the process requires special investigation. To ensure the convergence the right-hand sides should be subjected to certain general restrictions.

Now simple calculations show that a general solution of the system $(6.5_1)$ and $(6.5_2)$ can be represented by means of three analytic functions of $z$ in the form [7]

$$\overset{(n)}{U}_+ = -\frac{æ}{\pi}\int\int_D \frac{\Lambda(\zeta,\overline{\zeta})\varphi^1(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}} + \left(\frac{1}{\pi}\int\int_D \frac{\Lambda(\zeta,\overline{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}\right)\overline{\varphi'(z)} - \overline{\psi(z)}$$

$$+\frac{1}{8\mu h^2}\frac{\lambda+\mu}{\lambda+2\mu}\frac{1}{\pi}\int\int_D \frac{\overset{(n)}{F}_+(\zeta,\overline{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}, \qquad (6.6_1)$$

$$\overset{(n)}{U}_3 = f(z) + \overline{f(z)} - \frac{2}{\pi}\int\int_D \overset{(n)}{X}_3(\zeta,\overline{\zeta}) ln|\zeta - z| d\xi d\eta, \qquad (6.6_2)$$

where $\varphi^{'}(z)$, $f(z)$ and $\psi(z)$, are analytic functions of $z = x^1 + ix^2 \in D$, and $\zeta = \xi + i\eta$. Further,

$$\overset{(n)}{F}_{+}(z, \bar{z}) = -\frac{1}{\pi} \iint\limits_{D} \left( \frac{\overline{\overset{(n)}{X}_{+}}}{\bar{\zeta} - \bar{z}} - \frac{æ \overset{(n)}{X}_{+}}{\zeta - z} \right) d\xi d\eta, \quad \left( æ = \frac{\lambda + 3\mu}{\lambda + \mu} \right).$$

$D$ is the domain of the plane $Ox^1x^2$ onto which the midsurface $S$ of the shell $\Omega$ is mapped topologically.

Note that for $\Lambda = 1$ the expression for $\overset{(0)}{U}_{+}$ coincides with the well-known representation of Kolosov-Muskhelishvili [3]

$$U_{+} = æ\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}, \quad \left( \overset{(0)}{\chi} = 0 \right)$$

in the plane theory of elasticity.

## 7. The Basic Boundary Value Problems

The boundary conditions (5.4) and (5.5) for the approximation of order $N = 0$ for any $n$ have the form

(a) for the first basic boundary problem (in stresses)

$$\left\{ \begin{aligned} &\overset{(n)}{T}_{(ll)} + i \overset{(n)}{T}_{(ls)} = \tfrac{1}{2} \Bigg[ \left( \overset{(n)}{T}{}_1^1 - \overset{(n)}{T}{}_2^2 + i \overset{(n)}{T}{}_2^1 - i \overset{(n)}{T}{}_1^2 \right) \\ &\quad - \left( \overset{(n)}{T}_{11} - \overset{(n)}{T}_{22} + i \overset{(n)}{T}_{12} + i \overset{(n)}{T}_{21} \right) \left( \tfrac{d\bar{z}}{ds} \right)^2 \Bigg] = \\ &\quad = \overset{(n)}{d}_1 + i \overset{(n)}{d}_2, \ \ (L) \\ &\overset{(n)}{T}_{(ln)} = -Im \left( \overset{(n)}{T}_{+} \tfrac{d\bar{z}}{ds} \right) = \overset{(n)}{d}_3, \ \ (L) \end{aligned} \right. . \tag{7.1}$$

(b) for the second boundary problem (in displacements)

$$\left. \begin{aligned} &\overset{(n)}{u}_{(l)} + i \overset{(n)}{u}_{(s)} = i \overset{(n)}{u}_{+} \frac{d\bar{z}}{ds} = \overset{(n)}{e}_1 + \overset{(n)}{e}_2, \ \ (L) \\ &\overset{(n)}{u}_3 = \overset{(n)}{e}_3, \ \ (L). \end{aligned} \right\} \tag{7.2}$$

Here we have introduced the notation

$$\begin{aligned} &\overset{(0)}{f}_{(ll)} + i \overset{(0)}{f}_{(ls)} = d_1 + id_2, \quad \overset{(0)}{f}_3 = d_3, \\ &\overset{(0)}{g}_{(l)} + i \overset{(0)}{g}_{(s)} = e_1 + ie_2, \quad \overset{(0)}{g}_3 = e_3, \end{aligned}$$

note that

$$(d_i, \ e_i) = \sum_{n=0} \left( \overset{(n)}{d}_i, \ \overset{(n)}{e}_i \right) \varepsilon^n, \quad (i = 1, 2, 3),$$

and $L$ is the contour of the domain $D$ onto which the midsurface $S$ together with its boundary $\partial S = \Gamma$ is mapped topologically.

Here we present a general scheme of solution of boundary problems when the domain $D$ is a circle of radius $r_0$.

The second boundary problem (in displacements) for any $n$ takes the form

$$\overset{(n)}{u}_{+}|_{r_0} = \Big\{ -\frac{æ}{\pi} \int\int_D \frac{\Lambda(\zeta,\overline{\zeta})\varphi^1(\zeta)d\xi d\eta}{\bar{\zeta}-\bar{z}} + \Big(\frac{1}{\pi}\int\int_D \frac{\Lambda(\zeta,\overline{\zeta})d\xi d\eta}{\bar{\zeta}-\bar{z}}\Big)\overline{\varphi'(z)} - \overline{\psi(z)}\Big\}_{r_0} = \overset{(n)}{G}_{+}, \quad (| z | r_0) \quad (7.3_1)$$

$$\overset{(n)}{u}_{3}|_{r_0} = f(z) + \overline{f(z)}|_{r_0} = \overset{(n)}{G}_{3}. \ (| z |= r_0) \quad (7.3_2)$$

$$\big(n = 0, 1, \ldots, ; \ \ z = re^{i\theta}, \ \ \zeta = \xi + i\eta = \rho^{i\theta}\big),$$

where $\overset{(n)}{G}_{+}$ and $\overset{(n)}{G}_{3}$ are the known values containing solutions $\overset{(0)}{u}_{i}$, $\overset{(1)}{u}_{i}$, $\ldots$, $\overset{(n)}{u}_{i}$, $(i = 1, 2, 3)$ of the previous approximations.

Let $\Lambda(z, \ \bar{z})$ depend only on $r =| z |$; next $\varphi^{'}(z)$ and $\psi(z)$ are expanded in power series of the type

$$\varphi^{'}(z) = \sum_{k=0}^{\infty} a_k z^k, \ \ \psi(z) = \sum_{k=0}^{\infty} b_k z^k,$$

and the expression $\overset{(n)}{G}_{+}$ in the form of a complex Fourier series

$$\overset{(n)}{G}_{+} = \sum_{k=-\infty}^{\infty} A_k e^{ik\theta}.$$

Assuming that the above-mentioned series for $\varphi^{'}(z)$ and $\psi(z)$ converge not only inside the circle $| \ z \ |= r_0$, but also on the circumference $| \ z \ |= r_0$ and then substituting these expansions into $(7.3_1)$ and comparing the coefficients for $e^{ik\theta}$, we obtain

$$\left\{\begin{array}{l} a_1\alpha_0 + b_0 = -\bar{A}_0, \\ (æa_0 - \bar{a}_0)\alpha_0 = A_1 r_0, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ æ\alpha_k \dfrac{a_k}{r_0^{k+1}} = A_{k+1}, \end{array}\right. \cup \left\{\begin{array}{l} (a_2\alpha_0 + b_1)r_0 = -\bar{A}_{-1}, \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ (a_{k+1}\alpha_0 + b_k)r_0^k = -\bar{A}_{-k}, \end{array}\right.$$

This system allows one to find all but $a_0$ coefficients as follows:

$$a_k = \frac{1}{æ}\frac{r_0^{k+1}}{\alpha_k(r_0)}A_{k+1}, \ \ k \geq 1,$$

$$b_k = -\left(\frac{\bar{A}_{-k}}{r_0^k} + \frac{\alpha_0}{æ}\frac{r_0^{k+2}}{\alpha_{k+1}}A_{k+2}\right), \ \ (k \geq 0).$$

where

$$\alpha_k = 2\int\limits_0^{r_0} \rho^{2k+1}\Lambda(\rho)d\rho.$$

Coefficient $a_0$ is defined from the equality $(æa_0 - \bar{a}_0)\alpha_0 = A_1 r_0$ and that obtained by passing to the conjugated values $(æ\bar{a}_0 - a_0)\alpha_0 = \bar{A}_1 r_0$, whence

$$a_0 = \frac{r_0}{\alpha_0}\frac{æA_1 + \bar{A}_1}{æ^2 - 1}, \quad æ = \frac{\lambda + 3\mu}{\lambda + \mu} > 1.$$

If $\Lambda = 1$, then $\alpha_0 = r_0^2$, and for $a_0$ we obtain

$$a_0 = \frac{æA_1 + \bar{A}_1}{(æ^2 - 1)r_0},$$

which coincides with coefficient $\Lambda = 1$ of the second basic problem (in displacements) of the plane theory of elasticity [3].

Note that for cylindrical and conical shells $\Lambda = 1$, and for the spherical shell

$$\Lambda = \frac{4R^2}{(1 + z\bar{z})^2}, \quad z = tg\frac{\theta}{2}e^{i\varphi} = re^{i\varphi}, \quad r_0 = tg\frac{\theta_0}{2}.$$

Hence

$$\alpha_k = 8R^2\int\limits_0^{r_0}\frac{\rho^{2k+1}d\rho}{(1+\rho^2)^2} = \frac{1}{1+r_0^2} + (-1)^{k+1}k\left[ln(1+r_0^2) - \sum_{s=1}^{k}(-1)^s\frac{r_0^{2s}}{s}\right].$$

A solution of the boundary problem (7.3$_2$) is representable in the form of the Poisson integral,

$$\overset{(n)}{u}_3(r,\varphi) = \frac{1}{2\pi}\int\limits_0^{2\pi}\overset{(n)}{G}_3(\psi)\frac{r_0^2 - r^2}{r^2 - 2r_0 r cos(\psi - \theta) + r_0^2}d\psi.$$

Thus for any $n$ we can construct formal solutions of the second boundary problem, when $N = 0$.

Consider now the first basic problem (7.1) for any fixed number $n$ ($n \geq 0$).

Substituting the relations (6.4) into (7.1), we obtain

$$(\lambda + \mu)\overset{(n)}{\theta} + 2\mu\frac{\partial}{\partial\bar{z}}\left(\frac{1}{\Lambda}\overset{(n)}{u}_+\right)\frac{d\bar{z}}{dz} = \overset{(n)}{p}_+, \quad (|\, z \,| = r_0), \qquad (7.4_1)$$

$$\frac{2\mu}{\Lambda}\Big(\frac{\partial\overset{(n)}{u}_3}{\partial z}e^{i\varphi} + \frac{\partial\overset{(n)}{u}_3}{\partial\bar{z}}e^{-i\varphi}\Big) = \overset{(n)}{p}_3, \quad (|\, z \,| = r_0), \qquad (7.4_2)$$

$$\Big(\overset{(n)}{\theta} = \frac{1}{\Lambda}\Big(\frac{\partial\overset{(n)}{u}_+}{\partial z} + \overline{\frac{\partial\overset{(n)}{u}_3}{\partial\bar{z}}}\Big)\Big),$$

where $\overset{(n)}{p}_+$ and $\overset{(n)}{p}_3$ are the known values expressed in terms of the solution $\overset{(k)}{\boldsymbol{u}}$ $(k = 0, 1, \dots, n-1)$ of the previous approximations.

Consider the case of a spherical shell, whose midsurface is spherical segment of radius $R_0 sin\theta$, where $R_0$ is the radius of the sphere. Isometric coordinates on the sphere can be represented in the form,

$$z = x^1 + ix^2 = tg\frac{\theta}{2}e^{i\varphi} \;\;\Rightarrow\;\; ds^2 = \Lambda(z,\bar{z})dzd\bar{z}, \;\; \Lambda = \frac{4R_0^2}{(1+z\bar{z})^2}.$$

In this case a stereographic projection of the spherical segment of radius $R_0 sin\theta$ $\left(0 \le \theta \le \frac{\pi}{2}\right)$ from the South pole $\theta = \pi$ to the equatorial plane $\theta = \frac{\pi}{2}$ is a circle of radius $r_0 = tg\frac{\theta_0}{2}$.

For the components of the displacement vector we have the following complex representations:

$$\overset{(n)}{u}{}_+ = 4R^2\left\{-\text{æ}\int\limits_0^z \frac{\varphi'(\zeta)d\zeta}{(1+\zeta\bar{\zeta})^2} - \frac{z}{1+z\bar{z}}\overline{\varphi'(z)} - \overline{\psi(z)}\right\} + \widehat{u}_+,$$
$$\overset{(n)}{u}{}_3 = f(z) + \overline{f(z)} = \widehat{u}_3, \;\; \left(\text{æ} = \frac{\lambda+3\mu}{\lambda+\mu}\right),$$

where $\varphi'(z)$ $\psi(z)$ $f(z)$ are analytic functions in the domain $D(|z| \le r_0)$, and the values $\widehat{u}_+$ and $\widehat{u}_3$ are the particular solutions of systems of equations $(6.5_1)$ and $(6.5_2)$. In what follows, the index "$n$" will be omitted and the right-hand sides of the boundary conditions $(7.4_1)$ and $(7.4_2)$ will be denoted by $\widehat{p}_+$ and $\widehat{p}_3$.

Let the expansions

$$\varphi'(z) = \sum_{k=0}^{\infty} a_k z^k, \;\; \psi(z) = \sum_{k=0}^{\infty} b_k z^k, \;\; f(z) = \sum_{k=0}^{\infty} c_k z^k,$$
$$\widehat{p}_+ = \sum_{k=0}^{\infty} A_k e^{ik\varphi}, \;\; \widehat{p}_3 = \sum_{k=0}^{\infty} B_k e^{ik\varphi}, \;\; (B_{-k} = \bar{B}_k)$$

be valid. Then the boundary conditions $(7.4_1)$ and $(7.4_2)$ take the form

$$\sum_{k=0}^{\infty}\Big\{\left(a_k e^{ik\varphi} + \bar{a}_k e^{-ik\varphi}\right) r_0^k + (1+r_0^2)\left[2\text{æ} r_0^{k+2}\beta_k(r_0) a_k e^{ik\varphi} - \right.$$
$$\left. - \,\bar{a}_k r_0^k\left(k + \frac{r_0^k}{1+r_0^2}\right)e^{-ik\varphi} - \bar{b}_k r_0^{k-1}\left(k + (k+2)r_0^2\right)e^{-i(k+1)\varphi}\right]\Big\} =$$
$$= \frac{1}{2\mu}\sum_{k=-\infty}^{\infty} A_k e^{ik\varphi},$$
$$\sum_{k=0}^{\infty} k r_0^{k-1}(c_k e^{ik\varphi} + \bar{c}_k e^{-ik\varphi}) = \frac{1}{2\mu}\frac{R_0}{1+r_0^2}\sum_{k=-\infty}^{\infty} B_k e^{ik\varphi},$$

where

$$\beta_k(r)=\frac{1}{z^{k+2}}\int\limits_0^z \frac{(z-t)t^k dt}{(1+t\bar{z})^3}=-\frac{1}{2z\bar{z}}\Big\{\frac{1}{1+z\bar{z}}+(-1)^k\frac{k(k-1)}{(z\bar{z})^k}\Big[\ln(1+z\bar{z})$$

$$-\sum_{s=1}^{k-1}(-1)^s\frac{(z\bar{z})^s}{s}\Big]+(-1)^{k+1}\frac{k(k+1)}{(z\bar{z})^{k+1}}\Big[\ln(1+z\bar{z})-\sum_{s=1}^{k-1}(-1)^s\frac{(z\bar{z})^s}{s}\Big]\Big\}\Rightarrow$$

$$\Rightarrow\beta_k(z)=\sum_{s=1}^{k-1}(-1)^s\frac{(s+1)(s+2)}{2}\,\frac{(z\bar{z})^s}{(k+s+1)(k+s+2)}.$$

Comparing the coefficients for $e^{ik\theta)}$ and taking into account that the resultant vector and the principal moment are equal to zero, we obtain

$$a_k=\frac{A_k}{2\mu}\frac{1}{[1+2H(1+r_0^2)r_0^2]r_0^k},\quad (k=0,1,\dots)$$

$$b_k=\frac{1}{2\mu}\frac{1}{(1+r_0^2)(k+2r_0^2)r_0^{k-1}}\left[A_{k+1}\frac{1-[k+1+(k+2)r_0^2]}{1+2H(1+r_0^2)r_0^2\beta_{k+1}(r_0)}-\bar{A}_{-k-1}\right],$$

and for the coefficients $c_k$ we will have

$$c_k=\frac{1}{\mu}\frac{R_0}{1-r_0^2}\frac{B_k}{kr_0^{k-1}},\quad (k=1,2,\dots)$$

The condition

$$\frac{1}{2\pi}\int\limits_0^{2\pi}\widehat{p}_3(r_0,\varphi)d\varphi=0$$

is that of the existence of the Neumann problem for the harmonic functions. This implies that for $\overset{(n)}{u}_3$ we obtain the well-known Dini's formula

$$u_3(r,\varphi)=-\frac{r_0}{\pi}\frac{2R_0}{\mu}\frac{1}{1+r_0^2}\int\limits_0^{2\pi}\widehat{p}_3(r_0,\theta)ln|\sigma-z|d\theta+\text{const}$$

$$\left(\sigma=r_0e^{i\theta},\quad z=re^{i\varphi}\right).$$

## 8. Approximation of Order $N=1$

The case $N=1$ corresponds to the assumption when the picture of the stressed and strained state of the shell changes along the midsurface normal by the linear law. This case is close enough to the classical theory of shells, although does not exactly coincides with the latter. The classical theory is supplemented here in such a way that the obtained system of differential equations is consistent with six physical boundary conditions [1].

For the case $N=1$ we have

$$\mathbf{U}=\overset{(0)}{\boldsymbol{U}}+P_1\left(\frac{x_3}{h}\right)\overset{(1)}{\boldsymbol{U}},\quad \boldsymbol{\sigma}^i=\overset{(0)}{\boldsymbol{\sigma}}{}^i+P_1\left(\frac{x_3}{h}\right)\overset{(1)}{\boldsymbol{\sigma}}{}^i,\quad \mathbf{F}=\overset{(0)}{\boldsymbol{F}}+P_1\left(\frac{x_3}{h}\right)\overset{(1)}{\boldsymbol{F}},$$

$$\overset{(0)}{\boldsymbol{U}}{}' = \overset{(1)}{\boldsymbol{U}}, \ \ \overset{(1)}{\boldsymbol{U}}{}' = 0.$$

Introducing the notation

$$\overset{(0)}{\boldsymbol{U}} = \mathbf{U}, \ \ \overset{(1)}{\boldsymbol{U}} = \mathbf{V}, \ \ \overset{(0)}{\boldsymbol{\sigma}}{}^{i} = \mathbf{T}^i, \ \ \overset{(1)}{\boldsymbol{\sigma}}{}^{i} = \mathbf{S}^i, \ \ \overset{(0)}{\boldsymbol{F}} = \mathbf{X}, \ \ \overset{(1)}{\boldsymbol{F}} = \mathbf{Y},$$

from (5.1) we obtain the following system of equations of equilibrium for $N = 1$:

$$\frac{h}{\Lambda}\frac{\partial}{\partial z}(T_{11} - T_{22} + iT_{12} + iT_{21}) + h\frac{\partial}{\partial \bar{z}}(T_1^1 + T_2^2 + iT_2^1 - iT_1^2) - \\ -\varepsilon R(HT_+ + Q\bar{T}_+) + hX_+ = 0, \tag{8.1}$$

$$\frac{h}{\Lambda}\left(\frac{\partial T_+}{\partial z} + \frac{\partial \bar{T}_+}{\partial \bar{z}}\right) + \varepsilon R[HT_\alpha^\alpha + Re\overline{Q}(T_1^1 - T_2^2 + iT_2^1 + iT_1^2)] + hX_3 = 0,$$

$$\frac{h}{\Lambda}\frac{\partial}{\partial z}(S_{11} - S_{22} + iS_{12} + iS_{21}) + h\frac{\partial}{\partial \bar{z}}(S_1^1 + S_2^2 - iS_2^1 - iS_1^2) - \\ -\varepsilon R(HS_+ + Q\bar{S}_+) - 3T_+ + hY_+ = 0, \tag{8.2}$$

$$\frac{h}{\Lambda}\left(\frac{\partial S_+}{\partial z} + \frac{\partial \bar{S}_+}{\partial \bar{z}}\right) + \varepsilon R[HS_\alpha^\alpha + Re\overline{Q}(S_1^1 - S_2^2 + iS_2^1 + iS_1^2)] - 3T_3^3 + hY_3 = 0,$$

where

$$h(T_{11} - T_{22} + iT_{12} + iT_{21}) = 4\mu\Lambda\left(h\frac{\partial u^+}{\partial \bar{z}} - \varepsilon QRu_3\right) + \\ +2\Lambda Q\left\{(\overset{(0,0)}{I}_1 - H\overset{(0,0)}{I}_2)\left[(\lambda+\mu)(h\Theta - 2H\varepsilon u_3R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial u_+}{\partial z} - \varepsilon Hu_3R\right)\right] + \right. \\ + \overset{(0,0)}{I}_2 Q\left[(\lambda+\mu)Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon\overline{Q}u_3R\right) + (\lambda+3\mu)\overline{Q}\left(h\frac{\partial u^+}{\partial \bar{z}} - \varepsilon Qu_3R\right)\right] + \\ +(\overset{(0,1)}{I}_1 - H\overset{(0,1)}{I}_2)\left[(\lambda+\mu)(h\rho - 2H\varepsilon v_3R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial v_+}{\partial z} - \varepsilon RHv_3\right)\right] + \\ \left. + \overset{(0,1)}{I}_2 Q\left[(\lambda+\mu)Q\left(h\frac{\partial \overline{v^+}}{\partial z} - \varepsilon\overline{Q}v_3R\right) + (\lambda+3\mu)\overline{Q}\left(h\frac{\partial v^+}{\partial \bar{z}} - \varepsilon Qv_3R\right)\right]\right\},$$

$$h(T_1^1 + T_2^2 + iT_1^2 - iT_2^1) = 2(\lambda+\mu)(h\Theta - 2H\varepsilon u_3R) + \\ +2\left\{(\overset{(0,0)}{I}_1 - H\overset{(0,0)}{I}_2)\left[(\lambda+3\mu)\overline{Q}\left(h\frac{\partial u^+}{\partial \bar{z}} - \varepsilon Qu_3R\right) + \right.\right. \\ \left. + (\lambda+\mu)Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon\overline{Q}u_3R\right)\right] + \\ + \overset{(0,0)}{I}_2 Q\overline{Q}\left[(\lambda+\mu)(h\Theta - 2H\varepsilon u_3R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial u^+}{\partial z} - \varepsilon Hu_3R\right)\right] +$$

$$+(\overset{(0,1)}{I}_1 - H\overset{(0,1)}{I}_2)\left[(\lambda+3\mu)\overline{Q}\left(h\frac{\partial v^+}{\partial \bar z} - \varepsilon Q v_3 R\right) + \right.$$

$$\left. + (\lambda+\mu)Q\left(h\frac{\partial \overline{v^+}}{\partial z} - \varepsilon\overline{Q} v_3 R\right)\right] + \tag{8.3}$$

$$+ \overset{(0,1)}{I}_2 Q\overline{Q}\left[(\lambda+\mu)(h\rho - 2H\varepsilon v_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial v^+}{\partial z} - \varepsilon H v_3 R\right)\right]\Big\} + 2\lambda v_3,$$

$$hT_+ = 2\mu\left\{\left(h\frac{\partial u_3}{\partial \bar z} + \varepsilon\frac{Hu_+ + Q\overline{u_+}}{2}R\right) + \right.$$

$$+2Q\left[\overset{(0,0)}{I}_2\overline{Q}\left(h\frac{\partial u_3}{\partial \bar z} + \varepsilon R\frac{Hu_+ + Q\overline{u_+}}{2}\right) + \right.$$

$$+(\overset{(0,0)}{I}_1 - H\overset{(0,0)}{I}_2)\left(h\frac{\partial u_3}{dz} + \varepsilon\frac{\overline{Q}u_+ + H\overline{u_+}}{2}R\right) + \frac{1}{2}v_+$$

$$+ \overset{(0,1)}{I}_2\overline{Q}\left(h\frac{\partial v_3}{\partial \bar z} + \varepsilon R\frac{Hv_+ + Q\overline{v_+}}{2}\right) +$$

$$\left.\left. + (\overset{(0,1)}{I}_1 - H\overset{(0,1)}{I}_2)\left(h\frac{\partial v_3}{dz} + \varepsilon\frac{\overline{Q}v_+ + H\overline{v_+}}{2}R\right)\right]\right\},$$

$$hT_{(+)} = 2\mu\left\{\left(h\frac{\partial u_3}{\partial \bar z} + \varepsilon R\frac{Hu_+ + Q\overline{u_+}}{2}\right) + \left(1 + \frac{1}{3}\varepsilon^2 R^2 K\right)v_+\right.$$

$$\left. + \frac{\varepsilon R}{3}\left[Q\left(h\frac{\partial v_3}{\partial z} + \varepsilon R\frac{H\overline{v_+} + \overline{Q}v_+}{2}\right) - H\left(h\frac{\partial v_3}{\partial \bar z} + \varepsilon R\frac{Hv_+ + Q\overline{v_+}}{2}\right)\right]\right\},$$

$$hT_3^3 = \lambda(h\Theta - 2H\varepsilon u_3 R) + (\lambda+2\mu)\left(1 + \frac{1}{3}\varepsilon^2 R^2 K\right)v_3$$

$$+\frac{\lambda}{3}\varepsilon R\left[Q\left(h\frac{\partial \overline{v_+}}{\partial z} - \varepsilon RQv_3\right) + \overline{Q}\left(h\frac{\partial v^+}{\partial \bar z} + \varepsilon RQv_3\right) - H(h\rho - 2H\varepsilon R v_3)\right].$$

$$h(S_{11} - S_{22} + iS_{12} + iS_{21}) = 4\mu\Lambda\left(h\frac{\partial v^+}{\partial \bar z} - \varepsilon QRv_3\right) + 2\lambda RQv_3 +$$

$$+2\Lambda Q\left\{(\overset{(1,0)}{I}_1 - H\overset{(1,0)}{I}_2)\left[(\lambda+\mu)(h\Theta - 2H\varepsilon u_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial u_+}{\partial z} - \varepsilon H u_3 R\right)\right] + \right.$$

$$+ \overset{(1,0)}{I}_2\left[(\lambda+\mu)Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon\overline{Q}u_3 R\right) + (\lambda+3\mu)\overline{Q}\left(h\frac{\partial u^+}{\partial \bar z} - \varepsilon Q u_3 R\right)\right] +$$

$$+(\overset{(1,1)}{I}_1 - H\overset{(1,1)}{I}_2)\left[(\lambda+\mu)(h\rho - 2H\varepsilon v_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial v_+}{\partial z} - \varepsilon RHv_3\right)\right] +$$

$$\left. + \overset{(1,1)}{I}_2 Q\left[(\lambda+\mu)Q\left(h\frac{\partial \overline{v^+}}{\partial z} - \varepsilon\overline{Q}v_3 R\right) + (\lambda+3\mu)\overline{Q}\left(h\frac{\partial v^+}{\partial \bar z} - \varepsilon Q v_3 R\right)\right]\right\},$$

$$h(S_1^1 + S_2^2 + iS_1^2 - iS_2^1) = 2(\lambda+\mu)(h\rho - 2H\varepsilon u_3 R) - 2\lambda\varepsilon RHv_3 +$$

$$
\begin{gathered}
+2\left\{(\overset{(1,0)}{I}_1 - H\overset{(1,0)}{I}_2)\left[(\lambda+3\mu)\overline{Q}\left(h\frac{\partial u^+}{\partial \bar z} - \varepsilon Q u_3 R\right) + \right.\right. \\
\left. + (\lambda+\mu)Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon \overline{Q} u_3 R\right)\right] + \\
+ \overset{(1,0)}{I}_2 Q\overline{Q}\left[(\lambda+\mu)(h\Theta - 2H\varepsilon u_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial u^+}{\partial z} - \varepsilon H u_3 R\right)\right] + \\
+(\overset{(1,1)}{I}_1 - H\overset{(1,1)}{I}_2)\left[(\lambda+3\mu)\overline{Q}\left(h\frac{\partial v^+}{\partial \bar z} - \varepsilon Q v_3 R\right) + \right. \\
\left. + (\lambda+\mu)Q\left(h\frac{\partial \overline{v^+}}{\partial z} - \varepsilon \overline{Q} v_3 R\right)\right] + \qquad (8.4) \\
\left. + \overset{(1,1)}{I}_2 Q\overline{Q}\left[(\lambda+\mu)(h\rho - 2H\varepsilon v_3 R) + 2\mu\left(\frac{h}{\Lambda}\frac{\partial v_+}{\partial z} - \varepsilon H v_3 R\right)\right]\right\}. \\
hS_+ = 2\mu\left\{\left(h\frac{\partial v_3}{\partial \bar z} + \varepsilon\frac{Hv_+ + Q\overline{u_v}}{2}R\right) + \right. \\
+2Q\left[\overset{(1,0)}{I}_2\overline{Q}\left(h\frac{\partial u_3}{\partial \bar z} + \varepsilon R\frac{Hu_+ + Q\overline{u_+}}{2}\right) + \right. \\
+(\overset{(1,0)}{I}_1 - H\overset{(1,0)}{I}_2)\left(h\frac{\partial u_3}{dz} + \varepsilon\frac{\overline{Q}u_+ + H\overline{u_+}}{2}R\right) + \\
+ \overset{(1,1)}{I}_2\overline{Q}\left(h\frac{\partial v_3}{\partial \bar z} + \varepsilon R\frac{Hv_+ + Q\overline{v_+}}{2}\right) + \\
\left.\left. + (\overset{(1,1)}{I}_1 - H\overset{(1,1)}{I}_2)\left(h\frac{\partial v_3}{\partial z} + \varepsilon\frac{\overline{Q}v_+ + H\overline{v_+}}{2}R\right)\right] - \varepsilon R(Hv_+ - Q\overline{v_+})\right\}, \\
hS_{(+)} = 2\mu\left\{\left(h\frac{\partial v_3}{\partial \bar z} + \varepsilon R\frac{Hv_+ + Q\overline{v_+}}{2}\right) - 2H\varepsilon R v_+ + \right. \\
\left. + \varepsilon R\left[Q\left(h\frac{\partial u_3}{\partial z} + \varepsilon R\frac{H\overline{u_+} + \overline{Q}u_+}{2}\right) - H\left(h\frac{\partial u_3}{\partial \bar z} + \varepsilon R\frac{Hu_+ + Q\overline{u_+}}{2}\right)\right]\right\}, \\
hS_3^3 = \lambda(h\rho - 2H\varepsilon v_3 R) - \lambda\varepsilon RH(h\theta - 2H\varepsilon R u_3) - 2(\lambda+2\mu)H\varepsilon R v_3 + \\
+\lambda\varepsilon R\left[Q\left(h\frac{\partial \overline{u^+}}{\partial z} - \varepsilon R\overline{Q}u_3\right) + \overline{Q}\left(h\frac{\partial u^+}{\partial \bar z} - \varepsilon R Q u_3\right)\right].
\end{gathered}
$$

Here

$$
\Theta = \frac{1}{\Lambda}\left(\frac{\partial u_+}{\partial z} + \frac{\partial \bar u_+}{\partial \bar z}\right), \quad \rho = \frac{1}{\Lambda}\left(\frac{\partial v_+}{\partial z} + \frac{\partial \bar v_+}{\partial \bar z}\right),
$$

and the integrals of the type $\overset{(m,s)}{I}_i$ $(m,s=0,1;\ i=1,2)$ are calculated by formula (5.3) as follows:

$$\overset{(0,0)}{I}_1 - H\overset{(0,0)}{I}_2 = h\sum_{p=1}^{\infty}\frac{(\varepsilon R)^{2p-1}}{2p+1}\sum_{q=0}^{p-1}C_{2p-1}^{2q}H^{2p-2q-1}E^q,$$

$$\overset{(0,0)}{I}_2 = h^2\sum_{p=1}^{\infty}\frac{(\varepsilon R)^{2p-2}}{2p+1}\sum_{q=0}^{p-1}C_{2p-1}^{2q+1}H^{2p-2q-2}E^q,$$

$$\overset{(0,1)}{I}_1 - H\overset{(0,1)}{I}_2 = \frac{1}{3}(\overset{(1,0)}{I}_1 - H\overset{(1,0)}{I}_2) = h\sum_{p=0}^{\infty}\frac{(\varepsilon R)^{2p}}{2p+3}\sum_{q=0}^{p}C_{2p}^{2q}H^{2p-2q}E^q,$$

$$\overset{(0,1)}{I}_2 = \frac{1}{3}H\overset{(1,0)}{I}_2 = h^2\sum_{p=0}^{\infty}\frac{(\varepsilon R)^{2p-1}}{2p+3}\sum_{q=0}^{p-1}C_{2p}^{2q}H^{2p-2q-1}E^q,$$

$$\overset{(1,1)}{I}_1 - H\overset{(1,1)}{I}_2 = 3h\sum_{p=1}^{\infty}\frac{(\varepsilon R)^{2p-1}}{2p+3}\sum_{q=0}^{p-1}C_{2p}^{2q}H^{2p-2q-1}E^q,$$

$$\overset{(1,1)}{I}_2 = 3h^2\sum_{p=1}^{\infty}\frac{(\varepsilon R)^{2p-2}}{2p+3}\sum_{q=0}^{p-1}C_{2p+1}^{2q+1}H^{2p-2q-2}E^q.$$

Represent now the vectors **u** and **v** by means of the series with respect to the small parameter $\varepsilon$ in the form [4]

$$(\mathbf{u},\ \mathbf{v}) = \sum_{n=0}^{\infty}\Big(\overset{(n)}{\boldsymbol{u}},\ \overset{(n)}{\boldsymbol{v}}\Big)\varepsilon^n.$$

Introducing these expansions in (8.3) and (8.4) and substituting them into (8.1) and (8.2), we obtain the system of equations of equilibrium in terms of the components of the displacement vector, which for any $n$ has the form

$$\begin{cases} 4\mu h^2\dfrac{\partial}{\partial\bar z}\left(\dfrac{1}{\Lambda}\dfrac{\partial\overset{(n)}{u}_+}{\partial z}\right) + 2(\lambda+\mu)h^2\dfrac{\partial\overset{(n)}{\Theta}}{\partial\bar z} + 2\lambda h\dfrac{\partial\overset{(n)}{v}_+}{\partial\bar z} = \overset{(n)}{L}_+, \\ \mu h^2\nabla^2\overset{(n)}{v}_3 - 3\left[\lambda h\overset{(n)}{\Theta} + (\lambda+2\mu)\overset{(n)}{v}_3\right] = \overset{(n)}{M}_3, \end{cases} \tag{8.5}$$

$$\begin{cases} 4\mu h^2\dfrac{\partial}{\partial\bar z}\left(\dfrac{1}{\Lambda}\dfrac{\partial\overset{(n)}{v}_+}{\partial z}\right) + 2(\lambda+\mu)h^2\dfrac{\partial\overset{(n)}{\rho}}{\partial\bar z} - \\ \qquad -3\mu\left(2h\frac{\partial\overset{(n)}{u}_3}{\partial\bar z} + \overset{(n)}{v}_+\right) = \overset{(n)}{M}_+, \\ \mu h\left(\nabla^2\overset{(n)}{u}_3 + \overset{(n)}{\rho}\right) = \overset{(n)}{L}_3, \quad \left(\nabla^2 = \dfrac{4}{\Lambda}\dfrac{\partial^2}{\partial z\partial\bar z}\right), \end{cases} \tag{8.6}$$

where the values $\overset{(n)}{L}_i$ and $\overset{(n)}{M}_i$ $(i = 1, 2, 3)$ are expressed by means of solutions of the previous approximations, and hence are assumed to be known.

It should be noted that the left-hand side of the system (8.5) and (8.6) is analogous to I.N. Vekua's system of equations which he obtained for prismatic shells of constant thickness for the case $N = 1$, i.e. for the shells, whose midsurface is a plane. In our case the midsurface of shell is any smooth surface which first quadratic form is representable as

$$ds^2 = \Lambda(x^1, x^2)\left[(dx^1)^2 + (dx^2)^2\right], \quad \Lambda > 0.$$

The complex representation of a general solution of that system can be written as follows:

$$\begin{aligned}
\overset{(n)}{u}_+ &= -\frac{\lambda h}{6(\lambda+2\mu)}\frac{\partial\omega}{\partial\bar z} - \frac{5\lambda+6\mu}{3\lambda+2\mu}\frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\overline{\zeta})\varphi'(\zeta)d\xi d\eta}{\bar\zeta - \bar z} + \\
&+ \left(\frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\overline{\zeta})d\xi d\eta}{\bar\zeta-\bar z}\right)\overline{\varphi'(\zeta)} - \overline{\psi(z)}, \\
\overset{(n)}{v}_3 &= \omega - \frac{2\lambda h}{3\lambda+2\mu}[\varphi'(z) + \overline{\varphi'(z)}],
\end{aligned} \tag{8.7}$$

$$\begin{aligned}
\overset{(n)}{v}_+ &= i\frac{\partial\chi}{\partial\bar z} - 2h\overline{\Psi'(z)} - \frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\overline{\zeta})\Phi'(\zeta)d\xi d\eta}{\bar\zeta-\bar z} - \\
&- \left(\frac{1}{\pi}\iint_D \frac{\Lambda(\zeta,\overline{\zeta})d\xi d\eta}{\bar\zeta-\bar z}\right)\overline{\Phi'(z)} + \frac{2(\lambda+2\mu)h^2\overline{\Psi''(z)}}{3\mu}, \\
\overset{(n)}{v}_3 &= \Psi(z) + \overline{\Psi(z)} - \frac{1}{\pi h}\iint_D \Lambda(\zeta,\overline{\zeta})[\Phi'(\zeta) + \overline{\Phi'(\zeta)}]ln|\zeta - z|d\xi d\eta,
\end{aligned} \tag{8.8}$$

where $\varphi(z)$, $\psi$, $\Phi(z)$, $\Psi(z)$ are analytic functions of $z = x^1 + ix^2$, and $\omega$ and $\chi$ are the solutions of the following analytic equations:

$$\nabla^2\omega - \frac{3(\lambda+\mu)}{\lambda+2\mu)h^2}\omega = 0,$$
$$\nabla^2\chi - \frac{3}{h^2}\chi = 0.$$

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Author's address:

I. Vekua Institute of Applied Mathematics
I. Javakhishvili Tbilisi State University
2, University St., Tbilisi 0143
Georgia