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M. Gabidzashvili and Ts. Tsanava

ON THE ESTIMATES OF THE DEVIATION BY CESARO AND ABEL-POISSON MEANS IN WEIGHTED LORENTZ SPACES

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Let $\mathbf{T} = [-\pi, \pi)$ and $w : \mathbf{T} \to \mathbb{R}^1$ be an almost everywhere positive, integrable function. Let $f^*_w(t)$ be a nondecreasing rearrangement of $f : \mathbf{T} \to \mathbb{R}^1$ with respect to the Borel measure

$$w(e) = \int_e w(x)dx,$$

i.e.,

$$f^*_w(t) = \inf\left\{\tau \geq 0 : w\left(x \in \mathbf{T} : |f(x)| > \tau\right) \leq t\right\}.$$

Let $1 < p, s < \infty$ and let $L^{ps}_w(\mathbf{T})$ be a weighted Lorentz space, i.e., the set of all measurable functions for which

$$\|f\|_{L^{ps}_w} = \left(\int_{\mathbf{T}} \left(f^{**}(t)\right)^s t^{\frac{s}{p}} \frac{dt}{t}\right)^{1/s} < \infty,$$

where

$$f^{**}(t) = \frac{1}{t}\int_0^t f^*_w(u)du.$$

By $E_n(f)_{L^{ps}_w}$ we denote the best approximation of $f \in L^{ps}_w(\mathbf{T})$ by trigonometric polynomials of degree $\leq n$, i.e.,

$$E_n(f)_{L^{ps}_w} = \inf \|f - T_k\|_{L^{ps}_w},$$

where the infimum is taken with respect to all trigonometric polynomials of degree $k \leq n$. The weights w used in the paper are those which belong to

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the Muckenhoupt class $A_p(\mathbf{T})$, i.e., they satisfy the condition

$$\sup \frac{1}{|I|}\int_I w(x)dx \left(\frac{1}{|I|}\int_I w^{1-p'}(x)dx\right)^{p-1} < \infty, \qquad p' = \frac{p}{p-1}$$

where the supremum is taken with respect to all the intervals I with length $\leq 2\pi$ and $|I|$ denotes the length of I.

Let $T := [-\pi, \pi]$ and let

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty}(a_k coskx + b_k sinkx)$$

be the Fourier series of the function $f \in L^1(T)$.

The Cesàro mean of order $\alpha > 0$, σ_n^α, is defined as

$$\sigma_n^\alpha(f, x) = \frac{1}{\begin{pmatrix} n+\alpha \\ n \end{pmatrix}} \sum_{k=0}^{n} \begin{pmatrix} n-k+\alpha \\ n-k \end{pmatrix} A_k(x), \qquad \alpha > 0$$

where

$$A_0 = \frac{a_0}{2} \quad \text{and} \quad A_k(x) = a_k coskx + b_k sinkx.$$

Let also

$$u_r(f, x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k(x) r^k$$

be the Abel-Poisson means of function $f(x)$.

The following statement are true:

Theorem 1. *Let $1 < p < \infty$ and $1 < s \leq 2$ or $p > 2$ and $s \geq 2$. Let $w \in A_p(\mathbb{T})$. Then there exists a positive constant c such that*

$$\|f(x) - \sigma_n^\alpha(f, x)\|_{L_w^{ps}} \leq \frac{cps}{n} \left\{ \sum_{\nu=1}^{n} \nu^{\gamma-1} E_{\nu-1}^{\gamma}(f)_{L_w^{ps}} \right\}^{1/\gamma}$$

for arbitrary $f \in L_w^{ps}(\mathbb{T})$ and natural n, where $\gamma = min(s, 2)$.

Theorem 2. *Let $1 < p < \infty$ and $1 < s \leq 2$ or $p > 2$ and $s \geq 2$. Let $w \in A_p(\mathbb{T})$. Then there exists a positive constant c such that*

$$\|f(x) - u_r(f, x)\|_{L_w^{ps}} \leq c_{ps}(1-r) \left(\sum_{\nu=0}^{\infty} r^\nu (\nu+1)^{\gamma-1} E_\nu^\gamma(f)_{L_w^{ps}} \right\}^{1/\gamma}$$

for arbitrary $f \in L_w^{ps}(\mathbb{T})$ and natural n, where $\gamma = min(s, 2)$.

This results generalize the estimates obtained in [1].

References

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Authors' address:
Department of Mathematics
Georgian Technical University
77, M. Kostava St., Tbilisi 0175
Georgia