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Professor David Kveselava

(To the 100th Birthday Anniversary)

Short Biography

The childhood of David Kveselava went off in the village of Letsitskhvaie not far from Martvili (Georgia). He was born in the family of a local villager Aleqsandre Kveselava on August 25, 1911. The family was close-knit where national traditions and education were respected. Along with other virtues, the parents tried to inspire in their sons, David and Michael, the love to native land, dignity and diligence. Besides, children inherited naturally the thirst to knowledge. Actually in those early years their future successful creative stile of life was initiated.

In 1930, to give children a better education Kveselavas moved to Tbilisi. A year later David graduated from a secondary school and continued his education at the faculty of Physics and Mathematics of Tbilisi State University. His diligence and skills were not left unnoticed. In 1937, as a high

Michael Kveselava (1913–1995) became an accomplished philologist(German studies), writer and philosopher. He wrote books: “Faustian Paradigms” (1961), “Adam Mickiewicz” (1965), “A hundred and fifty days” (about World War II and the Nuremberg Trials, 1967-1971), “Poetical Integrals” (1977).

achiever student and diplomant, he called the attention of Niko Muskhelishvili who was at the time Chair of the State Examination Council. Due to Muskhelishvili's recommendation, David started his research activity under the supervision of Michael Alekseevich Lavrent'ev, the prominent Soviet mathematician and mechanic.

After graduating from the university, David continued his post-graduate course (aspirantura) at the Institute of Mathematics of the Georgian Branch of Academy of Sciences of the USSR (at that time Georgian Academy of Sciences did not exist independently). The years of graduate studies were both fruitful and interesting. His research activity was very intense, requiring a lot of energy and patience. In spite of such a heavy load, he always found time and grant attention to an active pedagogical activity. From the very first steps of his post-graduate studies he taught at Tbilisi Institute of Railway Engineering, Tbilisi State University and Gogebashvili Telavi Pedagogical Institute where he headed the Mathematical Department. It was the time when David was being formed as a professional teacher and lecturer with his unique manner of confident explanation of sometimes very complicated topics in his laconic and clear language (both Georgian and Russian). As time passed, his pedagogical skills and style got perfect. David's numerous students and listeners will remember forever his colorful speech and vivid image. In 1940, the All-Union Supreme Certifying Commission conferred David the rank of Associate Professor (Docent) for his successful pedagogical work. This was the year when the first triennial of David's creative work was summed up: he has completed his PHD (candidate dissertation) and defended it successfully. The title of the dissertation was "On the Theory of Conformal Mappings". The notion of "conformal" is adopted from the late Latin and means "like" or "similar". For example, the mapping of a sphere onto the plane is conformal, if the angle between two arbitrary directions emanating from any point of the sphere transforms into the same angle (in value) on the plane. Such mappings are somewhat related to creation of geographic maps, so that they didn't lose their meaning in cartography (and not only in cartography). If a plane is being transformed onto the plane, then the mapping is conformal, if an additional condition is satisfied. Namely, a circle should be transformed onto a circle, and the circumference should be transformed onto a circumference. However, here by circles and circumferences we mean not just usual ones, but those whose radii are infinitely small. It turned out that such mappings are nothing but analytic functions of a complex argument. A function is analytic in a neighborhood of a point, if it can be represented by the sum of infinitely many powers of the argument multiplied by a corresponding complex coefficients. And therefore, there naturally arose the necessity to compare the properties of conformal mappings on the one hand and the analytic functions on the other. David Kveselava faced the problem of estimation the mapping parameters when the difference between the image and pre-image domains was insignificant. The problem was studied before by the well-known experts. Professor Lavrent'ev wished his student to solve out the problem in much more greater generality. David jointly with his adviser succeeded in significant generalization of results of the prominent Russian mathematician A.Ostrogradskii. David's contribution was so important and interesting that Michael Alekseevich decided the paper should have been signed solely by David Kveselava. After the successful defense of the dissertation David was left at the Institute of Mathematics. His first position there was Academic Secretary of the institute. However, shortly he was appointed to the position of Senior Researcher, and in 1941 he got the corresponding official rank.

When it became clear that David possesses a talent of a scientific organizer, he was appointed to the position of Academic Secretary of Department of Natural Sciences of the Georgian Academy of Sciences. The administrative activity never hampered his research work.

In 1952, David defended his doctoral dissertation at Moscow Steklov Institute of Mathematics, the leading Soviet Mathematical Institute. The title of the dissertation was "Some Boundary Problems of the Function Theory and Singular Integral Equations". The decision of unprejudiced Council was unanimous in favor of David. Meanwhile David continued his pedagogical activity. He was an extraordinary lecturer at the university. Many remember his calm, but sometimes hot-tempered manner of teaching. Shortly, two years later, in 1954, he got the official rank of Professor.

October 8 of 1956 is a special day in David Kveselava's biography: he was appointed the director of newly created Computer Center of Academy of Sciences of Georgia. It was the time when science, engineering and technology started developing at a briskly pace. All the related branches such as computer engineering, computer mathematics and numerical analysis were facing new problems to handle the development. To solve out the emerged problems, in all developed nations new research centers and institutions were created. The Soviet Union of those days was not an exception. Under these incredible conditions everybody had his own opinion how to handle this new and unusual situation. Many things depended on the talent of a scientific organizer and managerial abilities of the principal. Heavens have showered gifts on David Kveselava. He gathered a strong team of young skilled scientific-technical personnel that created an effectively acting Computer Center. David Kveselava was the unchallenged leader of the institute till the end of his life. David Kveselava passed away on November 6, 1978. Besides huge organizational contribution, David Kveselava's scientific and research legacy consists of many scientific works and a monograph. The ideas and traditions laid by him are still alive: the Niko Muskhelishvili

Institute of Computational Mathematics continues its fruitful work as a part of the Georgian Technical University. The scientific community always highly appreciated David Kveselava's scientific and pedagogical merits. The title "Honored Georgian Scientist" as well as other awards were conferred on him. However, the main award is probably the great love and respect of his pupils and collaborators who will remember him forever.

Short Review of David Kveselava's Scientific Legacy

David Kveselava always was fair and favorable to every person irrespectively of his rank or age, in both personal and business relationships. All the above-said along with his natural skills in the teamwork left its mark on his creative work. A large part of his joint works with colleagues (with his teachers or students) represent excellent examples of scientific collaboration.

His results worked out in collaboration with his associates or solely by him belong to several branches of mathematics which are closely related to each other. As we have mentioned above, these include theories of conformal mappings and analytic functions of a complex variable, as well as the theory of singular integral equations and systems of integral equations. The problems arisen in these directions were prompted by the practical need, and many of them were of theoretical importance at that time. Nowadays, these directions belong to classical ones, and a solid part of them belongs to David Kveselava. The directions below are so interlaced, that their separation is a pretty difficult problem. Nevertheless, in order to make a short review of Professor Kveselava's legacy transparent, we split it into several parts. It is natural to start with his first results.

1. Methods of Approximations in Conformal Mappings

Here we speak of conformal mappings f and f_1 of mutually close simply connected domains D and D_1 respectively, given on the complex plane $z = x + iy$, onto the circle $|w| < 1$ and the estimation of $|f - f_1|$, the modulus of the difference of the mappings in question. In the joint with academician Lavrent'ev paper [1] the domain D of the complex plane z contains the origin, and its boundary lies completely in the ring $\theta < |z| < 1$, $\theta \geq 1/2$. Note that this sort of conditions does not cause a significant loss of generality. The parameter θ is not specified so far; the smaller the difference $1 - \theta$ is, the closer is the domain D to the unit circle D_1 of the z-plane. Obviously, the identity function $w = f_1(z) = z$ maps the unit circle D_1 of the z-plane onto the unit circle $|w| < 1$ of the w-plane. And if the function $w = f(z)$ also maps the domain D onto $|w| < 1$, then for any z with $|z| \leq \theta$ the following estimation holds

$$|f(z) - f_1(z)| = |f(z) - z| \leq k(1 - \theta) \log \frac{1}{1 - \theta}\,.$$

By means of this inequality the authors also approved analytically that the closer are the domains, the lesser is the difference between the corresponding mappings. Moreover, it turned out that at the neighborhood of the origin the difference is lesser than at a distant points. The coefficient on the left-hand side is an absolute constant. Also, the coefficient is an absolute constant, if the argument is inside the aforementioned circle, i.e.

$$|z| \leq \theta_1 \theta$$

where $0 < \theta_1 < 1$. In this case the estimation has the following refined form:

$$|f(z) - z| \leq k(1-\theta) \log \frac{1}{1-\theta_1} \, .$$

Here the transforming mapping solely is estimated. Also of interest are results related to inverse mappings when the unit circle is being conformally transformed onto two mutually close simply connected domains. One of the Kveselava's results in this direction sounds as follows: Let D and D_1 be ε-radially close domains such that D_1 is contained in D and let the functions f, $f(0) = 0$ and f_1, $f_1(0) = 0$, transform conformally the circle $|z| < 1$, respectively on the domains D and D_1. Then the following inequality holds

$$|f'(0)| \leq |f_1'(0)| + 4\varepsilon \, .$$

The well-known Lindelof principle deals with the sign of this variation only, and therefore, Kveselava's result represents a significant refinement of the principle.

The last result has an interesting application to the estimation of the modulus of the derivative in the case of an approximate conformal mapping. Let us single out one of them: If the boundary of D belongs to the ring $1 < |w| < 1 + \varepsilon$ and the function f, $f(0) = 0$, transforms conformly the circle $|z| < 1$ onto the domain D, then for $|z| < r_0$, $r_0 \geq 0.1$, we have

$$1 \leq |f(z)| \leq 1 + \varepsilon \, .$$

In the direction of conformal mappings we have also to mention the following Kveselava's theorem: Let D and D_1 be two star domains with respect to the origin $w = 0$ which are radially close. If the functions f, $f(0) = 0$, $f'(0) > 0$, and f_1, $f_1(0) = 0$, $f_1'(0) > 0$, transform the circle $|z| < 1$, conformly onto the domains D and D_1 respectively, then

$$|f(z) - f_1(z)| < M(r)\varepsilon \text{ for } |z| \leq r < 1 \, ,$$

where $M(r)$ depends only on r. He also has shown that the result cannot be extended to arbitrary Jordan domains.

D. Kveselava also has original and interesting results on the behavior of the derivatives of two conformal mappings on the joint part of their boundaries. We give here one of them. Let D and D_1 be neighboring domains with the right part γ of the joint boundary and let D and D_1

contain points z^0 and z_1^0 respectively. Assume that functions f, $f(z^0) = 0$, and f_1, $f_1(z_1^0) = 0$, transform conformly the domains D and D_1 onto the circle $|z| < 1$. Then for any point t of the arc γ the following inequality holds

$$|f'(t) f_1'(t)| \leq \frac{4}{\rho^2(t; z^0, z_1^0)} \,,$$

where $\rho(t; z^0, z_1^0)$ is the minimum between the distances $\rho(t; z^0)$ and $\rho(t; z_1^0)$ considered respectively in the domains D and D_1.

D. Kveselava has got significant results on conformal mappings by use of methods of the theory of integral equations. For finding needed conformal mappings (for both simply connected and multiply connected canonical domains) he introduced new integral equations. A significant advantage of these equations compared with analogous equations existed before his findings, was the greater possibilities of using numerical methods. D. Kveselava's students have elaborated the methods of approximate solutions to the integral equations in question and estimated the corresponding error. We stress that D. Kveselava has got explicitly the form of the moduli of conformal mappings of doubly-connected domains. In a joint paper with Z. Samsonia he has used this representation to show that for a sufficiently wide class of boundaries of ε-close doubly-connected domains D and D_1 the order of the difference of the conformal moduli equals ε.

2. Boundary Problems of the Theory of Analytic Functions

Linear boundary problems of the theory of analytic functions have been studied by Soviet mathematicians profoundly, and D. Kveselava took an active part in the study. In their joint paper D. Kveselava and N. Vekua found the conditions under which the Hilbert boundary value problem for several unknown functions can be solved out in quadratures. D. Kveselava has found advanced results for the Hilbert boundary problem in the case of a single unknown functions, open contour and discontinuous coefficients. The problem was solved out in several papers jointly with N. Muskhelishvili and included almost completely into his monograph “Singular Integral Equations”. One of the most important results of this cycle is the division of the set of solutions by classes and the introduction, in the case of open contour and discontinuous coefficients, of the related fundamental notion, the index of a boundary value problem. Their results here are as final and complete as those found before for the case of a closed contour and continuous coefficients. It should be noticed that D. Kveselava has got the solution to the Hilbert boundary value problem also for piece-wise meromorphic functions and mutually intersected domains. The last result has many both theoretical and practical applications. However Kveselava's most advanced results are related with shift included boundary problems for analytic functions. In them the boundary conditions represent the linear relations of the boundary values computed at different points of the boundary. Problems of this type go back to Riemann. Namely, he posed the problem of finding an analytic function of a complex variable, if on the boundary of the domain there is given an equation containing the real and imaginary parts of the function. Riemann came up with a series of conjectures over solvability of this problem, but he did not give a rigorous proof of the problem. It was Hilbert in the beginning of 20-th century who proved the conjectures in some particular cases. He considered a holomorphic function $\Phi = u + iv$ with the following boundary condition

$$\alpha u + \beta u = \gamma \,. \tag{1}$$

Several works due to Hilbert were devoted to this problem. One of them he showed that this problem can be solved out for the Laplace equation by reduction to two Dirichlet problems. The method was very sophisticated, however the possibilities of its applications are restricted: the problem is solvable completely only for simply connected domains. For problem (1) Hilbert has used the method of integral equations that reduces the problem to a singular integral involving the principal value of the Cauchy integral. However such equations were not studied well at that time. Because of this, while extending the Fredholm's theorems to singular equations, Hilbert got a wrong answer to problem (1).

Nevertheless, Hilbert's works were of great importance. For the first time it was shown a close relationship between the boundary value problems for analytic functions and singular integral equation with the Cauchy kernel.

Along with problem (1), Hilbert also considered the following problem: find two functions Φ^+ and Φ^- which for the given closed curve L satisfy the condition

$$\Phi^+(t) = G(t)\Phi^-(t) + g(t), \quad t \in L \,, \tag{2}$$

where Φ^+ is holomorphic inside of L, while Φ^- is holomorphic outside of L. Hilbert has solved this problem by means of the methods of integral equations, although he did not get any essential success.

After Hilbert's works the Riemann problem went in two directions: first, the problems posed by Hilbert that found further development with more general domains and coefficients; and there were also attempts of solving more general problems that Hilbert did not consider.

In 1907 Hilbert's student Charles Haseman posed the following problem: find two functions Φ^+ and Φ^- which along a given simple closed curve L satisfy the condition

$$\Phi^+[\alpha(t)] = G(t)\Phi^-(t) + g(t), \quad t \in L \,, \tag{3}$$

where Φ^+ is holomorphic inside of L, while Φ^- is holomorphic outside of L and α is a complex one-to-one function of a variable $t \in L$ which maps bijectively the curve onto itself. If $\alpha(t) = t$, then we obviously get Problem (2).

Problem (1) was generalized by Torsten Carleman, who in 1932 considered the following problem:

$$\Phi^+[\alpha(t)] = G(t)\Phi^+(t) + g(t), \quad t \in L\,, \tag{4}$$

where Φ^+ is holomorphic inside of L, while α, G and g are as in the previous problem.

Along with the aforementioned problems, D. Kveselava has considered the following ones:

$$\Phi^+[\alpha(t)] = G(t)\overline{\Phi^-(t)} + g(t), \quad t \in L\,, \tag{5}$$

$$\Phi^+[\alpha(t)] = G(t)\overline{\Phi^+(t)} + g(t), \quad t \in L\,. \tag{6}$$

Let us note that before Kveselava's works the Hilbert problem (2) was best and completely studied, whereas problems (3)–(6) were investigated pretty superficially. According to academician I. Vekua, D. Kveselava "...succeeded in making an essential step forward in their study. He managed to get results in the same complete form as those that existed in the case of Hilbert problem (2)... I believe, it is necessary to stress that by solving the listed problems D. Kveselava achieved outstanding scientific results. In this direction only Carleman has made a valuable step forward when he constructed simple integral equations for his problem. Whereas D. Kveselava has solved out the most complicated question: he has proved that the Carleman equation, as well as many other integral equations obtained for similar problems are soluble; based on his findings he managed to prove a series of significant theorems for problems (3)–(6)."

These results played an important role in further development of singular integral equation theory related to the boundary problems, and became a stimulating factor for other research works in this direction. Many authors have developed methods for generalization of D. Kveselava's theorems.Special attention was given to the case of unknown systems of functions.

We present here a result obtained by D. Kveselava, which represents a key for solving problems in the whole area. Let α_+ (α_-) be a differentiable homeomorphism of a simple closed Lyapunov contour L into itself which preserves (changes) the orientation of L. Assume also that $|\alpha_+(t)| > 0$ and $|\alpha_-(t)| > 0$ for $t \in L$ and α_+ on L satisfies the Holder condition. Then the elementary boundary problems on L

$$\Phi^+[\alpha_+(t)] = \Phi^-(t)\,,$$
$$\Phi^+[\alpha_-(t)] = \Phi^+(t)\,,$$

do not have analytic solutions except constants.

3. Theory of Singular Integral Equations

The solution of the Hilbert boundary value problem for an open contour or discontinuous coefficients has given an effective way for construction, in corresponding cases, of the theory of integral equations of the following form (they involve the principal value of the Cauchy integral):

$$\alpha(t)\varphi(t) + \frac{\beta(t)}{\pi i}\int\limits_L \frac{\varphi(\tau)d\tau}{\tau - t} + \frac{1}{\pi i}\int\limits_L K(t,\tau)\varphi(\tau)d\tau = f(t)\,,$$

and, as we have already remarked, the theory was given a completed form, as it was done before for the case of closed contour and continuous coefficients.

In Kveselava's works such a theory has been constructed in the following cases:

(i) The curve L consists of finitely many detached open smooth arcs, and the functions α, β, K belong to the Hölder class;

(ii) The curve L consists of finitely many detached closed smooth contours, and the functions α, β, K satisfy the Hölder condition everywhere on L except for the finitely many points of simple discontinuity;

(iii) The curve L consists of finitely many closed and open piece-wise smooth contours having finitely many intersections, and the functions α, β, K belong to the Hölder class.

Using in the related classes of solutions also the notion of corresponding indices, in the aforementioned cases made it possible to construct a complete theory of such integral equations. Nowadays these results on singular integral equations are regarded as a part of the classics.

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