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ON BELLMAN TYPE TRANSFORMS FOR DOUBLE TRIGONOMETRIC FOURIER SERIES

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In the present paper our discussion concerns the mapping property in the weighted Lebesgue spaces of Bellman [1] type transforms for double Fourier series. An analogous problem for the one-dimensional case has been considered in [2].

Almost everywhere, a positive summable function $\rho : \mathbb{R}^2 \rightarrow \mathbb{R}^1$ will be called a weighted function. By $L_\rho^p(\mathbb{R}^2)$ ($1 \leq p < \infty$) we denote the Banach space of all those measurable functions for which

$$\|f\|_{L_\rho^p(\mathbb{R}^2)} = \left(\int_{\mathbb{R}^2} |f(x, y)|^p \rho(x, y) \, dx dy \right)^{1/p} < \infty.$$

In the sequel, the weighted functions $v(x, y)$ and $w(x, y) = w_1(x)w_2(y)$ will be assumed to satisfy the conditions:

$$\sup \left(\int_0^a w_1^{1-p'}(x) \, dx \right)^{1/p'} \left(\int_0^b w_2^{1-p'}(y) \, dy \right)^{1/p'} \left(\int_a^\infty \int_b^\infty \frac{v(x, y)}{(xy)^p} \, dx dy \right)^{1/q} < \infty. \quad (1)$$

The following theorem is valid.

Theorem. *Let the condition (1) be fulfilled. Let the 2π -periodic with respect to each of variables, summable function $f \in L_w^p(\mathbb{R}^2)$ ($1 < p < \infty$) and its Fourier series have the form*

$$f(x, y) \sim \sum_{m, n=1}^{\infty} a_{mn} \cos mx \cos ny.$$

Moreover, let

$$A_{mn} = \sum_{i=m+1}^{\infty} \sum_{k=n+1}^{\infty} \frac{a_{ik}}{ik} + \frac{1}{2m} \sum_{k=n}^{\infty} \frac{a_{mk}}{k} + \frac{1}{2} \sum_{i=n}^{\infty} \frac{a_{in}}{i} - \frac{3}{4} \frac{a_{mn}}{mn}.$$

Then the double trigonometric series

$$\sum_{m, n=1}^{\infty} A_{mn} \cos mx \cos ny$$

is the Fourier series of some function $F \in L_v^p(\mathbb{R}^2)$, and there exists the positive constant c , such that

$$\|F\|_{L_v^p(\mathbb{R}^2)} \leq c \|f\|_{L_w^p(\mathbb{R}^2)}.$$

Similar statements are valid for double sine-series as well as for the series of type $\sum_{m, n=1}^{\infty} a_{mn} \sin mx \sin ny$.

Important tool used in proving our Theorem are the results obtained in [3].

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