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BOUNDEDNESS OF MULTIPLE CONJUGATE FUNCTIONS AND STRONG MAXIMAL FUNCTIONS IN WEIGHTED GRAND $L^{p)}$ SPACES

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Let $T^n = T \times T \times \cdots \times T$, where $T = [-\pi, \pi]$.

The weighted grand Lebesgue space $L_w^{p)}(T^n)$, $1 < p < \infty$ is a Banach function space of 2π-periodic with respect to each variable function $f : T^n \to R^1$ defined by the norm

$$\|f\|_{L_w^{p)}(T^n)} = \sup_{0<\varepsilon<p-1} \left(\frac{\varepsilon}{(2\pi)^n} \int\limits_{T^n} |f(x)|^{p-\varepsilon} w(x)dx \right)^{\frac{1}{p-\varepsilon}}.$$

The grand Lebesgue space $L^{p)}$ was introduced by T. Iwaniec and C. Sbordone [1].

We discuss the boundedness problem of the following integral operators:

$$\widetilde{f}(x) = \frac{1}{(2\pi)^n} \int\limits_{T^n} f(x_1 + s_1, \dots, x_n + s_n) \prod_{j=1}^{n} \operatorname{ctg} \frac{s_j}{2} ds, \quad x = (x_1, \dots, x_n)$$

-the multiple conjugate function and

$$M_s f(x) = \sup_{J \ni x} \frac{1}{|J|} \int\limits_J |f(y)|dy$$

-the strong maximal function. By J we denote n-dimensional rectangles with sides parallel to the coordinate axis and edges with length not greater than 2π.

Definition 1. A almost everywhere positive measurable function $w : R^n \to R^1$ is said to be class $\widetilde{A_p}(T^n)$ if

$$\sup_J \frac{1}{|J|} \int\limits_J w(x)dx \left(\frac{1}{|J|} \int\limits_J w^{1-p'}(x)dx \right)^{p-1} < \infty.$$

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Theorem 1. *Let $1 < p < \infty$. Then the following conditions are equivalent:*

(i) $\widetilde{f}$ *is bounded in* $L_w^{p)}$

(ii) M_s *is bounded in* $L_w^{p)}(T^n)$

(iii) $w \in \widetilde{A_p}(T^n)$.

Let σ_m^α, $m = (m_1, \ldots, m_n)$, $\alpha = (\alpha_1, \ldots, \alpha_n)$, $\alpha_j > 0$ $(j = 1, \ldots, n)$ be the Ćesaro means of multiple trigonometric Fourier series:

$$\sigma_m^\alpha(f, x) = \frac{\sum\limits_{j=0}^{m_1} \cdots \sum\limits_{j=0}^{m_n} A_{m_1 - j_1}^{\alpha_1 - 1} \cdots A_{m_n - j}^{\alpha_n - 1} S_{j, \ldots, j_n}(f, x)}{\prod\limits_{j=1}^{n} A_{m_j}^{\alpha_j}}$$

where $A_k^\beta = \binom{k}{\beta}$ and $S_{j_1, \ldots, j_n}$ are the rectangle partial sums of Fourier series.

Then we have

Theorem 2. *Let $1 < p < \infty$ and $w \in \widetilde{A_p}(T^n)$. Then*

$$\lim_{m \to \infty} \|\sigma_m^\alpha(f) - f\|_{p),w} = 0$$

for arbitrary $f \in L_w^{p)}(T^n)$.

Note that it is known $L_w^p \subset L_w^{p)} \subset L^{p-\varepsilon}$ with $0 < \varepsilon < p - 1$.

For the boundedness of Hardy-Littlewood maximal function in weighted grand $L^{p)}$ spaces we refer to [2]. For boundedness of Hilbert transform and Cauchy singular integral in $L_w^{p)}$ spaces was studied in [3] and [4] respectively.

Analogous problems in weighted Orlicz classes were investigated in [5].

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