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ON BELLMAN TYPE TRANSFORMS FOR DOUBLE TRIGONOMETRIC FOURIER SERIES

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In the present paper our discussion concerns the mapping property in the weighted Lebesgue spaces of Bellman type transforms for double Fourier series. An analogous problem for the one-dimensional case has been considered in [2].

Almost everywhere, a positive summable function $\rho : \mathbb{R}^2 \rho R^1$ will be called a weighted function. By $L^p_\rho(R^2)$ $(1 \le p < \infty)$ we denote the Banach space of all those measurable functions for which

$$\|f\|_{L^p_\rho(R^2)} = \Big(\int\limits_{R^2} |f(x,y)|^p \rho(x,y)\,dxdy\Big)^{1/p} < \infty.$$

In the sequel, the weighted functions $v(x,y)$ and $w(x,y) = w_1(x)w_2(y)$ will be assumed to satisfy the conditions:

$$\sup_{a,b>0}\Big(\int\limits_0^a w_1^{1-p'}(x)\,dx\Big)^{1/p'}\Big(\int\limits_0^b w_2^{1-p'}(y)\,dy\Big)^{1/p'}\Big(\int\limits_a^\infty\int\limits_b^\infty \frac{v(x,y)}{x^p y^p}\,dxdy\Big)^{1/p'} < \infty, \tag{1}$$

$$\sup_{0<y<\pi}\sup_{a>0}\Big(\int\limits_a^\infty \frac{v(x,y)}{x^p}\,dx\Big)^{1/p}\Big(\int\limits_0^a w_1^{1-p'}(x)\,dx\Big)^{1/p'} < \infty, \tag{2}$$

$$\sup_{0<x<\pi}\sup_{b>0}\Big(\int\limits_b^\infty \frac{v(x,y)}{y^p}\,dy\Big)^{1/p}\Big(\int\limits_0^b w_2^{1-p'}(y)\,dy\Big)^{1/p'} < \infty. \tag{3}$$

The following theorem is valid.

Theorem. *Let the conditions* (1), (2) *and* (3) *be fulfilled. Let the 2π-periodic with respect to each of variables, summable function $f \in L^p_w(R^2)$ $(1 < p < \infty)$ and its Fourier series have the form*

$$f(x,y) \sim \sum_{m,n=1}^\infty a_{mn}\cos ny.$$

Moreover, let

$$A_{mn} = \sum_{i=m+1}^\infty \sum_{k=n+1}^\infty \frac{a_{ik}}{ik} + \frac{1}{2m}\sum_{k=n}^\infty \frac{a_{mk}}{k} + \frac{1}{2}\sum_{i=n}^\infty \frac{a_{in}}{i} - \frac{3}{4}\frac{a_{mn}}{mn}.$$

Then the double trigonometric series

$$\sum_{m,n=1}^\infty A_{mn}\cos mx\cos ny$$

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is the Fourier series of some function $F \in L^p_v(R^2)$, and there exists the positive constant c, such that

$$\|F\|_{L^p_v(R^2)} \le c\|f\|_{L^p_w(R^2)}.$$

Similar statements are valid for double sine-series as well as for the series of type $\sum\limits_{m,n=1}^{\infty} a_{mn}$.

Important tool used in proving our Theorem are the results obtained in [3].

References

1. R. Bellman, A note on a theorem of Hardy on Fourier constants. *Bull Amer. Math. Soc.* **50**(1944), 741–744.
2. Ts. Tsanava, On mapping properties of Bellman transform in weighted Lebesgue spaces. *Proc. A. Razmadze Math. Inst.* **137**(2005), 141–143.
3. A. Meskhi, On two-weighted inequalities for multiple Hardy-type operators. *Proc. A. Razmadze Math. Inst.* **136**(2004), 149–153.

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