I N T E R N A T I O N A L C O N F E R E N C E

CONTINUUM MECHANICS AND RELATED PROBLEMS OF ANALYSIS

to Celebrate the 70th Anniversary
of the Georgian National Academy of Sciences
&
the 120th Birthday of its First President
Academician Nikoloz (Niko) Muskhelishvili

BOOK OF ABSTRACTS

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"Tbilisi University?! This is just a mirage! It is virtually impossible to express even the simplest terminology of sciences like mathematics, chemistry and biology in Georgian that has no appropriate tradition!" – argued numerous opponents of the foundation of the University in Tbilisi.

Ivane Javakhishvili and his associates refuted this skepticism, and as early as November 1918 Professor Andrea Razmadze delivered his first lecture in Mathematical Analysis in perfect Georgian. Later, a young graduate of Moscow University Archil Kharadze communicated mathematics in Georgian in the same elegant way. General Andrea Benashvili, still wearing his uniform, did likewise in Astronomy. Soon they were joined by Giorgi Nickoladze and Nikoloz Muskhelishvili.

Shalva Nutsubidze recalls: "Andrea Razmadze, then the Dean of the Physics and Mathematics Faculty, entered my room followed by an energetic-looking young man. – He has just come from Petrograd where he had been engaged in scientific work. I need your consent to offer him a position. He seems talented and energetic. Andrea Razmadze's request was of course granted. The young scientist fully justified Andrea Razmadze's faith in him. That man was Nickoloz Muskhelishvili whom we all know now."

* * *

Andrea Razmadze, Giorgi Nikoladze, Nikoloz Muskhelishvli and Achil Kharadze – "the Great Four" – were the founders of the Georgian Mathematics School. It is difficult to imagine the amount of work done by "the four" in a decade. On top of the intensive pedagogical work, they had to establish and refine scientific mathematical terminology in Georgian, to write and publish the first original textbooks in their native language and to form foundations of scientific research in various branches of mathematics.

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In 1929, Andrea Razmadze passed away unexpectedly. Giorgi Nikoladze passed away in 1931. The whole burden fell upon Nikoloz Muskhelishvili, Archil Kharadze and upon some young mathematicians who had graduated from Tbilisi University. The latter were shown great examples of devotion and service to their country by their senior colleagues who never stopped their research and pedagogical work at Tbilisi State University and Georgian Polytechnic Institute.

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During his career, Niko Muskhelishvili has worked as the Dean of the University Polytechnic Faculty, the Dean of the Physics and Mathematics Faculty, as the Pro-Rector of the Georgian Polytechnic Institute, the Chair in Theoretical Mechanics, as well as the Head of the Physics, Mathematics and Mechanics Institute, which had been founded at the University by his initiative. With his usual energy and enthusiasm, he also continued his pedagogical work. The older generation still remember his lectures in analytic geometry, theoretical mechanics, and the theory of differential equations

Niko Muskhelishvili wrote an original textbook in analytic geometry which was published several times and was widely regarded as one of the main University textbooks. Originality is also a distinctive feature of his "Course in Theoretical Mechanics" which was published in two parts "Statics" and "Kinematics" in 1926 and 1928 respectively and which later appeared in a second edition. Niko Muskhelishvili started to work on mathematical terminology soon after returning to his country. In the preface of "Mathematical Terminology" (1944, Russian-Georgian part), its editor professor Vukol Beridze wrote: "A particularly great contribution to the terminology is due to Academician N. Muskhelishvili who checked each word and tried to achieve the maximum accuracy and conformity between a mathematical notion and the term that describes it."

It should be noted here, that the universally used term "toloba" (equality) and the naturally derived from it "utoloba" (inequality) and "gantoleba" (equation) were introduced by Niko Muskhelishvili in the early twenties.

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In 1922, Niko Muskhelishvili's book "Applications des intégrates analogues à celles de Cauchy à quelques problémes de la physique mathématique" was published in French in Tbilisi. This was in a sense a predecessor of his fundamental monograph "Some Basic Problems of the Mathematical Theory of Elasticity" (1933) which was based on the lectures delivered by the author in 1931-32 for the staff of the Leningrad Seismologic Institute and for PhD students of the Physics and Mathematics Institute as well as the Mathematics and Mechanics Institute of the Leningrad University.

The monograph soon gained popularity and its author became recognised as a prominent expert in elasticity theory. The same year,1933, Muskhelishvili was elected a Corresponding Member of the Academy of Sciences of the USSR, and in 1939 he became a Full Member of the Academy. At the same time, he served as the Chairman of the Georgian Branch of the Academy of Sciences of the USSR.

When the Academy of Sciences was established in Georgia in 1941, Muskhelishvili was unanimously elected its President. At the very first meeting of the Academy on 27 February 1941, Muskhelishvili ended his speech as follows: "Unfortunately today's festive mood is spoiled by the feeling that the scientist who had been looking forward to this great day with an utmost admiration is no longer among us. There is no doubt that had Ivane Javakhishvili been alive, he would have taken the high position that I am honoured to take now.”

An extended second edition of “Some Basic Problems of the Mathematical Theory of Elasticity” was published in 1935, and its author was awarded a Stalin Prize in 1941. Muskhelishvili received the same Prize in 1946 for his other well known monograph “Singular Integral Equations”. Before that, in 1945 Academician Nikoloz Muskhelishvili was awarded the title of a “Hero of Socialist Labour”.

* * *

Both monographs have been translated and published abroad in many languages. Many complimentary reviews have been written about them.

* * *

Muskhelishvili’s scientific work was recognized by dozens of prizes and awards. In particular, the Turin Academy of Sciences awarded him in 1969 its international prize and gold medal “Modesto Paneti”. Our fellow countryman was the first Soviet and the sixth world scientist whose scientific achievements were marked with this high award. “This prize was absolutely unexpected – said the scientist to the correspondent of the newspaper “Komunisti”,– I am delighted to receive this high recognition. The Turin Academy is one of the oldest Academies in Italy. Many of my works are connected with the works of Italian mathematicians. Italy was the first foreign country where my work was published upon presentation by the great Italian mathematician Vito Voltera”.

* * *

Muskhelishvili was buried on Mount Mtatsminda - the burial place of Georgia’s most revered sons and daughters. A prize bearing Muskhelishvili’s name was created in his memoriam, the Institute of Computational Mathematics was named after him, and his monument was erected on Chavchavadze Avenue near the house where Muskhelishvili lived from 1941 to 1976 till the last day of his life. A memorial plaque with a bas-relief image of the First President of the Georgian Academy of Sciences marks the house.

Nikoloz Muskhelishvili’s son, Doctor of Technical Sciences, Professor Guram Muskhelishvili never neglected the room where his father used to work. This room is also like a memorial with photographs on the walls, with medals and awards on special stands and books on the shelves... Guram Muskhelishvili and his daughters Olga and Marina are looking with great care after their father’s and grandfather’s things and after the books he used...

* * *

Here is how Guram Muskhelishvili recalls his renowned father: “I’ll start from a distant. While living in St Petersburg, my father used to spell his name as Muskhelov, and the books bought there and his published articles were signed as Muskhelov. Many Georgian scientists and public figures wrote their names in a Russian way in those days. Soon after coming back to Georgia, father asked Ivane Javakhishvili about changing his name back to Georgian. Javakhishvili replied in his usual polite way: “choosing your name is entirely up to you, but I have to say that Muskhelishvili is better, this is a Georgian name and it is second to none”. By the way, our ancestors were called Muskheli, but under the influence of Ivane Javakhishvili’s authority my father chose the name Muskhelishvili.

My father liked working at night. I remember, once he sat at his desk in the evening, before going to school in the morning I saw him still sitting at the table and working, and when I returned home from school I saw the same scene...

Still being a student, my father developed a habit of making not only a list of the scientific works that were of interest to him, but also of writing their short summaries in a special notebook. He never changed this habit. The purpose of his first work trip abroad was to acquaint himself with and to acquire scientific literature. He brought a lot of books from Germany and France. Back then, I mean in the twenties, we were very poorly supplied with specialised literature.

I am often asked why I did not follow in my father’s footsteps, why I chose to become a physicist. I had a great desire to become a radio engineer, but it was wartime and I could not go to Leningrad, so I enrolled in the Physics and Mathematics Faculty of Tbilisi University. Initially, I liked mathematics and was going to carry on studies in this field, but my father told me “you’ve got good hands and it would be better to become a physicist”. I followed his advice. By the way, he loved physics himself and he enjoyed reading classical works. Here are Einstein’s “Relativitätstheorie” in German and Newton’s “Philosophiæ Naturalis Principia Mathematica” also in German. The latter was bought in 1921 by Niko Ketskhoveli who later presented it to my father. Look how laconically it is signed: “to Niko from Niko. 1946.”

I graduated from the University in the field of Physics. Don’t assume that being Niko’s son meant any preferential treatment To tell the truth, my teachers never spared me and always had high expectations for me. I remember how strict my father’s student and his good friend Ilia Vekua was at the examination; he often visited us and naturally he knew me quite well. Others behaved in much the same way and sometimes tested me for hours. Later I learned that my father asked them to test me as strictly as possible.

My father himself taught me two subjects, analytic geometry and differential equations. In the first one I got the top mark. I derived one of the formulae in an original way and he liked that. As for the other subject A friend of mine and I were taking the exam together in this very room. My friend passed it in five minutes and got the top mark, while I got a “fail” also in five minutes and in a very peculiar way too: when my father heard my answer, he stood up and left the room, which meant that the exam was over.

My father treated books with great care, and he taught me and my children to behave in the same way. You have probably noticed that he got many of his books rebound. Quite a few of these books are here too.

My father loved poetry immensely and it is natural that he often read verses and poems. He loved Rustaveli, Barathashvili, Pushkin, but I think his favourite poet was Barathashvili. Father read Dostoevsky and Leskov with great enthusiasm, and he enjoyed reading Pushkin and Gogol to his grandchildren.

I would like to add that my father read a lot in French, especially Anatole France, Alphonse Daudet and French translations of Conan Doyle. He loved other writers too, and it is difficult to single out someone, but I remember he particularly enjoyed reading Charles Dickens, and he liked "Napoleon" by Tarle.

My father was not indifferent to Georgian folklore, especially to proverbs and shairi (a short form of a witty verse like a pun). He often made a pun with his friends Niko Keckhoveli and Mikheil Chiaureli, and I was asked to leave the room of course. As for the proverbs, he knew quite a few, but his favourite one was: "If an aubergine had wings it would have been a swallow".

My father loved hunting, especially on quails around Manglisi and around his native Matsevani. His other passion or as they say today, hobby was carpentry. We used to have lots of things made by him, we still have some of them..."

* * *

Here is a prediction of a great Russian applied mathematician, Academician Aleksey Krylov for the Georgian mathematical school in 1939:

"My dear friends! Kupradze, Mikeladze, Gorgidze, Nodia!

In Moscow, I met N. Muskhelishvili who had been unanimously elected a Full Member of the Academy of Sciences of the USSR. Nikolai Ivanovich is the founder of the brilliant Georgian mathematical school, and you are his first and closest colleagues - the pioneers in this field.

I had a pleasure of rising a glass with Nikolai Ivanovich to your health and the prosperity of the Georgian mathematical society, and of expressing my deep belief that like the life-giving elixir of the vines of the Tsinandali and Mukuzani vineyards surpasses in its natural qualities the produce of the vines of Bordeaux and Sauternes, so the output of the Georgian mathematical school established by the genius N. Muskhelishvili and his colleagues – Kupradze, Vekua, Mikeladze, Rukhadze, Gorgidze, Nodia – will develop rapidly and its scientific merits will become comparable to those of the schools of Lagrange and Cauchy."

Academician A. Ishlinski wrote in 1997: "The Georgian people should rightly be proud of the world-wide recognition of the achievements of the Georgian school of Mathematics and Mechanics established in this century, the founder of which is N. Muskhelishvili. There are many celebrated names among the representatives of this school: I. Vekua, V. Kupradze, K. Marjanishvili, A. Gorgidze, and others.

A brief chronology of Nikoloz Muskhelishvili's life and work

Nikoloz (Niko) Muskhelishvili was born on 16 February 1891 in Tbilisi in the family of a military engineer General Ivane Muskhelishvili and Daria Saginashvili. He spent most of his childhood in the village Matsevani of Tetrickharo region where his maternal grandfather Alexander Saginashvili lived.

In 1909, Muskhelishvili finished the Second Classical Gymnasium in Tbilisi. The same year, he enrolled in the Physics and Mathematics Faculty of St Petersburg University.

In 1914, he graduated with Distinction from the Physics and Mathematics Faculty of St Petersburg University specializing in Mathematics, and he was invited to continue his postgraduate studies in Theoretical Mechanics at the same University.

In 1915, Muskhelishvili, jointly with his scientific supervisor professor Guri Kolosov, published his first scientific work in the proceedings of the Imperial Electrotechnical Institute: "On the equilibrium of elastic circular disks under the influence of stresses applied at the points of their encirclement and acting in their domains" (Russian), Izv. Electrotekhnich. Inst., Petrograd, 12 (1915), 39–55 (jointly with G.V. Kolosov). The work was about a particular problem in elasticity theory. Then and later, Muskhelishvili's research interests were mostly in the field of elasticity theory and, more generally, in the field of mechanics and mathematical physics.

In 1916–1919, Muskhelishvili published three works. From 2 March to 2 June 1919, he passed with Distinction all his Magister exams, while being heavily involved in teaching at the same time.

In 1920, Muskhelishvili returned to Tbilisi and started working at Tbilisi State University.

On 1 September 1920, the Scientific Board of the Faculty of Mathematical and Natural Sciences elected Muskhelishvili as the Chair in Mechanics, while on 29 October, the Board of Professors elected him as a Professor.

In 1926–1928, Muskhelishvili was the Dean of the Polytechnic Faculty of Tbilisi University. In 1928, the Georgian Polytechnic Institute was established on the basis of the Faculty, where Muskhelishvili was the Pro-Rector in Education in 1928–1930, and the Chair in Theoretical Mechanics in 1928-38.

In 1933, Muskhelishvili was elected a Corresponding Member of the Academy of Sciences of the USSR. The same year a research Institute of Mathematics and Physics was established under his leadership at Tbilisi University. In 1935, a separate Institute of Mathematics was created, and in 1937 it first passed into the system of the Georgian Branch of the Academy of Sciences of the USSR and then in 1941 – into the system of the Academy of Sciences of Georgia.

In 1939, Muskhelishvili was elected a Full Member of the Academy of Sciences of the USSR, and in 1942-53 and in 1957-72 he was a member of the Presidium of the Academy of Sciences of the USSR.

In 1920–1962, Muskhelishvili was the Chair in Theoretical Mechanic, and in 1962–1971 the Chair in Continuum Mechanics at Tbilisi University.

In 1941, the Academy of Sciences of Georgia was established, and Nikoloz Muskhelishvili was its President until 1972 and the Honorary President from 1972 to 1976. From 1945 to the end of his life he was the Director of A. Razmadze Institute of Mathematics of the Academy of Sciences of Georgia.

In 1941, a Stalin Prize of the First Degree was awarded to Muskhelishvili's monograph "Some Basic Problems of the Mathematical Theory of Elasticity" (Russian, 1939), an earlier edition of which was published by the Academy of Sciences of the USSR in 1933. The monograph has been published five times and translated into many languages.

In 1945, Muskhelishvili was awarded the title of a "Hero of Socialist Labour".

In 1946, Muskhelishvili's second monograph "Singular Integral Equations" (Russian) was published and he was awarded a Stalin Prize for it.

In 1957–1976, he was the Chairman of the USSR National Committee for Theoretical and Applied Mechanics.

In 1952, Muskhelishvili was elected a Member of the Bulgarian Academy of Sciences, in 1960 – a Member of the Academy of Sciences of Poland, in 1967 – a Foreign Member of the Academy of Sciences of German Democratic Republic (Berlin), in 1961 – a Member of the Academy of Sciences of Armenia, in 1972 – a Member of the Academy of Science of Azerbaijan.

In 1969, Turin Academy of Sciences awarded Muskhelishvili its international prize "Modesto Paneti"; in 1970 he was awarded a Gold Medal of the Slovak Academy of Sciences, and in 1972 – the highest award of the Academy of Sciences of the USSR, the M. Lomonosov Gold Medal.

Nikoloz Muskhelishvili passed away on 15 July 1976. He is buried at the Pantheon of Georgian writers and public figures at Mama David church on Mount Mtatsminda.

- The Institute of Computational Mathematics of the Academy of Sciences of Georgia, the Kutaisi Polytechnic Institute, Tbilisi state school No. 55 and Manglisi state school have been named after Muskhelishvili.

- The Academy of Sciences of Georgia introduced the Muskhelishvili Prize for research in Mathematics, Mechanics and Physics in 1977.

- Muskhelishvili Scholarships were established for undergraduate and postgraduate students.

- Muskhelishvili's bust was put up in Tbilisi University.

- His museum was opened in his flat.

- His monument was erected on Chavchavadze Avenue.

Muskhelishvili's works were devoted to the following four basic problems of mechanics and mathematics:

1. The plane problems of elasticity theory.

2. Torsion and bending of homogeneous and composite beams.

3. Boundary value problems for the harmonic and biharmonic equations.

4. Singular integral equations and boundary value problems of the theory of analytic functions.

The study of these problems has had a major influence on the further development of several branches of mathematics and mechanics.

Muskhelishvil's methods in plane elasticity theory were applied and further developed in the works of S. Mikhlin, D. Sherman, and others. With the help of these methods, many problems that arise in industry were solved in the works G. Savin, D. Vainberg, and others. Muskhelishvili's results were applied and further developed in the theory of contact problems by L.Galin, A. Kalandia, I. Karcivadze, I. Shtaerman, and others. Applications to problems of torsion and bending of beams developed in various directions in the works of A. Gorgadze, A. Rukhadze, and others. Muskhelishvili's ideas have had a major impact on the work on boundary value problems of the theory of analytic functions and singular integral equations carried out in the Soviet Union (by T. Gakhov, I. Vekua, N. Vekua, A. Bitsadze, D. Kveselava, B. Khvedelidze, L. Magnaradze, G. Manjavidze, and others).

The same ideas have firmly established themselves in the general theory of elliptic partial differential equations (works of I. Vekua, B. Khalilov, and others). In particular, they have found important applications in shell theory.

Muskhelishvili's works enjoy wide popularity among a large number of foreign experts. Large parts of monographs by A. Green and W. Zerna (England), I. Sokolnikoff (USA), I. Babushka, K. Rektoris (Czech Republic, Slovakia) and others are devoted to a detailed exposition of Muskhelishvili's methods and results.

Translation from Georgian by **G. Lezhava**

English translation edited by **E. Shargorodsky**

International Symposia dedicated to the scientific activities of N. Muskhelishvili

1. Problems of Continuum Mechanics.(Russian) Izdat. AN SSSR, Moskva, 1961. (edited also in the USA: Problems of Continuum Mechanics. SIAM, Philadelphia, Pennsylvania, 1961).
2. Applications of the theory of functions to continuum mechanics. (Russian). Proc. Intern. Symp. in Tbilisi, September 17–23, 1963, Nauka, Moscow, 1965, t.1; t.2;
3. Continuum Mechanics and Related Problems of Analysis. (Russian). Nauka, Moscow, 1972.
4. Proc. of Symposium in Continuum Mechanics and Related Problems of Analysis. (Russian). (Tbilisi, September, 23–29, 1971), Metsniereba, Tbilisi, 1973, t.1; 1974,t.2.
5. Continuum Mechanics and Related Problems of Analysis. Proc. the Intern. Symp. Dedicated to the Centenary of Academician N. Muskhelishvili. Tbilisi, Republic of Georgia, 6–11 June, 1991.

Basic Publications

(i) Monographs

1. Applications des integrales analogues à celles de Cauchy quelques problèmes de la physique mathématique. Tiflis, Edition de l'Université de Tiflis, Imprimerie de l'Etat, 1922.
2. Some basic problems of the mathematical theory of elasticity. Basic equations, a plane problem, torsion and bending (Foreword by Acad. A.N. Krilov). (Russian) Acad. Sci. USSR, Leningrad, 1933.
3. Some basic problems of the mathematical theory of elasticity. Basic equations, the plane problem, torsion and bending. (Russian) 2nd ed., revised and enlarged. Acad. Sci. USSR Publ. House, Moscow–Leningrad, 1935.
4. Singular integral equations, boundary value problems of the function theory and some of their applications to mathematical physics. (Russian), Moscow–Leningrad, 1946.
5. Some basic problems of the mathematical theory of elasticity. Basic equations, the plane theory of elasticity, torsion and bending. 3rd ed., revised and enlarged, Moscow–Leningrad, 1949.
6. Some basic problems of the mathematical theory of elasticity. Groningen, P. Noordhoff, 1953, XXXI. (English translation of the 3rd Russian edition of 1949).
7. Singular integral equations. Groningen, P. Noordhoff, 1953.

8. Some basic problems of the mathematical theory of elasticity. Basic equations, the plane theory of elasticity, torsion and bending. 4th ed., revised and enlarged. (Russian) Acad. Sci. USSR Publ. House, 1954.
9. Cîteva probleme fundamentale ale teorei matematice á elasticotatii, v. 1–3. Bucharest. Inst. Studii Rom.-Sov., 1956. (Translates into Romanian from the 3rd Russian edition of 1949).
10. Some basic problems of the mathematical theory of elasticity. Peking, 1958, XVII (Translated into Chinese from the 4rd Russian ed. of 1954).
11. Singular integral equations. Boundary value problems of the theory of functions and some of their applications to mathematical physics. (Russian) 2nd ed., revised. Fizmatgiz, Moscow, 1962.
12. Some basic problems of the mathematical theory of elasticity. Groningen, P. Noordhoff, 1963, XXXI.
13. Singulare Integralgleichungen. Randwertprobleme der Funktionentheorie und Anwendungen auf die mathematische Physik. Berlin, Akademie-Verlag, 1965, XIV. (German translation of the 2nd Russian edition of 1962).
14. Some basic problems of the mathematical theory of elasticity. Basic equations, the plane theory of elasticity, torsion and bending. (Russian) 5th ed., revised and enlarged. Nauka, Moscow, 1966.
15. Singular integral equations. Boundary value problems of the theory of functions and some of their applications to mathematical physics. (Russian) 3rd ed., revised and enlarged. Nauka, Moscow, 1968.
16. Einige Grundanfgaben zur mathematischen Elastizitätstheorie. Leipzig, 1971. (Translation into German from the 5th Russian ed. of 1966).
17. Singular integral equations. Boundary value problems of the theory of functions and some of their applications to mathematical physics. (Georgian) Metsniereba, Tbilisi, 1982.
18. Application of Cauchy type integrals for some problems of mathematical physics. (Georgian) Metsniereba, Tbilisi, 1992.

(ii) Articles

1. On the equilibrium of elastic circular disks under the influence of stresses applied at the points of their encirclement and acting in their domains. (Russian), Izv. Electrotechnique. Inst., Petrograd, 12(1915), 39–55 (jointly with G.V. Kolosov).
2. On thermal stresses in the plane problem of the theory of elasticity. (Russian), Izv. Electro-technique. Inst., Petrograd, 13(1916),23–37.
3. On defining a harmonic function by conditions given on the contour. (Russian) Zh. Fiz.-Mat ob-va, Perm Univ., 1918, (1919), Issue I, 89–93.

4. Sur l'intégtation de l'équation biharmonique. Izv. Ross. Akad. Nauk, 13(1919), No. 12–15, 663–686.
5. Sulla deformazione piana di un cilindro elastico isotropo. Atti Accad. Lincei, 31(1922), 5 ser. No. 12, 548–551.
6. Sur l'equilibre des corps élastiques soumis à l'action de la chaleur. Bull. Univ. Tiflis, 1923, No. 3, 17–26.
7. Solution of an integral equation occurring in the theory of black radiation. (Russian) Zh. Russk. Fiz.-Khim. Ob-va, Part Phys., 56(1924), issue I, 30–39.
8. Sur la solution du problème biharmonique pour l'aire extérieure à une ellipse. Math. Zeitschr. Bd. 26,1927 No.5, 700–705.
9. Sur l'intégration approchee de l'équation biharmonique. C. R. Acad. Sci., 185(1927), No. 22, 1184–1186.
10. Sur les orbites périodiques et les lignes géodésiques fermees. Atti Accad. Lincei, 5(1927), ser. 6, No. 10, 769–773.
11. On periodic orbits in closed geodesic lines (abstract). (Russian) Trans. All-Russian Math. Congr. Moscow, April 27 – May 4, 1927, Moscow-Leningrad, 1928, 189.
12. On some contour problems of plane hydrodynamics. (Russian) Proc.of All-Russian Math. Congr. Moscow–Leningrad, 1928, 262.
13. Sur le problème fondamental d'hydrodynamique à deux dimensions. Atti Accad. Lincei, 7(1928), ser. 6, No. 12, 995–1002.
14. Zum problem der Torsion der homogenen isotropen Prismen. Izv. Tiflis. polit. in-ta, 1(1929), part 1, 1–20.
15. Sur le problème de torsion des cylindres élastiques isotropes. Atti Accad. Lincei, 9(1929), ser. 6, No. 4, 295–300.
16. Nouvelle méthode de rèduction du problème biharmonique fondamental à une equation de Fredholm. C. R. Acad. Sci., 192(1931), No. 2, 77–79.
17. Théorèmes d'existence relatifs au problème biharmonique et aux problèmes d'élasticité à deux dimensions. C. R. Acad. Sci., 192(1931), No. 4, 221–223.
18. To the problem of torsion and bending of elastic bars composed of various materials. (Russian). Izv. AN SSSR, OMEN, 1932, iss.7, 907–945.
19. To the problem of torsion and bending of composite elastic beams. (Russian) Izv. Inzh. Inst. Gruzii, 1932, iss. I, 123–127.
20. Sur le problème de torsion des poutres élastiques composées. C. R. Acad. Sci., 194(1932), No. 17, 1435–1437.

21. Recherches sur des problèmes aux limites relatifs à l'équation biharmonique et aux équations de l'élasticité à deux dimensions. Math. Ann., 1932, Bd. 107, No. 2, 282–312.

22. Solution of a plane problem of the theory of elasticity for a solid ellipse. (Russian) PMM, I(1933), is. I, 5–12.

23. Praktische Lösung der fundamentalen Randwertaufgaben der Elastizitätstheorie in der Ebene fur einige Berandungsformen. Z. angew. Math. und Mech., 1933, Bd. 13, No. 14, 264–282.

24. Sur l'équivalence de deux méthodes de reduction du problème plan biharmonique à une équation intégrale. C. R. Acad. Sci., 196(1933), No. 26, 1947–1948 (avec V. Fock).

25. A new general method of the solution of the basic boundary value problems of the plane theory of elasticity. (Russian) DAN SSSR, 3(1934), No. 1, 7–11.

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Plenary Talks

The Mixed and Contact Problems of Elasticity Theory

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The investigations of the problems of interaction of elastic and rigid bodies and of the related problems of the theory of cracks (inside the body) started from the early 40s of the past century just after the elaboration of the theory of boundary value problems of analytic functions and singular integral equations. Important results in the theory of mixed boundary problems have been obtained in the works of N. Muskhelishvili, L. Galin, I. Schtaerman, D. Sherman, A. Lur'e, G. Irwin etc.

The investigations of contact problems dealing with the interaction of massive deformable bodies and thin-shelled elements (stringers, inclusions), the problems of the crack theory, when the crack takes its origin on the body's boundary or on the interface of a piecewise homogeneous elastic body started in the early 60s. Fundamental results obtained in this direction belong to E. Melan, V. Koiter, E. Buell, E. Brown, G. Irwin, H. Bueckner, A. Khrapkov, V. Vorovich, G. Popov, E. Reissner, H. Buffler, R. Myki, S. Stenberg, N. Arutyunyan, B. Abramyan, V. Alexandrov, R. Bantsuri etc.

From the applied viewpoint, one of the important problems is that of finding equi-strong contours, i.e., the problems of stress concentration control on the boundary of the hole of an elastic body. These problems take their origin in the 60s of the past century and are reflected in the works of G. Cherepanov, N. Banichuk, G. Ivanov, D. Kosmodam'janskii, H. Neiber etc. After intensive investigations in this direction the different methods of exact and approximate solutions have been developed, for example, the method of orthogonal polynomials, the asymptotical method, the method of reduction to the Riemann problem, the factorization method, the method of integral transformations, and so on.

During last five years we have considered:

1. The mixed and contact problems connected with the interaction between different deformable bodies and elastic thin-shelled elements of variable geometric or physical parameters. The investigation of statical and dynamical boundary-contact problems for bodies with another elastic elements in the framework of different elastic properties and phenomenological theories of the material.

2. The problems of elasticity theory for a domain with partially unknown boundary (equi-strong contour finding) connected with optimal stress distribution by the selection of a body shape.

Taylor Expansion and Sobolev Spaces

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In the lecture a new characterization of Sobolev spaces $W(m, p, R(n))$, $m \geq 1$, in the form of a pointwise inequality will be discussed. The functions considered are a priori assumed only in $L(p)$, $p > 1$. This inequality reveals the – local and global – polynomiallike behaviour of these functions. If time allows various consequences of this characterization will be discussed.

The Method of Transference Lemma in Problems on Rearrangements of Summands of Function Theory

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The method allows to establish the convergence of a functional series under rearrangement in terms of the series convergence for an assignment of signs. The method of transference lemma allowed us to find

(i) A series of new maximum inequalities for rearrangements of vector summands that go back to Garsia's inequalities;

(ii) A general theorem on the sum range of a conditionally convergent series in an infinite-dimensional space (the Riemann–Levy–Steinitz type theorems);

(iii) Generalizations of the Nikishin type theorems on almost surely convergent rearrangements of a function series;

(iv) Theorems related to Uljanov's conjecture on the uniform convergence of a rearrangement of the Fourier trigonometric series of a continuous periodic function;

(v) Theorems related to to the Kolmogorov conjecture (open since 1920-s) on the system of convergence of an orthonormal system.

Recent Developments in Analysis of Uncertainty in Theoretical and Applied Mechanics

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This lecture reviews some recent achievements in uncertainty analysis of engineering structures including stochasticity, anti-optimization and fuzzy sets. Special emphasis is placed on subtle points such as formulation of stochastic variational principles for determination of the probabilistic characteristics of structures with attendant formulation for the stochastic FEM; choice of interval analysis versus ellipsoidal modeling for anti-optimization; safety factor evaluation within the fuzzy sets methodology. It contrasts various approaches with the view of making an appropriate selection based on available experimental data.The lecture advocates for combined theoretical. numerical and experimental approach to introduce uncertainty concepts in both the theory and practice of applied mechanics.

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Matrix Spectral Factorization and Wavelets

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Matrix Spectral Factorization Theorem asserts that *if* $S(t) = \big(f_{ij}\big)_{i,j=\overline{1,r}}$ *is an* $r \times r$ *positive definite* (*a.e.*) *matrix function with integrable entries* $f_{ij} \in L_1(\mathbb{T})$, *defined on the unit circle* $\mathbb{T}$, *and if the Paley–Wiener condition*

$$\log\det S(t) \in L_1(\mathbb{T}) \tag{1}$$

is satisfied, then (1) *admits a* (*left*) *spectral factorization*

$$S(t) = S^+(t)S^-(t) = S^+(t)\big(S^+(t)\big)^*, \tag{2}$$

where S^+ *is an* $r \times r$ *outer analytic matrix function from the Hardy space* H_2 *and* $S^-(z) = \big(S^+(1/\overline{z})\big)^*$, $|z| > 1$. It is assumed that (2) holds a.e. on $\mathbb{T}$. A spectral factor $S^+(z)$ is unique up to a constant right unitary multiplier. The sufficient condition (1) is also a necessary one for the factorization (2) to exist.

This theorem was proved by Wiener in 1958. Since then the spectral factorization has became an important tool in solution of various applied problems in Control Engineering and Communications, and challenging problem became the actual approximate computation of the spectral factor $S^+(z)$ for a given matrix-function $S(t)$. Not surprisingly, dozens of papers has been addressed to the solution of this problem.

Recently a new effective method of matrix spectral factorization has appeared in [1], and we are going to describe this method in our talk. Unitary matrix functions of certain structure plays decisive role in our method to obtain a spectral factor S^+, and it turned out that these unitary matrix functions are closely related with wavelet matrices. Consequently the proposed method makes it possible to construct compact wavelets in a fast and reliable way and to completely parameterize them [2].

It will be presented also an elementary proof of the polynomial matrix spectral factorization theorem, spectral factorization of rank-deficient matrices, and their connection to wavelet matrices.

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Hamiltonian Form of the Maximum Principle in Optimal Control

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In my talk, I want to discuss a specific property of maximum principle, its native Hamiltonian form, canonically inherent in the formulation of the principle. It is always there, regardless of the nature of the given problem, be it regular or strongly singular, such as linear systems.

As a result of its universal Hamiltonian form, maximum principle admits an invariant geometric formulation, tough its initial form was purely analytic. The corresponding geometry turns out to be a very simple and basic geometry canonically generated by the optimal problem under consideration. It is my intention to describe this geometry.

Recent Advances in Riemann–Hilbert Problem

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A number of new results on the Riemann–Hilbert boundary value problem and Riemann-Hilbert monodromy problem will be presented most of which have been obtained by the authors and their collaborators. Specifically, we'll discuss in some detail the new versions of the Riemann–Hilbert boundary value problem and Riemann–Hilbert monodromy problem in the context of compact Lie groups developed

in the last two decades. The basic results on solvability and Fredholm theory of such problems will be presented and a few typical examples will be given. The Riemann–Hilbert boundary value problem in the context of generalized analytic functions will be also discussed.

Applications of Integral Equation Methods to a Class of Fundamental Problems in Mechanics and Mathematical Physics

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The monograph *Singular Integral Equations* by N. I. Muskhelishvili was published originally in Russian in 1946 and was revised and translated into English in 1958. In this monograph, the solution of the Dirichlet problem is expressed in terms of the potential of a simple layer, which leads to a Fredholm integral equation of the first kind. This new approach introduced by Muskhelishvili in 1946 for solving boundary value problems by using integral equations of the first kind has made significant contribution 30 years later to the development of variational methods for boundary integral equations and their numerical discretizations. The later is known as the boundary element method and has become one of the most popular numerical schemes in nowadays.

To pay a high contribute to Muskhelishvili in celebrating his 120th birthday anniversary, this lecture discusses boundary integral equations of the first kind and its applications to a class of fundamental problems in elasticity, fluid mechanics and other branches of mathematical physics. Applications are drawn from various disciplines including topics such as singular perturbation theory for viscous flow past an obstacle, boundary variational inequalities for contact problems in elasticity, coupling procedure and domain decomposition for interface problems in non homogeneous medium. The presentation of these topics indicates also the chronological order of the development of the Muskhelishvili's method concerning first kind integral equations and its generalizations.

Mathematical Models of Elastic Cusped Shells, Plates, and Rods

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Nikoloz Muskhelishvili's famous monograph in elasticity theory [1] had and has a great worldwide influence on investigations in the corresponding fields of mechanics and mathematics. His student and distinguished successor I. Vekua constructed hierarchical models of shells which are the natural continuation of the ideological program of the Georgian Mathematical school founded by N. Muskhelishvili. If we ignore members containing unknown so called moments and their first order derivatives in the governing system of prismatic shells, then the system obtained can be divided in two groups. The first one will contain systems of the 2D elasticity, while the second one will contain Poisson's equations. This fact makes possible to apply Muskhelishvili's methods based on the theory of analytic functions of one complex variable. In 1977 I. Vekua [2] wrote: "If we consider shells with cusped edges, then the thickness vanishes on the boundary or on it's part. In this case we have an elastic system of that equations with degenerations on the boundary. At present, the investigation of the class of such equations is carried out rather intensively (cf. A. Bitsadze, M. Keldysh, S. Tersenov, G. Fichera). However, in the study of the equations generally, and the above system, in particular, only the first steps are made (cf. G. Jaiani)". The present updated survey is mainly devoted to contribution of Georgian Scientists (in alphabetic order) G. and M. Avalishvili, N. Chinchaladze, D. Gordeziani, G. Jaiani, S. Kharibegashvili, N. Khomasuridze, B. Maistrenko, D. Natroshvili, N. Shavlakadze, and G. Tsiskarishvili in this field. Some of these results are obtained with R. Gilbert, A. Kufner, B. Miara, P. Podio-Guidugli, B.-W. Schulze, and W.L. Wendland within the framework of international projects. As a background along with I. Vekua's hierarchical models, Timoshenko's geometrically nonlinear and Kirchoff-Love's models of plates of variable thickness are used [3].

An exploratory survey with a wide bibliography of results concerning elastic cusped shells, plates, and rods, and cusped prismatic shell-fluid interaction problems one can find in [4].

In the present talk we mainly confine ourselves to cusped prismatic shells and rods.

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Consistent Theories of Isotropic and Anisotropic Plates

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The three-dimensional equations of the linear theory of elasticity are approximated by a two-dimensional plate theory in the following manner. The displacements are developed into a Fourier series in thickness direction. As basis, we use either monomes or scaled Legendre polynomials. The coefficients of the series expansion are the unknowns of our problem. We use the kinematic relations to calculate the strains and in turn Hooke's law to evaluate the strain-energy density and the potential of external forces. Both are integrated with respect to the thickness directions. After a suitable introduction of dimensionless quantities a so-called plate parameter c^{2n} evolves with $c^2 = \frac{h^2}{12a^2}$ (h and a are characteristic lengths in thickness and plate-plane direction, respectively). Constitutive relations, i.e., the relations between stress resultants and strains are derived and the field equations as well as the boundary conditions follow from the principle of minimum potential energy resulting in an infinite number of unknowns.

The question arises: where to cut off the series expansion? Consistent plate theories are obtained, if all governing equations are uniformly approximated to the same order m, i.e., all terms multiplied by c^{2n}, $n = 0, 1, \ldots, m$, are retained and all terms multiplied by c^{2n}, $n > m$, are neglected. In addition, during the reduction of the system of differential equations to one or two "main" differential equations, terms of the order c^{2n}, $n > m$, are neglected likewise. For $m = 0$, it turns out that the approximation admits only a rigid-body translation in thickness direction and two rigid-body rotations with respect to the two in-plane axes. For $m = 1$, the classical Kirchhoff-Love plate theory evolves without any a-priori assumptions including Kirchhoff's "Ersatzscherkräfte". Reissner-Mindlin type of plate theories are derived for $m = 2$. In the talk, various second-order plate theories existing in the literature are compared with our consistent theory. We investigate monotropic materials (the symmetry plane of the material coincides with the mid plane of the plate) and introduce in turn further material symmetries (orthotropy, transversal isotropy, isotropy).

Boundary Value Problems for Analytic and Harmonic Functions in New Function Spaces

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Basing on N. Muskhelishvili's ideas we study BVPs for analytic and harmonic functions in new setting.

Our lecture deals with the solution of above mentioned problems in the frame of grand Lebesgue and variable exponent Lebesgue spaces. The theory of these spaces was intensively studied in the last decade and it still continuous to attract the researches due to various applications.

The following tasks will be discussed:

i) to reveal the influence air recent results on mapping properties of non-linear harmonic analysis operators in new function spaces.

ii) to give the solution of the Riemann problem on linear conjugation of analytic functions in the class of Cauchy type integrals with the density from grand Lebesgue spaces.

iii) to solve the Dirichlet problem for harmonic functions, real parts from variable exponent Smirnov classes analytic functions. The problem is solved in domains with nonsmooth boundaries. Our goal is to give a complete picture of solvability, to expose the influence of the solvability conditions; in all solvability cases to construct solutions in explicit form.

Singular Perturbation Problems in Periodically Perforated Domains

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This talk is devoted to the analysis of boundary value problems on singularly perturbed domains by an approach which is alternative to those of asymptotic analysis and of homogenization theory.

Such approach has been applied to linear and nonlinear boundary value problems and to linear eigenvalue problems and is based on potential theory, on functional analysis and for certain aspects on harmonic analysis.

We consider a periodic nonlinear boundary value problem depending on a positive parameter ϵ which describes the singular perturbation and we assume that the problem degenerates when ϵ tends to zero.

Then the goal is to describe the behaviour of the solutions when ϵ is close to 0. We do so in terms of analytic functions and of singular but known functions of the parameter ϵ such as ϵ^{-1} or $\log \epsilon$.

Based on joint work with Paolo Musolino.

Buckling from a Membrane Stress State

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Depending on the prebuckling stress state of structures or structural members, a distinction between different modes of loss of stability can be made. Classical Euler-buckling e.g. occurs from a prebuckling stress state of pure compression. Torsional flexural buckling, on the other hand, usually originates from a combination of axial compression and bending. As a limiting case, it may either start from a prebuckling state of pure bending or pure compression. The prebuckling stress state of shells generally represents a combination of membrane and bending stresses. Buckling from a pure membrane stress state represents a limiting case of this situation.

Focussing on this limiting case, a necessary and sufficient condition for bifurcaiton buckling from a membrane stress state is derived in the frame of the Finite Element Method. The basis for this derivation is a condition resulting from disintegration of the derivative of the mathematical formulation of the so-called consistently linearized eigenvalue problem, with respect to a dimensionless load parameter.

It it shown that, from a mechanical viewpoint, a membrane stress state is a necessary condition for treatment of loss of stability as a linear stability problem as well as for consideration of linear prebuckling paths. The aforementioned condition for bifurcation buckling from a membrane stress state also clarifies the misconception that linear stability analysis and linear prebuckling paths are mutually conditional.

Sensitivity analysis of bifurcation buckling of a two-hinged arch, subjected to a uniformly distributed static load, by varying the geometric form of its axis serves the purpose of non-trivial verification of the derived condition for the special case of a thrust-line arch.

Integral Operators in New Function Spaces

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The goal of our talk is to present recent results of Georgian mathematicians regarding mapping properties of a wide range of integral operators of harmonic analysis in so called non-standard Lebesgue spaces (variable exponent Lebesgue spaces, grand Lebesgue and Morrey spaces etc). In particular, we are focused on weighed boundedness criteria for maximal, singular and potential operators is these spaces.

In recent years it was realized that the classical function spaces are no more appropriate spaces to solve a number of contemporary problems naturally arising, e.g., in solvability problems of PDEs. Actual need to introduce and study such spaces from various viewpoint became apparent. One of such spaces is the variable exponent Lebesgue space. We aim to discuss the two-weight problem for maximal, fractional and Calderón–Zygmund singular operators in Lebesgue spaces with non-standard growth. Other spaces of our interest are Iwaniec–Sbordone grand Lebesgue spaces with weights and their generalizations. The necessity of introduction and study of these spaces was realized because of their rather essential role in various fields, in particular, in the integrability problem of Jacobian under minimal hypothesis. It should be emphasized that these spaces are non-reflexive, non-separable and non-invariance of rearrangement. The boundedness weighted criteria of potential and singular operators of various type in grand Lebesgue and Morrey spaces will be discussed. The results were mainly derived by V. Kokilashvili and the speaker and were published in [1–7].

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On Extension of the Muskhelishvili and Vekua–Bitsadze Methods for the Geometrically and Physically Nonlinear Theory of Non-Shallow Shells

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In this paper the 3-D geometrically and physically nonlinear theory of non-shallow shells are considered. Using the reduction methods of I. Vekua and the method of a small parameter, 2-D system of equations is obtained. By means of the Muskhelishvili and Vekua–Bitsadze methods, for any approximation of order N the complex representation of the general solutions are obtained [1, 2, 3, 4].

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Some New Hardy-Type Inequalities with Applications

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First we present some ideas and results from the papers [5] and [6]. In particular, we will give a surprisingly elementary proof of "all" Hardy inequalities for finite intervals for the case with power weights, show that these inequalities in fact are equivalent and where we keep the sharp constant in all situations. After that we will briefly describe some new results concerning characterizations of the validity of weighted Hardy-type inequalities to hold. For example the usual characterization conditions (Tomaselli, Talenti, Muckenhoupt, Bradley, Kokilashvili, Mazya, Rosin, Persson, Stepanov, Gurka, etc.) in fact may be replaced by infinitely many other alternative but equivalent conditions (see e.g. [1]). Finally, we will present some applications to modern Homogenization Theory and its connection to material science and tribology. The results above can not be found in the existing literature on Hardy-type inequalities, see e.g. the books: [2]–[4].

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On Nonstandard Contact Interactions

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The standard theory of contact interactions cannot accommodate line or point concentrations. However, such concentrations are often to be expected; indeed, at least within the framework of linear isotropic elasticity, their study has a long history. A generalization of the standard theory that would apply whatever the material response is badly needed. I shall discuss some examples, assembled in collaboration with F. Schuricht [1], that may help to delineate such a theory.

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Bernoulli Free-Boundary Problems

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A Bernoulli free-boundary problem is one of finding domains in the plane on which a harmonic function simultaneously satisfies the homogeneous Dirichlet and a prescribed inhomogeneous Neumann boundary conditions. The boundary of such a domain is called a free boundary because it is not known a priori. The classical Stokes waves provide an important example of a Bernoulli free-boundary problem. Existence, multiplicity or uniqueness, and smoothness of free boundaries are important questions and their solutions lead to nonlinear problems.

The talk, based on a joint work with J. F. Toland, will examine an equivalence between these free-boundary problems and a class of nonlinear pseudo-differential equations for real-valued functions of one real variable, which have the gradient structure of an Euler-Lagrange equation and can be formulated in terms of the Riemann–Hilbert theory. The equivalence is global in the sense that it involves no restriction on the amplitudes of solutions, nor on their smoothness.

Non-existence and regularity results will be described and some important unresolved questions about how irregular a Bernoulli free boundary can be will be formulated.

Integrability in Quantum Theories and Applications

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I review the recent developments in regard to the relation between quantum integrable systems and supersymmetric gauge theories. Examples include the Elliptic Calogero–Moser quantum many body system for which the description of the exact quantum spectrum will be presented using the methods of supersymmetric gauge theories.

A Riemann Surface Approach for Diffraction from Rational Wedges

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This work aims at the explicit analytical representation of acoustic, electromagnetic or elastic, time-harmonic waves diffracted from wedges in R^3 in a correct setting of Sobolev spaces. Various problems are modelled by Dirichlet or Neumann boundary value problems for the 2D Helmholtz equation with complex wave number. They have been analyzed before by several methods such as the Malinzhinets method using Sommerfeld integrals, the method of boundary integral equations from potential theory or Mellin transformation techniques. These approaches lead to results which are particularly useful for asymptotic and numerical treatment. Here we develop new representation formulas of the solutions which are based upon the solutions to Sommerfeld diffraction problems. We make use of symmetry properties, which require a generalization of these formulas to Riemann surfaces in order to cover arbitrary rational angles of the wedge. The approach allows us to prove well-posedness in suitable Sobolev spaces and to obtain explicit solutions in a new, perhaps surprising, form provided the angle is rational, i.e., equals $2\pi m/n$ where m, n are natural. The talk is based upon joint work with T. Ehrhardt and A. P. Nolasco.

Factorization of Matrix Functions: a Survey

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Factorization of matrix functions plays crucial role in consideration of singular integral and convolution type equations, boundary value problems for analytic functions, integrable systems, elasticity, scattering, random matrix theory, etc. Groundbreaking results concerning the factorization were obtained by N. Muskhelishvili (see, e.g., [3]) and were further systematically developed in the works of M. Krein, I. Gohberg [2], I. Simonenko [4], to name only the most profound figures. The state of the matter towards the end of last century is described in [1].

In this talk we will attempt, time permitting, to give an overview of the further progress in the factorization theory, not reflected in [1]. We will address both the existential side of the issue, and the explicit/constructive factorization. The differences, as well as the similarities, with the factorization of scalar valued functions will be emphasized. Special attention will be paid to the almost periodic (AP) case, in which the differences manifest themselves already at the existential level.

We will also address the factorization problem in the ordered abelian group setting, allowing in particular to consider the classical Wiener–Hopf factorization and the AP factorization from the unified point of view.

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Some Finite Element Approaches for Contact/Obstacle Problems

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The first part of the talk deals with dual formulations for unilateral contact problems with Coulomb friction. Starting from the complementary energy minimization problem, Lagrangian multipliers are introduced to include the governing equation, the symmetry of the stress tensor as well as the boundary conditions on the Neumann and contact boundary. Since the functional arising from the friction part is nondifferentiable an additional Lagrangian multiplier is introduced. This procedure yields a dual-dual formulation of a two-fold saddle point structure. Two different Inf-Sup conditions are introduced to ensure existence of a solution. The system is solved with a nested Uzawa algorithm.

In the second part of the talk a mixed hp-time discontinuous Galerkin method for elastodynamic contact problem with friction is considered. The contact conditions are resolved by a biorthogonal Lagrange multiplier and are component-wise decoupled. On the one hand the arising problem can be solved by an Uzawa algorithm in conjunction with a block-diagonalization of the global system matrix. On the other hand the decoupled contact conditions can be represented by the problem of finding the root of a non-linear complementary function. This nonlinear problem can in turn be solved efficiently by a semi-smooth Newton method. The second method can also be applied to parabolic obstacle problems, e.g. pricing American put options.

In all cases numerical experiments are given demonstrating the strengths and limitations of the approaches.

The talk is based on a joint work with M. Andres and L. Banz.

Water Waves in the Context of Shape Optimisation Problems

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The water wave problem will be formulated as a question of finding the shape of a domain for which the natural energy functional is critical in the sense of the calculus of variations. It will be shown that when a minimizer exists it gives a water wave, but that sometimes a minimiser does not exist. The ramifications of this observation will be discussed.

Commutative Algebras of Toeplitz Operators in Action

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We will discuss a quite unexpected phenomenon in the theory of Toeplitz operators on the Bergman space: the existence of a reach family of commutative C^*-algebras generated by Toeplitz operators with non-trivial symbols. As it turns out the smoothness properties of symbols do not play any role in the commutativity, the symbols can be merely measurable. Everything is governed here by the geometry of the underlying manifold, the hyperbolic geometry of the unit disk. We mention as well that the complete characterization of these commutative C*-algebras of Toeplitz operators requires the Berezin quantization procedure.

These commutative algebras come with a powerful research tool, the spectral type representation for the operators under study. This permit us to answer to many important questions in the area.

Continuum Mechanics I:
Theory of Elasticity and Hydromechanics

Mixed Variational Principle of Creeping Theory with Taking into Consideration the Damage, Corrosion and the Volume Deformation

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Nowadays more attention is attracted by investigators to the problem of determining the voltage-deformable condition in the continuous mediums for creeping with taking into consideration the damage and corrosion. First of all it is because of high requirements introduced to the quality and reliability of metallic constructions in condition of continuous high-temperature charging which are in aggressive environment. In this case the process of physico-chemical transformations can lead to the volume change of the body.

In the present paper the variational method of mixed type is formulated for the creeping with simultaneous taking into consideration the process of corrosion, damage and volume change θ, when the immediate deformation ε^I satisfies the equations of conditions of the flan theory type. Using the notations of the paper [1], we construct the functional

$$R = \int\limits_V \left\{ \dot{\sigma}^{ij}\dot{\varepsilon}_{ij} - \frac{1}{2}\left(\dot{\varepsilon}^I_{ij} + 2\dot{p}_{ij}\right)\dot{\sigma}^{ij} - \dot{\theta}\delta_{ij}\dot{\sigma}^{ij} + \lambda_\omega\left(\frac{1}{2}\dot{\omega}^2 - \dot{\omega}\varphi\right) + \right.$$
$$\left. + \lambda_c\left[\frac{1}{2}\dot{c}^2 - \dot{c}\,\mathrm{div}\left(D\nabla c\right)\right]\right\} dV - \int\limits_{S_\sigma} \dot{\bar{T}}^i \dot{u}_i \, dS - \int\limits_{S_u} \dot{T}^i \left(\dot{u}_i - \dot{\bar{u}}_i\right) dS.$$

It is supposed here that on the part of the surface S_σ forces are given and on the part of the surface S_u displacements are given ($S = S_\sigma \bigcup S_u$). It is proved that the stationary value $\delta R = 0$, when the values $\dot{\sigma}^{ij}, \dot{u}_i, \dot{\omega}$ and $\dot{c}$ vary independently and in this case $\varepsilon_{ij} = 0,5\left(\nabla_i u_j + \nabla_j u_i\right)$, and $\dot{\varepsilon}^f_{ij} = H_{ijkm}\dot{\sigma}^{km} + \dot{p}_{ij} + \dot{\theta}\delta_{ij}$ as Euler equation reduces to the equilibrium equation $\nabla_j \sigma^{ij} = 0$, boundary conditions $\bar{T}^i = \sigma^{ij}n_j, \forall\, x^k \in S_\sigma$ and $\bar{u}_j = u_i \;\forall\, x^k \in S_u$ and kinetic equation

$$\dot{c} = \mathrm{div}\left(D\nabla c\right), \quad \dot{\omega} = \varphi\left(\sigma^{\alpha\beta}, \omega, c\right).$$

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The Application of Methods of Linear and Nonlinear Waves to Investigation of Unsteady Space Deterministic Problems for Mechanics and Electrodynamics of Continua and to Stochastic, Point and Distributed, Processes

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We present our results on linear and nonlinear weak waves propagation in continua, containing generalization of known Poincare-Lighthill-Kuo method, in gas dynamics called method of Lighthill-Whitham, on plane and space diffraction waves problems and caustics, when singularities of linear solutions are avoided by methods of derivation of nonlinear evolution equations, in same first order as linear solution, describing waves vicinities, and their solutions by matching with linear solution, as well as, additionally, of approximate satisfaction of shock waves conditions. All these results are obtained for arbitrary ideal media, including inhomogenous magnetogasdynamics, magneto-elasticity, electrodynamics, pieso-electrics, two-component various media, and also are applicable for description of arbitrary systems of nonlinear hyperbolic differential equations with variable coefficients. For linear homogenous media method of solution of boundary value unsteady plane and space problems is developed by method of integral transformations of Laplace and Fourier, method of Wiener–Hopf, coupled with method of Plemeli–Gilbert–N.P. Vekua of reduction of problem to solution of Fredholm systems, whose numerical solution gives simultaneously effective method of factorization of matrices, and at last by bringing solution for originals to close form of Smirnov–Sobolev. The last one allows also obtain fundamental solution for mentioned homogenous media as well as for hyperbolic systems with constant coefficients and precise results previously obtained by direct application of mentioned method of complex solutions form to problem of determination of fundamental solutions in anisotropy elastic plane. In nonlinear problems methods of derivation of nonlinear Shrödinger equations for above mentioned media, describing amplitudes of quasi-monochromatic waves, so called modulations equations for envelope waves, with application to mentioned media, on account also dispersion and dissipation, are also developed. Analytic solutions for axial-symmetric Gaussian nonlinear beams with application to optics and acoustics for all mentioned media, as well as for two counter propagating beams, which can be applied in descriptions of modern experimental problems, for example in models for elementary particles interactions are obtained. During five last years we spend to application of these ideas of nonlinearisation to known linear diffusion equations for probabilities of the Markove stochastic processes, which is essential in vicinity of waves of probabilities, whose speeds after averaging, are equal to inclinations of mean curves of processes. Solutions of various extremely economics, genetics, seismic, physics problems, whose probabilities are obtained by methods of shock waves, may be smoothed by diffusivity, also is retained third order derivative and is obtained solution of two-dimensional stochastic plane problem for nonlinear beams of probability are obtained. By above mentioned methods models of prognosis of economical crisis, of solution of nonlinear variant of known linear "Black-Sholes" equation for options with application to practical curves of markets, genetics problems, physical problems, traffic flow are proposed. For above mentioned media stochastic space problems using nonlinear variants of Fokker–Plank functional equations for probabilities and their solutions by methods of H. Haken and any other waves methods are also solved. We participated with talks at international conferences: "Chaos 2009, Greece", "Chaos 2011, Greece", "Computer Mechanics, Alushta 2009, 2011", "Nonequillibrium processes, Alushta, 2010", "Probabilities and Statistics, Leiptzig, 2010" and furthermore, full texts of talk are exchanged with their participants.

Boundary Value Problems of the Plane Theory of Termoelastisity with Microtemperatures for a Half-Plane

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The present paper deals with to the two-dimensional version of statics of the linear theory of elastic materials with inner structure whose particles, in edition to the classical displacement and temperature fields, possess microtemperatures. Some problems of the linear theory of thermoelasticity with microtemperatures will be considered in the upper half-plane. Let on the boundary of the half-plane one of the following boundary conditions be given: a) displacement vector, microtemperature vector and the temperature, b) displacement vector, microtemperature vector and a linear combination of normal component of microtemperature vector and a normal derivative of temperature, c) displacement vector, tangent components of microtemperature vector, a normal components of microstress vector and temperature, d) displacement vector, normal components of microtemperature vector, tangent components of microstress vector and the normal derivative of temperature. Using Fourier transform, these problems are solved explicitly (in quadratures).

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High Frequency Homogenization

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It is highly desirable to be able to create continuum equations that embed a known microstructure through effective or averaged quantities such as wavespeeds or shear moduli. The methodology for achieving this at low frequencies and for waves long relative to a microstructure is well-known and such static or quasi-static theories are well developed. However, at high frequencies the multiple scattering by the elements of the microstructure, which is now of a similar scale to the wavelength, has apparently prohibited any homogenization theory. Many interesting features of, say, periodic media: band gaps, localization etc occur at frequencies inaccessible to averaging theories. Recently we have developed an asymptotic approach that overcomes this limitation, and continuum equations are developed, even though the microstructure and wavelength are now of the same order. The general theory will be described and applications to continuum, discrete and frame lattice structures will be outlined. The results and methodology are confirmed versus various illustrative exact numerical calculations showing that theory captures, for instance, all angle negative refraction, ultra refraction and localised defect modes.

Dynamics of the Defect Structure of Solids Under Variable External Load

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The basic position of modern ideology of earthquake engineering consist in that the construction should be designed so that suffers almost no damage at an earthquake the occurrence of which is most probable in given area during the lifetime of the construction. At less probable, hence stronger earthquakes, the construction should receive a different damage rate. This statement essentially based on the so-called Performance Based Design, the ideology of the 21st century. In the article attention is focused on the fact that the modern ideology of earthquake engineering assigns structures to a dangerous zone in which their behavior is defined by processes of damage and destruction of materials, which in essence is nonequilibrium process and demands application of special refined methods of research. In such conditions use of ratios that correspond to static conditions of loading to describe the process of damage of materials appears to be unfounded. The article raises the question of necessity of working out a new mathematical model of behavior of materials and structures at rapid intensive influences. How are the macroscopic parameters of the material changing under external stress varying in time? How do these changes relate to micro- and mesoscopic transformations in the bulk of the material? Can the material periodically restore its original structure when the external stress is removed? What alloy or solid solution is able to prevent the nucleation and propagation of microcracks most effectively?

How can the mean free path for magistral crack propagation be reduced? Can we stop the destruction of the material by artificial generation of additional microcracks acting as traps and crack stoppers? How the formation mechanism of these traps is related to the properties of the magistral crack? Can the magistral crack itself generate these crack stoppers? Solving such problems requires a self-consistent approach, a joint study of both micro- and mesoscopic processes in a complex crystal structure.

Signorini's Problem with Natural Nonpenetration Condition in Elasticity

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In the recent work the contact problem of an elastic anisotropic unhomogeneous body with a rigid body (frame) is considered. Usually such contact by *Signorini boundary conditions* is described including normal displacement and normal stress (also the tangential components if the friction arise between bodies). These conditions are derived after some linearizations and other simplifications of the *Natural Nonpenetration Condition*:

$$x_3 + u_3 \le \psi(x' + u'), \tag{1}$$

where $u = (u_1, u_2, u_3)$ is a displacement vector, $x = (x_1, x_2, x_3)$ belongs to the contact part of boundary of the elastic body, $u' = (u_1, u_2)$, $x' = (x_1, x_2)$ and ψ describes the contact surface of the frame.

Our aim is to avoid the simplification procedure which distancing the mathematical model from the physical processes, and describe the mentioned contact by the initial Nonpenetration Condition (1). The only assumption to this end is that ψ be concave and continuous.

Suppose that the elastic body is subjected under volume and external forces, then the Condition (1) leads to the variational inequality on the close convex set. When the body is fixed by part of its boundary (i.e., we have the Dirichlet condition on this part of boundary), then the variational inequality has a unique solution. Without this condition the necessary result of the existence of solution is obtained. When $\psi \in C^2(R^2)$, then we write the boundary conditions corresponding to the variational inequality. The stability of solution is also obtained when the problem is uniquely solvable and $\psi \in C^0(R^2)$.

Stresses Near the Absolutely Rigid Thin Inclusion in the Orthotropic Plane

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It is well known that the two-dimensional problem of the elasticity theory for an isotropic plane containing defects can be reduced to the Riemann problem for one or two functions using the Kolosov–Muskhelishvili complex potentials under certain conditions. Such problems can be solved in closed form. Therefore, the method of discontinuous solutions of the elasticity theory equations is very effective. The solution of mentioned class of problems is reduced to the solution of the system of singular integral equations allowing the closed solutions. Complex potentials are ineffective for anisotropic plane, in particular for orthotropic plane.

Based on the discontinuous solutions of equations of the elasticity theory both for an orthotropic plane and for an isotropic plane, the complex functions reducing the two-dimensional problem for an orthotropic plane with defects on the one of the principal directions of orthotropy to the Riemann problem for one or two functions are considered in the Chapter I. Some particular cases are being of special attention.

Particularly, the closed solutions of the following problems are constructed:

a) two-dimensional problem of stress state of the elastic orthotropic plane, containing the absolutely rigid thin inclusion with length a on one of the principal direction of materials' orthotropy $y = 0$ on the interval $(0, a)$ and semi-infinite crack on the interval $(-\infty, 0)$;

b) two-dimensional problem of stress state of the elastic orthotropic plane with semi-infinite crack on one of the principal direction of materials' orthotropy $y = 0$ on the interval $(-\infty, a)$ and the absolutely rigid thin inclusion with length a welded on the lower edge of crack on the interval $(0, a)$;

c) two-dimensional problem of stress state of the elastic orthotropic plane with the slit on the interval $(-a, 0)$ and the absolutely rigid thin inclusion on the interval $(0, a)$ of one of the principal direction of orthotropy $y = 0$.

Complete Solutions of Saint-Venan's Problems for Two-Layer Confocal Elliptic Tube

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The word "Complete" in the title means that different materials composing a confocal tube have Poisson's different ratios. For composite bodies N. Muskhelishvili introduced a concept of three auxiliary problems about plane deformation whose solution enables us to reduce solution of the three-dimensional Saint-Venan's problems to plane problems of theory of elasticity.

Here three auxiliary problems are solved for confocal composite ring. It should be noted that unlike concentric circles, where only 4 coefficients are unknown, here for determination of complex potentials 16 coefficients are needed. In spite of this, all unknown values are determined explicitly and complex potentials are determined in closed form. This circumstance gave us possibility to solve the considered problems effectively and to reduce three-dimensional problems to plane ones.

As concrete cases, solutions of problems are obtained when the smaller semi-axis of the inner ring is zero, i.e. we obtain solution of the problem when the composite tube has a cut via focuses along the whole length.

Some Thermoelasticity Problems for Cylindrical Bodies with Non-Classical Conditions on the Surface

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There are a number of elasticity problems which can be called non-classical due to the fact that boundary conditions at a part of the boundary surface are either overdetermined or underdetermined, or the conditions on the boundary are linked with the conditions inside the body.

Using the method of separation of variables the following non-classical three-dimensional thermoelasticity problems are stated and analytically solved in the present paper.

In a generalized cylindrical system of coordinates (Cartesian system of coordinates, circular, elliptical, parabolic and bipolar cylindrical system of coordinates) thermoelastic equilibrium of finite bodies bounded by coordinate surfaces of this coordinate system is considered. Either symmetry or antisymmetry conditions or those similar to the above ones are defined on the lateral surfaces of the involved body. The upper and the lower boundaries of the body are free from stress.

The problem is to choose temperature distribution on the upper and lower boundaries of the body, so that normal or tangential displacements would take a priory defined values on those boundary surfaces. Hence, in addition to stresses, either normal or tangential displacements are given on the upper and lower surfaces.

The studied problems significantly differ from the above-mentioned non-classical problems and are of great applicational importance.

Null Lagrangians in the Field Theories of Continuum Mechanics

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Although the continuum field theory based on the variational principle of least action in the most cases provides an elegant and rather fruitfull mean of studying a wide range of problems known from continuum physics it involves with its exploitation several intrinsic difficulties. Within the context of the field theories of continuum mechanics it has long been known that there exist Lagrangians that satisfy the Euler-Lagrange differential equations identically with respect to all field variables representing physical fields in a space-time of an arbitrary finite dimension. Thus such Lagrangians have no connections with real physical fields at all and can be associated with the empty space-time manifold wherein physical fields are realized. These are known as Lagrangians of an empty space-time or null Lagrangians. It can be demonstrated that this happens when the Lagrangian is the total divergency of a vector field of the space-time coordinates, field variables and their gradients. It is then seen that additive null Lagrangians do not touch the differential field equations of continuum theories but do affect the constitutive equations of these theories and modify the infinitesimal condition of the action integral invariance. The latter is closely related to variational symmetries of the field equations and conservation laws for physical fields due to the Noether theory.

It is usually suggested that the inclusion of null Lagrangians in formulations of the continuum field theories may be of use in continuum mechanics but this has not been pursued yet. By now the notion of null Lagrangians has only been used in continuum mechanics occasionally.

In the present study the exact forms for a Lagrangian in a space-time of an arbitrary finite dimension to be null are given and further specifications of their antisymmetric general forms for a three-dimensional space and the Minkowski 4-space-time are derived aiming at their application to problems of the classical elasticity and cross-coupled hyperbolic thermoelasticity and development of new physically meaningful constitutive models of the heat transport as undamped heat waves propagating at finite speed.

A few examples of the use of a novel null Lagrangians technique in the field theories of continuum thermomechanics by constructing physically reasonable Lagrangian functions containing one or more summands which are null are discussed.

On Complex Potentials and on the Application of Related to Them M. G. Krein's Two Integral Equations in Mixed Problems of Continuum Mechanics

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The methods of complex potentials, in general, those of the boundary value problems of the theory of analytic functions are well-known and widely used in various problems of solid mechanics and mathematical physics. These methods are especially efficient in combination with the methods of the theory of integral transformations and integral equations.

In the present work we suggest a way different from that suggested by N. I. Muskhelishvili in his fundamental monograph, i.e., a way of representation of a general solution of the governing system of the plane theory of elasticity by means of complex potentials and consider applications of two integral M.G. Krein's equations to the solution of one class of mixed problems of continuum mechanics which are connected with complex potentials of the corresponding physical fields.

The Dynamic Contact Problem which Reduce to the Singular Integral Equation with Two Fixed Singularities

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The solving of the problem on the longitudinal share harmonic oscillations of the elastic strip, which is linked with the elastic halfspace, is given in the report. By the method of the integral transformations the initial boundary problem is reduced to the integral equation with regard to the contact stress in the domain of the strip and the halfspace adhesion.

This integral equation have, besides the singularity in the Cauchy's form, the two immovable singularities in the ends of the integration interval. One of the main results of the report is the numerical method of this equation solving. The proposed method takes into consideration the real singularity of the solution and is based on the special quadrature formulas for the singular integrals' calculation. The obtained approximate solution give the opportunity to investigate numerically the influence of the oscillations' frequency and elastic constants of the strip and the halfspace on the stress' distribution.

Using a Nonlinear Static "Pushover" Analysis for Design the High-Rise Building on Seismic Action

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The static pushover procedure has been presented and developed over the past twenty years by various researches. The seismic demands are computed by nonlinear static analysis of the structure subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a predetermind target displacement is reached. Both the force distribution and target displacement are based on a vary restrictive assumptions, i.e. a time-independent displacement shape Under incrementally increasing loads various structural elements yield sequentially. Consequently, at each event, the structure experiences a loss in stiffness. Using a pushover analysis, a characteristic nonlinear force-displacement relationship can be determined. First stage of nonlinear static analysis of building is construction of the "capacity" curve, which represents the relation between base shear force and control node displacement. A non-linear static (pushover) analysis is used to characterize the response of the structure typically by means of a "capacity curve". The expected peak displacement, or "target displacement", is determined by means of an "equivalent" single-degree-of-freedom oscillator, whose properties are derived from the capacity curve.

The main output of a pushover analysis is in terms of response demand versus capacity. If the demand curve intersects the capacity envelope near the elastic range, then the structure has a good resistance. If the demand curve intersects the capacity curve with little reserve of strength and deformation capacity, then it can be concluded that the structure will behave poorly during the imposed seismic exitation.

The main output of a pushover analysis is in terms of response demand versus capacity. If the demand curve intersects the capacity envelope near the elastic range, then the structure has a good resistance. If the demand curve intersects the capacity curve with little reserve of strength and deformation capacity, then it can be concluded that the structure will behave poorly during the imposed seismic exitation.

The process of constructing the "capacity" curve is illustrated by the three story-two bays concrete plane frame for triangular and rectangular patterns distribution of forces.

Plane mathematical model of 45-storey dual (frame-wall) RC building with the core consisting of RC walls by a linear analysis was designed on the spectral method according to Eurocode-8. For the construction of the "capacity" curve for 45-storey RC building was used the first form of vibrations, which corresponds to a triangular distribution of forces in height, using "Seismostract". Version 4.0.3. 2007.

On Some Problems of Instability and Bifurcations in the Heat-Conducting Flows Between Two Rotating Permeable Cylinders

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The instability of a viscous heat-conducting flows between two rotating cylinders heated up to different temperatures is investigated. It is assumed that the flow is subjected to the action of a radial flow through the cylinder walls and a radial temperature gradient.

The talk presents the results of investigation of the neutral curves which separate the instability regions from the stability ones for various width values of the gap between cylinders, also the regimes, arising after the main flows lose their stability.

Pulsation Flow of Incompressible Electrically Conducting Liquid with Heat Transfer

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In the last years heat phenomena in pipes under the action of extremal magnetic fields attracted special interest. In paper [1] a stationary flow of the electro-conductive viscous incompressible liquid between two isothermal walls with heat exchange is studied under the action of external homogeneous magnetic field on heat exchange during constant drop of pressure and constant loss of liquid. The essential relationship between the temperature of liquid flow, Nusselt number of heat exchange, the value of external magnetic field and electric-conductivity of the walls of flat pipe is revealed. In paper [2], in contrast to [1], a non-stationary flow of viscous incompressible liquid between two parallel walls with thermal exchanges is studied. The flow of velocity distribution has parabolic character. In works [3, 6–9], analytical solutions of the Navier–Stokes equations are obtained for non-stationary motion and heat exchange. The flow of liquid is considered between two parallel walls and in a circular pipes under the action of external homogeneous magnetic field.

In this paper, in contrast to [1, 2, 3–9], a pulsating flow of electro-conductive viscous incompressible liquid in a flat pipe is studied taking into account the Joule heating $\left(M^2\left(\frac{\partial h}{\partial \xi}\right)^2\right)$ and the friction heating $\left(\left(\frac{\partial u}{\partial \xi}\right)^2\right)$. The flow of liquid is initiated by pulsating motion of the walls and the corresponding pulsating pressure drop given by the following law: $-\frac{1}{\rho}\frac{\partial p}{\partial z} = Ae^{-i\omega t}$. The change of the temperature in the flat pipe is accomplished in a pulsating manner. The exact solutions of the Navier–Stokes and heat exchange equations are obtained. The resulting criteria of similarity characterize the oscillatory motion of the liquid caused by viscosity forces under the action of external homogeneous magnetic field.

When pulsating flow of electro-conductive liquid is caused by pulsating drop of the pressure, for $2\alpha\tau = 0^0$, under increase of the magnetic field and of the parameter α on the axis of the flat pipe (when $S \to 0$ and α is a finite number), the difference in temperatures between the initial pulsating distribution and the final pulsating distribution decreases, while the difference in temperatures increases with the decrease of α and M.

For $2\alpha\tau = 0^0$ and $2\alpha\tau = \frac{\pi}{4}$, under increase of the external magnetic field and α, the friction on the wall increases, while leakage of liquid decreases.

When $A_0 = B_0 = 0$, $D = 1$, $\alpha \to 0$ and S is a finite value, the obtained results coincide with those given in the paper [1].

In general, pulsating motion caused by the pulsating drop of pressure causes decrease of temperature of the electro-conductive liquid.

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Solution of a Mixed Problem of the Linear Theory of Elastic Mixture for a Polygon a Part of Which Boundary Is an Equi-Strong Arc

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In the present work we consider a mixed boundary value problem of statics of the linear theory of elastic mixtures for an isotropic polygon one side of which has a cut of unknown shape.

On the entire boundary $\sigma_s = 0$, and the vector U_n takes constant values on the linear part of the boundary, moreover $\sigma_n = P$, on an unknown contour, where $P = (P_1, P_2)^T$ is known constant vector,

$$
\begin{aligned}
U_n =& \Big(u_1 n_1 + u_2 n_2, u_3 n_1 + u_4 n_2 \Big)^T, \\
\sigma_n =& \Big((Tu)_1 n_1 + (Tu)_2 n_2, (Tu)_3 n_1 + (Tu)_4 n_2 \Big)^T, \\
\sigma_s =& \Big((Tu)_2 n_1 - (Tu)_1 n_2, (Tu)_4 n_1 - (Tu)_3 n_2 \Big)^T,
\end{aligned}
$$

u_k and $(Tu)_k$, $k = 1, 2, 3, 4$ are partial displacement and stress' vectors components respectively, and $n = (n_1, n_2)^T$ is the unit vector of the outer normal.

Owing method of the Kolosov–Muskhelishvili the problem is reduced to the Riemann–Hilbert problem for a half-plane.

The stressed state of the body is defined and the equation of the unknown contour is obtained, for the condition that so-called the tangential normal stress vector on those boundary takes one and the same constant value. Using the obtained results, the problem with an unknown boundary for polygons weakened by holes in the presence of symmetry are solved.

To Uniform Systems of Equations of Continuum Mechanics and Some Mathematical Problems for Thin-Walled Structures

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We propose a uniform dynamic system of pseudo-differential equations which contains as a particular case Navier–Stokes, Euler equations, systems of PDEs of solid mechanics, Newton's law for viscous flow, generalized Hooke's law, the mass and principle of energy conservations. Such unique representation of this system allows us to prove that the nonlinear phenomena observed in problems of solid mechanics can also be detected in Navier–Stokes type equations, and vice versa.

One of the most principal objects is a system of nonlinear DEs constructed by von Kármán. In spite of this in 1978 Truesdell expressed an idea about neediness of "Physical Soundness" of this system. Based on [1], the method of constructing such anisotropic inhomogeneous 2D nonlinear models of von Kármán–Mindlin–Reissner (KMR) type for a binary mixture of porous, piezo and viscous elastic thin-walled structures with variable thickness is given, by means of which terms take quite determined "Physical Soundness". The corresponding variables are quantities with certain physical meaning: averaged components of the displacement vector, bending and twisting moments, shearing forces, rotation of normals, surface efforts. By choosing parameters in the isotropic case from KMR type system von Kármán DEs as one of the possible models is obtained. Further for generalized transversal elastic plates from KMR the unified representations for all refined theories are obtained which have the convenient form for applying directly the methods developed by Muskhelishvili, Vekua and other authors as these models are the systems of Cauchy–Riemann DEs type. Further these equations are constructed taking into account the conditions of equality of the main vector and moment to zero. Then we prove that for the global estimation of remainder transition vector under to thickness has the 6th order on sufficient smooth classes of admissible solutions or the best Sard's type constants for another one.

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Development Approaches of Academic Muskhelishvili in Researches of Belarusian Mechanics

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The first regular researches connected with construction of solutions of problems mechanics on the basis of methods and approaches of academician N.Muskhelishvili, on department of mechanics of Belorussian State University have begun in 1970 under the leadership of I.A. Prusov.

I.A. Prusov are received fundamental results in elaboration and application of one of the most effective methods of the theory of complex variable functions - a method of linear interface of N.I. Muskhelishvili – to the solution of boundary problems of stationary heat conductivity and thermoelasticity of an anisotropic body. In particular, for the solution of the basic boundary problems of the theory of a bend of isotropic and anisotropic plates. A number of two-dimensional problems of a filtration is solved. The thermoelastic complex potentials created by I.A. Prusov allow to receive simple and effective solutions of the basic boundary problems of anisotropic bodies and had the further development in works of his pupils and followers.

The next direction of researches in which methods and the approaches developed in works by N. Muskhelishvili are actively used, is connected with researches of M.A. Zhuravkov and his colleagues and pupils with reference to solutions construction of various classes of problems of mechanics of rocks and massifs, the hydrogeomechanics, the connected problems of geomechanics and gas dynamics, mechanics of underground constructions etc.

So, analytical solutions for definition of the stress-strain state of a heavy massif with several underground constructions of any cross-section contour are constructed. Approaches to the solutions of some classes of geomechanics problems for the biphasic environment, connected with studying gasdynamic phenomena are offered. The solution of a crack stability problem in a heavy massif in the conditions of a complex stress state is received, etc.

Researches in such perspective directions of modern mechanics, as biomechanics and nanomechanics are begun on department last years. One of directions of researches is connected with construction of numerically-analytical solutions for modeling and calculation of biomechanical systems. For example, numerically-analytical models for studying state of system "a jaw - artificial limbs", "a jaw - correcting devices" etc.

Methods of N. Muskhelishvili are used at giving of several basic and special courses for students of mechanics.

Continuum Mechanics II: Shells, Plates, and Beams

On a Numerical Solution of a Problem of Non-Linear Deformation of Elastic Plates Based on the Refined Theory

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We consider a numerical solution of a problem of non-linear deformation of elastic plates based on the refined theory. This theory takes into account the deformations which may not be homogeneous along the shifts. The obtained numerical results are compared with that of based on the other theory.

Stability of Plates with Holes

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In the present paper the problem of stability of a rectangular plate on the elastic basis with a circular hole is considered. The rectangular plate lies on the elastic basis of Winkler's type. Compressive forces act on two short sides of rectangular plate. The problem is solved by the energy method involving the theory of functions of complex variable. The solution of the problem is based on V. G. Naloeva's experimental works, where it was established, that the stability of a plate with a hole is only slightly different from the case of a plate without a hole. Depending on character of fixing of edges of a rectangular plate function of bending of a plate choose at stability loss. During the solution of a problem of stability the energy method in the form of Bryan's criterion is used. As is known, at using of energy criterion of Bryan it is necessary to determine initial to a critical condition of the two-connected plate compressed by in regular intervals allocated loading with certain intensity. The solution of a two-dimensional problem for to an initial condition is made by methods of the theory of functions of the complex variable. Determining of stress-strain state of a compressed two-connected plate is reduced to a finding of two complex potentials in the form of Loran series with unknown coefficients. Unknown coefficients are determined from corresponding boundary conditions of a two-dimensional problem.

Numerical realization of a problem of stability is made by the Rayleigh–Ritz method. For various geometrical parameters of a considered problem critical parameters of loading have been determined and corresponding diagrammes are made.

The Vibration of the Shallow Spherical Panel with the Account of the Inner Rotation

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The problem is investigated on the base of the reduced model of the Cosserat's theory. The generalization of Hook's law is suggested in the form [1].

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk} + J\,\frac{\partial^2\omega_{ij}}{\partial t^2}\,.$$

Here the new notations are introduced: ω_{ij} are the components of the inner rotation tensor, J is the coefficient of rotation inertia. The other rotations are well known [2]. The equations of the spherical panel vibrations are received by using the Kirchhoff–Love's hypothesis. These equations are bringing to one separate equation for the shear vibrations and the system of the two coupled equations for longitudinal and bending vibrations. The influence of the inner rotation on the frequencies of the shear and bending vibration is investigated.

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Vibration of an Elastic Plates with Variable Thickness on a Basis of the Refined Theories

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In 1955 I. Vekua raised the problem of investigation of elastic cusped plates, whose thickness on the whole plate or on a part of the boundary vanishes. Such bodies, considered a 3D ones, may occupy 3D domains with, in general, non-Lipschitz boundaries. In practice cusped plates are often encountered in spatial structures with partly fixed edges, e.g., stadium ceilings, aircraft wings, submarine wings, etc., in machine-tool design, as in cutting-machines, planning-machines, in astronautics, turbines, and in many other application fields of engineering. Mathematically, the corresponding problems lead to non-classical, in general, boundary value and initial-boundary value problems for governing degenerate elliptic and hyperbolic systems in static and dynamical cases, respectively (for corresponding investigations see, e.g., the survey in [1]). Some satisfactory results are achieved in this direction in the case of Lipschitz domains but in the case of non-Lipschitz domains there are a lot of open problems. To consider such problems is a main part of the objectives of the present talk which is organized as follows: in the first section special flexible cusped plates vibrations on the base of the classical (geometrically) non-linear bending theories [2] is investigated; in the second part concrete problems for cusped plates for Reisner–Mindlin type models are studied (case of constant thickness is considered, e.g., in [3]).

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$\mathbb{R}$-Linear Problem and its Application to Composites with Imperfect Contact

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We consider a conjugation problem for harmonic functions in multiply connected circular domains. This problem is rewritten in the form of the R-linear boundary value problem by using equivalent functional-differential equations in a class of analytic functions. It is proven that the operator corresponding to the functional-differential equations is compact in the Hardy-type space. Moreover, these equations can be solved by the method of successive approximations under some natural conditions. This problem has applications in mechanics of composites when the contact between different materials is imperfect. Its given information about effective conductivity tensor with fixed accuracy for macroscopic isotropy composite material.

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Classical Bending Theories for Orthotropic Beams

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Current beam bending theories are almost exclusively dominated by the classical Euler-Bernoulli and Timoshenko beams models. The present contribution suggests a simple analytical procedure, within context of plane linear orthotropic elasticity, for generating beam bending theories *ad libitum*. Level of accuracy is increasing recursively with first and second order theories coinciding with Euler-Bernoulli theory and nearly with the Timoshenko theory, respectively.

Currently used methods to determine the Airy stress function employ expansion in powers of axial coordinate, or a Fourier series solution. Here, by contrast, we represent the stress function as a series in increasing order of derivatives of bending moment. The coefficients of each term are self induced functions of the transverse coordinate that transcend any particular load distribution.

An effective, user friendly, recursive procedure is derived for finding the transverse functions, thus providing the solution for Airy stress function at increasing levels of accuracy. Compliance with stress boundary conditions over the long faces of the beam leads to the complete analytical solution for stress components.

Turning to beam kinematics, we show how displacement data, at the beam supports, is accounted for in an averaged Saint-Venant type fashion. In particular, we derive a universal relation for first and second order bending rigidities and examine comparison with both Euler-Bernoulli and Timoshenko beam theories.

A few instructive examples are illustrated and accuracy assessed against known solutions obtained by either power expansion or Fourier series. It is argued that the present method determines leading terms of beam bending response by elegant physically motivated relations.

Exact Two- and Three-Dimensional Models Beams, Plates and Shells of Layered Composites and Nanostructures

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In this paper we discuss beams, plates and shells from layered composites and nanostructures. Within the bounds of each layer a composite material is anisotropic. A junction of

* This work is based on MSc research at the Technion.

layers ensures continuity of corresponding stresses and strains while crossing from one layer to another. In addition, elasticity constants and densities could vary continuously via the layers thickness. The composite and nano-material is linearly-elastic. There are no restrictions on a shape of the structural member and thickness of the layers. Stress and strain state under static and dynamic loads is studied. General formulation for bounded electro-, magnetic- and thermo-elastic phenomenon in composites is offered.

The mathematical model of the composite structure is created by using the method of initial functions (MIF), which is an analytical method of mechanics of solids [1].

Main equations of the method connect the stresses and strains on one of coordinate surfaces of the body with characteristics of the strain and stress state in an arbitrary point of the body in the form: $U = LU^0$, where $L = [L_{ij}]$ is a matrix of differential operator-functions. In a curvilinear orthogonal coordinate: $\alpha,\ \beta,\ \gamma : U = \{u_\alpha, u_\beta, u_\gamma, \tau_{\gamma\alpha}, \tau_{\gamma\beta}, \sigma_\gamma, \sigma_\alpha, \sigma_\beta, \tau_{\alpha\beta}\}$, $U = \{u^0_\alpha, u^0_\beta, u^0_\gamma, \tau^0_{\gamma\beta}, \tau^0_{\gamma\alpha}, \sigma^0_\gamma\}$.

These equations allow reducing the complex three-dimensional problem of the elasticity theory of an anisotropic body to more simple two-dimensional one, and with approximation of functions to the system of three algebraic equations with regard to unknown Fourier coefficients. Note that a number of governing equations for layered systems do not increase. Using the two- and three-dimensional models of layered composites and nanostructures enable to discover such peculiarities in the stress and strain state which cant be estimated by numerical methods and approximate (non-classical) theories. In addition, the method of initial functions allow to state and solve problems with common positions and be ensure to value the accuracy of solutions. For a large class of problems it is possible to obtain exact solutions without employing any hypothesis in stress and strain state.

Some specific problems of composite mechanics in exact states are considered: vibrations of a beam of graphene, bending of layered and non-homogeneous plates under static and dynamic loads; behavior of layered circular cylindrical shell under moving loads; analysis of stress and strain state of circular cylindrical shells including winding ones. Peculiarities in deformations of composite structures are examined.

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The Buckling of a Long Multilayer Viscoelastic Shell Under the Action of Nonuniform External Pressure

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The buckling instability of a long multilayer linearly viscoelastic shell, composed of different materials and loaded with a nonuniformly distributed external pressure of given intensity

$$q = q_0 \left(1 + \mu \sin^2 \varphi\right)$$

is investigated where q_0- is the loading parameter, $\mu > 0$ is the parameter describing the nonhydrostatic of compression. By neglecting the influence of fastening of its end faces, the initial problem is reduced to an analysis of the loss of load-carrying capacity of a ring of unit width separated from the shell. The new problem is solved by using a mixed-type variational method [1], allowing for the geometric nonlinearity with respect to the deflection, and the azimuthal direction is taken into account, together with the Rayleigh–Ritz method. The creep kernels are taken exponential with equal indices of creep. As an example, a two- and a three-layer ring with a structure symmetric with respect to its midsurface is considered, and the effect of its physicomechanical and geometrical parameters on the critical time of buckling instability of the ring is determined. In particular, it is shown, that the critical time for a two-layer ring are greater than for a three-layer one. It is found that, by selecting appropriate materials, more efficient multilayer shell-type structures can be created.

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Some Problems of Thermostability of Shells of Revolution, Close by Their Forms to Cylindrical Ones, with an Elastic Filler

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The stability of shells of revolution, which by their form are close to cylindrical ones, with elastic filler, is considered. The shell is under the action of torques applied to the shell ends as uniformly distributed tangential stresses, external normal pressure distributed uniformly over the whole lateral shell surface and heating. The shell is assumed to be thin and elastic. Temperature is uniformly distributed over the shell body. The elastic filler is modeled by the Winkler base, and its extension depending on heating is neglected. We consider the shells of middle length the form of whose middle surface generatrix is described by the parabolic function. We consider the shells of positive and negative Gaussian curvature. The boundary conditions at the shell ends correspond to free support, admitting some radial displacement in a subcritical state.

Formulas to define critical loading and forms of wave formation depending on temperature, elastic base rigidity and on amplitude of deviation of the shell from the cylindrical form, are given. Universal curves of dependence of the critical torque on the reduced circumferential contractive stress are obtained. The reduced circumferential stress is a sum of the normal stress from the action of external pressure and of that from the action of temperature and constraint of the elastic filler. It is shown that considerable increase of the filler rigidity under high temperatures leads to significant decrease of critical external pressure, which is undesirable for resolving power of the shell. Moreover, it is found that if a cylindrical shell is under the action of external pressure close to its critical value, then the shell for a small convexity may additionally endure some finite value of torque, whereas for an infinitely small concavity it loses its stability.

Nonlinear Parametric Vibrations of Viscoelastic Medium Contacting Cylindrical Shell Stiffened with Annular Ribs and with Respect Lateral Shift

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Resistance, vibration and strength analysis of thin-walled structural elements of the shell-type contact with the media play an important role in the design of modern devices, machines and soruzheny. Nonlinear deformation of cylindrical shells under the action of various kinds of dynamic loads monograph [1]. Matters relating to the stability of ribbed cylindrical shells without protection papers [2]–[4]. In a geometrically nonlinear formulation, nonlinear oscillations supported by a cross-system edges of a cylindrical shell with viscoelastic filler without shear are studied in [5].

In the paper, a problem on parametric vibration of a stiffened circular ribs cylindrical shell contacting with external viscoelastic medium and situated under the action of internal pressure is solved in a geometric nonlinear statement by means of the variation principle. Lateral shift of the shell is taken into account. Influences of environment have been taken into account by means of the Pasternak model. Dependencies of dynamical stability area on the construction parameters are given on the plane “load-frequency”.

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Problems of the Plane Theory of Elasticity and of Bending of Thin Plates with Partially Unknown Boundaries

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In the plate with a hole it is important to investigate the concentration of stresses near the hole contour. The tangential-normal stresses-in case of plane elasticity theory or the tangential-normal moments in case of bending of thin plates, can reach at some point such values that cause destruction of plates or the formation of plastic zones near the hole. Proceeding from the above-said, the following problem was posed: Given load applied to the plate's boundary, it is required to choose such hole shape at which boundary the maximum value of tangential normal stress or tangential-normal moment would be minimal in comparison with the all other holes. It is proved that such condition is valid provided that tangential normal stress and tangential-normal moments are constant at that hole. As is known, such holes are called equistable ones.

Problems of finding equistable holes for an infinite plane were considered by N. B. Banichuk, G. P. Cherepanov, S. B. Vigdergauz, N. Neuber and other, for the case where tensile or compressive stress act at infinity.

G.P. Cherepanov proved that the tangential-normal stresses of plate weakened by hole with equistable contour is less than 40% of maximum tangential normal stress of a plate weakened by circular hole.

The problems of equistable contours for finite doubly-connected domains bounded by broken lines and a sought smooth contour were considered by R. Bantsuri and R. Isakhanov. The problems reduced to the Riemann–Hilbert and Carleman type problems for the circular ring, whose solutions are given in such form, that it is difficult to construct require contours.

In our report we consider the problem of plane elasticity theory for a isosceles trapezium with partially unknown boundary. To solve this problems it is used the methods of function theory of complex variable.

Mathematical Models of Thin Shells on the Basis of Asymmetric Theory of Elasticity

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Asymmetric (micropolar, moment) theory of elasticity is a mathematical model of studying stress-strain state in bodies with obviously expressed structure at micro- and nano-levels. One of the important problems of applied asymmetric theory of elasticity is the construction of mathematical models of thin plates and shells. In this paper on the basis of qualitative results of the solution of boundary-value problem of three-dimensional asymmetric theory of elasticity in thin regions [1], following rather general hypotheses are formulated:
a. During the deformation initially linear and normal to the median surface fibers rotate freely in space at an angle as a whole rigid body, without changing their length and without remaining perpendicular to the deformed middle surface.
The hypothesis is mathematically written as follows $(i = 1, 2)$:

$$\begin{aligned} &V_i = u_i(\alpha_1, \alpha_2) + \alpha_3 \psi_i(\alpha_1, \alpha_2),\ V_3 = w(\alpha_1, \alpha_2), \\ &\omega_i = \Omega_i(\alpha_1, \alpha_2),\ \omega_3 = \Omega_3(\alpha_1, \alpha_2) + \alpha_3 \iota(\alpha_1, \alpha_2). \end{aligned} \tag{1}$$

b. In the generalized Hook's law the force stress σ_{33} can be neglected in relation to the force stresses σ_{ii} $(i = 1, 2)$.
c. Using the assumption of thin-walled shell, it is assumed $1 + \frac{\alpha_3}{R_i} \approx 1 \quad (i = 1, 2)$.
d. During determination of deformations, bending-torsions, force and moment stresses, first for the force stresses σ_{3i} and moment stress μ_{33} we shall take:

$$\sigma_{3i} = \overset{0}{\sigma}_{3i}(\alpha_1, \alpha_2), \quad \mu_{33} = \overset{0}{\mu}_{3i}(\alpha_1, \alpha_2). \tag{2}$$

After determination of above mentioned quantities, values of σ_{3i} and μ_{33} are retrieved from (2). The latter formula is derived, in it's turn, by integrating of the correspondent equilibrium equations under the constraint that the average quantities with respect to the shell's thickness are equal to zero.

On the basis of accepted hypotheses a model of micropolar thin shells with free fields of displacements and rotations and, using condition $\omega = \frac{1}{2} rot \mathbf{u}$, a model of micropolar shells with constrained rotation are constructed.

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Singular Integral and Pseudodifferential Equations

On the Fredholm Property of Wiener–Hopf and Wiener–Hopf–Hankel Operators with Piecewise-Almost Periodic Symbols on Weighted Lebesgue Spaces

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We consider Wiener-Hopf, Wiener–Hopf plus Hankel, and Wiener–Hopf minus Hankel operators on weighted Lebesgue spaces and having piecewise almost periodic Fourier symbols. The main results concern conditions to ensure the Fredholm property and the lateral invertibility of these operators. In addition, under the Fredholm property, conclusions about the Fredholm index of those operators are also discussed.

Fredholmity Criteria for a Singular Integral Operator on an Open Arc in Spaces with Weight

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In this paper we study a singular integral operator (shortly SIO) with the Cauchy kernel

$$\mathbf{A}\varphi(t)=a(t)\varphi(t)+\frac{b(t)}{\pi i}\int_{\Gamma}\frac{\varphi(\tau)d\tau}{\tau-t},\quad A\ :\ \mathbf{H}^0_{\mu}(\Gamma,\rho)\to\mathbf{H}^0_{\mu}(\Gamma,\rho) \tag{1}$$

and Hölder continuous coefficients $a,\ b\in\mathbf{H}_{\mu}(\Gamma)$ in the space $\mathbf{H}^0_{\mu}(\Gamma,\rho)$ of Hölder continuous functions with an exponential "Khvedelidze" weight $\rho:=(t-c_1)^{\alpha}(t-c_2)^{\beta}$, $0<\mu<1$, $\mu<\alpha$, $\beta<\mu+1$. The underlying contour $\Gamma=\overset{\smile}{c_1c_2}$, is an open arc with the endpoints c_1 and c_2.

It is well known, that for the operator in (1) to be Fredholm the condition $\inf\limits_{t\in\Gamma}|a(t)\pm b(t)|\neq 0$ is only necessary. Necessary and sufficient condition is the so called "Arc Condition", which means that some chords of a circle, depending on the exponents of the space α, β and μ, connecting the disjoint endpoints of the graph $a(c_1) \pm b(c_1)$ and $a(c_2) \pm b(c_2)$, does not cross zero 0. The "Arc Condition" was found by I. Gohberg and N. Krupnik in 1965 for the Lebesgue spaces $\mathbb{L}_p(\Gamma, \rho)$ with an exponential "Khvedelidze" weigh $\rho := (t-c_1)^\alpha (t-c_2)^\beta$, $1 < p < \infty$, $1/p - 1 < \alpha,\ \beta < 1/p$ (also see the earlier paper by H. Widom for $p = 2$). The result was carried over in 1970 to the Hölder spaces with weight $\mathbf{H}^0_\mu(\Gamma, \rho)$ by R. Duduchava in his doctor thesis.

Based on the Poincare-Beltrami formula for a composition of singular integral operators and the celebrated N. Muskhelishvili formula describing singularities of Cauchy integral, the formula for a composition of weighted singular integral operators ($-1 < \gamma,\ \delta < 1$)

$$S_\gamma S_\delta \varphi = \varphi(t) \ + \ i \cot \pi(\gamma - \delta) \left[S_\gamma \ - \ S_\delta \right] (\varphi), \qquad S_\delta \varphi(t) := \frac{t^\delta}{\pi i} \int_0^\infty \frac{\varphi(\tau)}{\tau^\delta (\tau - t)} d\tau \tag{2}$$

is proved. Using the obtained composition formula (2), the localization (which means "freezing the coefficients") we derive the criterion of fredholmity of the SIO (1) (the "Arc Condition") by looking for the regularizer of the operator $\mathbf{A}$ in the form $\mathbf{R} = a^* I + b^* S_\gamma$, $a^* = a(a^2 - b^2)^{-1}$, $b^* = -b(a^2 - b^2)^{-1}$ and choosing appropriate γ. To the composition $\mathbf{RA}$ is applied the formula (2) and coefficients of non-compact operators are equated to 0 to get $\mathbf{RA} = I + T$, where T is compact. The "Arc condition" follows. Further the index formula and the necessity of the "Arc condition" are proved by using a homotopy and the stability of the index of a Fredholm operators.

Absolutely similar results with a similar approach are obtained for SIO (2) with continuous coefficients in the Lebesgue spaces with a "Khvedelidze" weight $\mathbb{L}_p(\Gamma, \rho)$.

R–Linear Problem for Multiply Connected Domains and Alternating Method of Schwarz

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We study the R–linear conjugation problem for multiply connected domains by a method of integral equations. The method differs from the classical method of potentials. It is related to the generalized alternating method of Schwarz which is based on the decomposition of the considered domain with complex geometry onto simple domains and subsequent solution to boundary value problems for simple domains. Convergence of the method of successive approximations is based on the paper [1].

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Method of Discrete Singularities for Solution of Singular Integral and Integro-Differential Equations

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The common approach, named the method of discrete singularities by Prof. I. K. Lifanov for solution of the integral equations on the finite interval containing Cauchy type singular integral, is proposed. This method can be applied to the singular integral equations of a first and second kind, to equations with generalized Cauchy kernel, as well as to integro-differential equations.

The direct methods of solution of the integral equations are very effective when the obtaining of the closed solution is impossible. The application of Gauss type quadrature formulas for solution of the singular integral equation of first kind is well known. Mainly, the solutions unbounded at the both ends were considered.

The method of discrete singularities is the common method for solution of singular integral equations on finite interval when unknown function can be represented as product of unknown Hölder continuous function and Jacobi polynomials weight function $(1-t)^{\alpha}(1+t)^{\beta}$ ($Re(\alpha,\beta)>-1$). The method is based on the quadrature formula of Gauss type for singular integral. The method of discrete singularities was successfully applied to solution of the wide class of mixed boundary value problems of the elasticity theory. The solving process of the considered problems was reduced to solution of a singular integral equations of the second kind with real or complex coefficients, as well the equations with generalized Cauchy kernel, known also as equations with fixed singularities.

Application of Muskhelishvili's Method for Solution of Singular Integral Equations for Contact Problems with Adhesion

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Formulations of the contact problem with adhesion assume existence of region in contact area where contacting bodies are rigidly bounded with each other. Contact problems with adhesion come conventionally under two groups: problems with fixed contact area and problems of quasistatic indentation with increasing contact area:

Full adhesion
N. I. Muskhelishvili (1935)
V. M. Abramov (1937)

Partial adhesion
L. A. Galin (1945)
S. V. Falkovich (1945)

Indentation with adhesion
L. E. Goodman (1962)
V. I. Mossakovskyi (1963)
(incremental procedure)

For the second group of plane problems with full contact the following boundary conditions take place

$$u(x,a) = \varphi(x), \quad v(x,a) = g(x); \;\; x \in [-a, b] \tag{1}$$

where $\varphi(x)$ – unknown tangential boundary displacement, $g(x)$ – shape of indenter (stamp), a – size of contact area $[-a, b]$ as loading parameter, $b = b(a)$.

In the case of elastic half plane boundary conditions (1) lead to a singular integral equation for contact stresses. According to N. I. Muskhelishvili such equation can be solved in the class of bounded solutions under condition

$$\int_{-a}^{b} \frac{g'(x) + i\varphi'(x)}{(a+x)^{(1-i\tau)/2}(b-x)^{(1+i\tau)/2}}\, dx = 0; \;\; \tau \equiv \frac{1}{\pi}\ln(3-4\nu) \tag{2}$$

which can be considered as Volterra type equation in function $\varphi(x)$. This function permits to derive contact stresses by taking integrals only and solve the contact problem without use of incremental procedure. Equation (2) can be applied to nonsymmetric formulations of the contact problem with adhesion (I. A. Soldatenkov. Impression with adhesion of a punch into an

elastic half-plane under a tangential load // J. Appl. Mathematics and Mechanics. 1999, v. 63, No. 1, pp. 101–107).

Singular Integrals and Pseudo Differential Equations

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In author's papers (see [1]) a certain multidimensional singular integral defined by Bochner kernel was introduced. It was shown with the help of such integral one can obtain the solution formulas for a model elliptic pseudo differential equation in a cone. Here one considers some variations for such singular integral which can be used for studying solvability of pseudo differential equations in domains with cusp points.

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Differential Equations and Applications

Mixed Problems and Crack-Type Problems for Strongly Elliptic Second-Order Systems in Domains with Lipschitz Boundaries

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We consider two classes of problems for a strongly elliptic second-order system in a bounded n-dimensional domain Ω with Lipschitz boundary Γ, $n \geq 2$. For simplicity, we assume that the Dirichlet and Neumann problems in Ω are uniquely solvable.

1. Mixed problems. In the simplest case, the boundary Γ is divided into two parts Γ_1 and Γ_2 by a closed Lipschitz $(n-1)$-dimensional surface, with the Dirichlet and Neumann conditions on Γ_1 and Γ_2, respectively. The problem is uniquely solvable in the simplest spaces H^s (with the solution in $H^1(\Omega)$) and (the regularity result) in some more general Bessel potential spaces H^s_p and Besov spaces B^s_p. Equations on Γ are obtained equivalent to the problem. For this, we use analogs N_1 and D_2 f the Neumann-to-Dirichlet operator N and the Dirichlet-to-Neumann operator D on parts Γ_1 and Γ_2 of Γ.

The operators N_1 and D_2 are connected with Poincaré–Steklov type spectral problems with spectral parameter on a part of Γ. In the selfadjoint case, the eigenfunctions form a basis in the corresponding spaces, and in the non-selfadjoint case they form a complete system. If Γ is almost smooth (smooth outside a closed subset of zero measure), the eigenvalues of self-adjoint problems have natural asymptotics.

2. Problems with boundary or transmission conditions on a non-closed surface Γ_1, which can be extended to a closed Lipschitz surface Γ. In elasticity problems, Γ_1 is a crack, and in problems of acoustics and electrodynamics, it is a non-closed screen. For simplicity, we assume that Γ lies on the standard torus.

The results are similar to those indicated above. The corresponding operators are restrictions A_1 to Γ_1 of the single layer potential type operator A on Γ and H_1 to Γ_1 of the hypersingular operator H on Γ. For the corresponding spectral problems, the results are similar to those indicated above.

The references, mainly to Georgian and German papers, will be indicated in the talk.

Cauchy Problem for Some Systems of Hyperbolic equations with Damping Terms

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Let's consider the system of semilinear hyperbolic equations

$$u_{1_{tt}} + u_{1_t} + (-1)^{l_1} \Delta^{l_1} u_1 = \lambda_1 |u_1|^{p-1} |u_2|^{q+1} u_1, \quad t > 0, \quad x \in R^n, \tag{1}$$

$$u_{2_{tt}} + u_{2_t} + (-1)^{l_2} \Delta^{l_2} u_2 = \lambda_2 |u_1|^{p+1} |u_2|^{q-1} u_2, \quad t > 0, \quad x \in R^n, \tag{2}$$

$$u_k(0, x) = \varphi_k(x), \quad u_{k_t}(0, x) = \psi_k(x), \quad k = 1, 2, \quad x \in R^n. \tag{3}$$

If $\lambda_1 < 0$, $\lambda_2 < 0$, then for any $\varphi_k(x) \in W_2^{l_k}(R^n)$, $\psi_k(x) \in L_2(R^n)$, $k = 1, 2$ problem (1)–(3) has a unique solution

$$(u_1, u_2) \in C([0, \infty); W_2^{l_1}(R^n) \times W_2^{l_2}(R^n)) \cap C^1([0, \infty); L_2(R^n) \times L_2(R^n)).$$

Let

$$\lambda_1 > 0, \quad \lambda_2 > 0, \tag{4}$$

$$p > 0, \quad q > 0; \quad \frac{p+1}{l_1} + \frac{q+1}{l_2} > \frac{2}{n} + r(p, q), \tag{5}$$

where $r(p, q) = \frac{1}{l_1}$, if $p > 2$, $q \geq 0$; $r(p, q) = \frac{p}{2l_1} + \frac{2-p}{2l_2}$, if $0 \leq p \leq 2$, $q > 0$.

Theorem. *Let conditions* (4)–(5) *be satisfied. Then there exists* $\delta_0 > 0$ *such that for any* $(\varphi_k, \psi_k) \in U_\delta^{l_k} = \left\{(u, v) : \|u\|_{W_2^{l_k}(R^n)} + \|u\|_{L_1(R^n)} + \|v\|_{L_2(R^n)} + \|v\|_{L_1(R^n)} < \delta, \ k = 1, 2\right\}$ *problem* (1)–(3) *has a unique solution* (u_1, u_2)*:* $u_k \in C([0, \infty); W_2^{l_k}(R^n)) \cap C^1([0, \infty); L_2(R^n))$*,* $k = 1, 2$*, which satisfies the following estimates:*

$$\sum_{|\alpha|=r} \|D^\alpha u_k(t, \cdot)\|_{L_2(R^n)} \leq C(1+t)^{-\frac{n+2r}{4l_k}}, \quad t > 0, \quad r = 0, 1, \ldots, l_k,$$

$$\|u_{k_t}(t, \cdot)\|_{L_2(R^n)} \leq C(1+t)^{-\min(1+\frac{n}{4l_k}, \gamma_k)},$$

where $\gamma_k = \frac{n}{2} \sum\limits_{i=1}^{2} \frac{\rho_{ki}}{l_i} - r_k$, $k = 1, 2$.

Next we discuss the counterpart of the condition (5).

On the Principle of a Priori Boundedness for Boundary Value Problems for Systems of Nonlinear Generalized Ordinary Differential Equations

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Consider the nonlinear boundary value problem

$$dx(t) = dA(t) \cdot f(t, x(t)), \quad h(x) = 0, \tag{1}$$

where $A : [a, b] \to R^{n\times n}$ is a matrix-function with bounded total variation components, $f : [a, b] \times R^n \to R^n$ is a vector-function belonging to the Carathéodory class $K_{n,A}$, $h : BV_n \to R^n$ is a continuous operator satisfying the condition $\sup\{\|h(x)\| : \|x\|_s \le \rho\} < \infty$ $(\rho > 0)$, and BV_n is the normed space of all bounded total variation vector-functions $x : [a, b] \to R^n$ with the norm $\|x\|_s = \sup\big\{\|x(t)\| : t \in [a, b]\big\}$.

A vector-function $x \in BV_n$ is said to be a solution of the system $dx(t) = dA(t)\cdot f(t, x(t))$ if $x(t) = x(s) + \int\limits_s^t dA(\tau) \cdot f(\tau, x(\tau))$ for $a \le s \le t \le b$, where the integral is understand in the Lebesgue–Stieltjes sense.

Definition. The pair (P, l) of a matrix-function $P \in K_{n\times n,A}$ and a continuous operator $l : BV_n \times BV_n \to R^n$ is said to be consistent if: (i) for any fixed $x \in BV_n$ the operator $l(x, .) : BV_n \to R^n$ is linear; (ii) for any $z \in R^n$, x and $y \in BV_n$ and for $\mu(\alpha)$-almost all $t \in [a, b]$ $(\alpha(t) \equiv \|Var_a^t(A)\|)$ the inequalities $\|P(t, z)\| \le \xi(t, \|z\|)$, $\|l(x, y)\| \le \xi_0(\|x\|_s) \cdot \|y\|_s$ hold, where $\xi_0 : R_+ \to R_+$ is a nondecreasing function, and $\xi : [a, b] \times R_+ \to R_+$ is a function, measurable and integrable with respect to the measure $\mu(\alpha)$ in the first argument and nondecreasing in the second one; (iii) there exists a positive number β such that for any $y \in BV_n$, $q \in L_{n,A}$ and $c_0 \in R^n$, for which the condition $\det\big(I_n + (-1)^j d_jA(t) \cdot P(t, y(t))\big) \ne 0$ $(t \in [a, b],\ j = 1, 2)$ holds, an arbitrary solution x of the boundary value problem $dx(t) = dA(t) \cdot \big(P(t, y(t)) \cdot x(t) + q(t)\big)$, $l(x, y) = c_0$ admits the estimate $\|x\|_s \le \beta(\|c_0\| + \|q\|_{L,\alpha})$.

Theorem. *Let there exist a positive number ρ and a consistent pair (P, l) of a matrix-function $P \in K_{n\times n,A}$ and a continuous operator $l : BV_n \times BV_n \to R^n$ such that an arbitrary solution of the problem $dx(t) = dA(t) \cdot \big(P(t, x(t)) \cdot x(t) + \lambda[f(t, x(t)) - P(t, x(t))] \cdot x(t)\big)$, $l(x, x) = \lambda[l(x, x) - h(x)]$ admits the estimate $\|x\|_s \le \rho$ for any $\lambda \in]0, 1[$. Then problem (1) is solvable.*

To a considerable extent, the interest to the theory of generalized ordinary differential equations has been stimulated by the fact that this theory enables one to investigate ordinary differential, impulsive and difference equations from a unified point of view.

Defect Numbers of the Dirichlet Problem for the Properly Elliptic Equation

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Let D be the unit disk of the complex plane with boundary $\Gamma = \partial D$. We consider in D the fourth order properly elliptic equation

$$\frac{\partial}{\partial \bar{z}}\Big(\frac{\partial}{\partial \bar{z}} - \mu\,\frac{\partial}{\partial z}\Big)\Big(\frac{\partial}{\partial z} - \nu_1\,\frac{\partial}{\partial \bar{z}}\Big)\Big(\frac{\partial}{\partial z} - \nu_2\,\frac{\partial}{\partial \bar{z}}\Big)u = 0, \tag{1}$$

where μ, ν_1, ν_2 are complex constants such that $\mu\nu_1\nu_2 \neq 0$, $|\mu| < 1$, $|\nu_1| < 1$, $|\nu_2| < 1$, $\nu_2 = r\nu_1$, $0 < r < 1$. We seek the solution u of equation (1) from the class $C^4(D)\bigcap C^{(1,\alpha)}(D\bigcup\Gamma)$, which on the boundary Γ satisfies the Dirichlet conditions

$$\frac{\partial^k u}{\partial r^k}\Big|_{\Gamma} = f_k(x,y), \quad k = 0,1, \quad (x,y) \in \Gamma. \tag{2}$$

Here $f_k \in C^{(1-k,\alpha)}(\Gamma)$ $(k = 0,1)$ are prescribed functions on Γ, $\frac{\partial}{\partial r}$ is a derivative with respect to module of the complex number $(z = re^{i\varphi})$.

The problem (1), (2) is Fredholmian (see [1]). In the paper [2] were found the necessary and sufficient conditions of the unique solvability of this problem, and the formulas for the determination of the defect numbers of the problem, if the unique solvability failed. Further, in [3] it was proved, that in some cases the defect numbers of the problem (1), (2) are equal to one. In this paper we find another representation of the unique solvability conditions for the problem (1), (2) which helps us to prove, that in this case also the defect numbers of the problem are equal either to zero or to one.

Theorem 1. *Let $z = \mu\nu_1$. The problem* (1), (2) *is uniquely solvable if and only if*

$$f_k(z) \equiv z^{k-2}(1-r^{k-1}) + z^{k-3}(1-r^{k-2}) + \cdots + (1-r) \neq 0, \quad k = 3,\dots. \tag{3}$$

If the conditions (3) *fail then $f_m(z) = 0$ for only one $m > 2$, and the homogeneous problem has one linearly independent solution, which is polynomial of order $m+1$.*

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Mixed Boundary-Value Problems of Piezoelectricity in Domains with Cracks

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We investigate the solvability and asymptotic properties of solutions to spatial mixed boundary-value problems of piezoelectricity for homogeneous anisotropic bodies with cracks. Using the potential theory and theory of pseudodifferential equations on a manifold with boundary, the existence and uniqueness theorems are proved. The complete asymptotics of solutions are obtained near the crack edges and near the lines where the different boundary conditions collide. We analyze singularity properties of solutions and the corresponding stresses. We have found an important special class of transversally-isotropic bodies for which the oscillating singularities vanish and the singularity exponents are calculated explicitly with the help of the eigenvalues of a special matrix associated with the principal homogeneous symbol matrix of the corresponding pseudodifferential operator. It turned out that these singularity exponents essentially depend on the material constants.

On the Radiation Condition for Anisotropic Maxwell's Equations

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We consider Maxwell's equations in an anisotropic media, when the dielectric permittivity ε and the magnetic permeability μ are 3×3 matrices. We formulate a Silver–Müller type radiation condition at infinity which ensures the uniqueness of solutions when permittivity and permeability matrices are real valued, symmetric, positive definite and proportional $\varepsilon = \kappa\mu$, $\kappa > 0$.

Screen-Type Boundary-Value Problems for Polymetaharmonic Equations

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The Riquier and Dirichlet screen-type boundary-value problems are considered for the polymetaharmonic equation $(\Delta + k_1^2)(\Delta + k_2^2)u = 0$. We investigate these problems by means of the potential method and the theory of pseudodifferential equations, prove the existence and uniqueness of solutions and establish their regularity properties in Sobolev–Slobodetski spaces. In particular, it is shown that solutions of the Dirichlet screen boundary-value problem has higher smoothness (H^2-smoothness) than solutions of the Riquier screen-type problems, which belong to the space H^1, in general. We analyse the asymptotic behaviour of solutions near the screen edge and prove that solutions to the Riquier problems have $C^{\frac{1}{2}}$-smoothness, while a solution to the Dirichlet problem has $C^{\frac{3}{2}}$-smoothness.

Segregated Direct Boundary-Domain Integral Equations for Variable-Coefficient BVPs in Exterior Domains

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A *variable-coefficient* PDE on a three-dimensional unbounded domain under the mixed boundary conditions on the compact boundary, is reduced to four different segregated direct BDIE systems, which are analysed in weighted Sobolev (Beppo Levi type) function spaces suitable for infinite domains. Equivalence of three of the BDIE systems to the original BVPs and the invertibility of the BDIE operators of these three systems are proved. Fredholm properties of the fourth system are studied as well. This analysis is based on the unique solvability in the weighted Sobolev spaces of the variable-coefficient BVPs in unbounded domains, that is also proved. The discussed results extend the results obtained in [1]–[4] for interior domains in Sobolev spaces without weights, and are partly available in [5].

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The Simple Layer Potential Approach to the Dirichlet Problem: an Extension to Higher Dimensions of Muskhelishvili Method and Applications

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Some time ago I proposed a method for studying the following integral equation of the first kind

$$\int_{\partial\Omega} \varphi(y)\,|x-y|^{2-n} d\sigma_y = \psi(x), \quad x \in \partial\Omega, \tag{1}$$

where Ω is a simply connected bounded domain of $\mathbb{R}^n$ with a sufficiently smooth boundary.

The main idea is to take the differential of both sides in (1):

$$\int_{\partial\Omega} \varphi(y)\, d_x|x-y|^{2-n} d\sigma_y = d\psi(x), \quad x \in \partial\Omega. \tag{2}$$

This equation is a singular integral equation of a new kind, since the unknown is a scalar function while the data is a differential form of degree 1. If we denote by J the singular integral operator on the left hand side of (2), J can be considered as a linear and continuous operator from $L^p(\partial\Omega)$ into $L^p_1(\partial\Omega)$ (the space of differential forms of degree 1, whose coefficients belong to L^p in every local coordinate systems), $1 < p < \infty$.

After constructing a reducing operator J', i.e. an operator $J' : L^p(\partial\Omega) \to L^p_1(\partial\Omega)$ such that $J'J = I + K$ is a Fredholm operator from $L^p(\partial\Omega)$ into itself, one can apply the general theory of reducible operators. This leads to an existence theorem in suitable spaces for (1).

This method can be considered as an extension of the classical one which Muskhelishvili gave in the plane for the equation

$$\int_{\partial\Omega} \varphi(\zeta)\, \log|z-\zeta|\, ds_\zeta = \psi(z), \quad z \in \partial\Omega. \tag{3}$$

It is clear that an existence theorem for equation (1) leads to a representation theorem for the solution of the Dirichlet problem for Laplace equation by means of a simple layer potential.

Slightly less obviously it leads also to a double layer potential representation for the Neumann Problem.

In this talk, after recalling this method for Laplace equation, I will illustrate some recent results concerning the Elasticity and Stokes systems in multiply connected domains. These last results have been obtained jointly with Drs. Vita Leonessa and Angelica Malaspina.

Transmission and Interface Crack Problems of Thermoelasticity for Hemitropic Solids

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Technological and industrial developments, and also recent important progress in biological and medical sciences require the use of more general and refined models for elastic bodies. In a generalized solid continuum, the usual displacement field has to be supplemented by a microrotation field. Such materials are called micropolar or Cosserat solids. They model composites with a complex inner structure whose material particles have 6 degree of freedom (3 displacement components and 3 microrotation components). Recall that the classical elasticity theory allows only 3 degrees of freedom (3 displacement components). In the mathematical theory of hemitropic elasticity there are introduced the asymmetric force stress tensor and moment stress tensor, which are kinematically related with the asymmetric strain tensor and torsion (curvature) tensor via the constitutive equations. All these quantities are expressed in terms of the components of the displacement and microrotation vectors. In turn the displacement and microrotation vectors satisfy a coupled complex system of second order partial differential equations of dynamics [1], [2].

The main goal of our investigation is to study the Dirichlet and Neumann type boundary transmission and interface crack problems of the theory of elasticity for piecewise homogeneous hemitropic composite bodies of arbitrary geometrical shape with regard to thermal effects. We consider the differential equations corresponding to the time harmonic dependent case, the so called pseudo-oscillation equations and develop the boundary integral equations method to obtain the existence and uniqueness results in various Hölder $(C^{k,\alpha})$, Sobolev–Slobodetski (W^s_p) and Besov $(B^s_{p,q})$ functional spaces. We study regularity properties of solutions at the crack edges and characterize the corresponding stress singularity exponents.

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A Dirichlet Problem in Weighted Spaces and a Uniqueness Theorem for Harmonic Functions

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Let B be the class of harmonic functions u in the upper half-space $R^3_+ = \{(x,y,z),\ z>0\}$, satisfying the condition $|u(z)| < M,\ z > z_0 > 0$, where M is a constant, depending on z_0, $L^1(\rho)$ be the space of measurable on R^2 functions integrable with respect to the weight $\rho(x,y) = (1+|x|+|y|)^\alpha,\ \alpha \geq 0$. We consider the following Dirichlet type problem in R^3_+:
Problem D. Let $f \in L^1(\rho)$. Find a harmonic function $u \in B$ such that the following boundary condition is satisfied

$$\lim_{z\to 0} \|u(x,y,z) - f(x,y)\|_{L^p(\rho)} = 0,$$

where $\|.\|_{L^p(\rho)}$ is the norm in the space $L^1(\rho)$:
The problem D in the half-plane at $p=1,\ \alpha \leq 0$ in more general conditions, imposed on the weight function is investigated in [1] and [2]. It is shown that it has a solution for any function $f \in L^p(\rho)$ and the solution is found in explicit form.

Theorem 1. *Let $\alpha \geq 1$ and $f \in L^1(\rho)$. The problem D is solvable if and only if the function f satisfies the conditions*

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)u^m v^n\,du dv = 0,\ \ 0 \leq m+n \leq [\alpha]-1.$$

Theorem 2. *Let u be a bounded harmonic function in R^3_+ and*

$$\sup_{(x,y,z)\in R^3_+} |u(x,y,z)|e^{|x|+|y|} < \infty.$$

Then $u \equiv 0$.

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Asymptotic Expansion of Solution of Cauchy Problem for Barenblatt–Zheltov–Kochina Equation at Large Time

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At studying filtration of fluid in fissure layer by Barenblatt–Zheltov–Kochina there was received an equation unsolved to time derivative

$$(\eta\Delta_{n+m} - 1)\, D_t u\,(x,t) + \aleph\Delta_n u\,(x,t) = 0, \tag{1}$$

where $n = 3$, $m = 0$, Δ_{n+m} is Laplacian with respect to $x = (x_1, x_2, \ldots, x_{n+m})$ in R_{n+m} Euclidean space, η, $\aleph$ are constants with certain physical interpretations. The different boundary problems were studied by the indicated authors.

In this paper the asymptotic expansion of solution of Cauchy problem as $t \to +\infty$ for arbitrary nonnegative integer n, m, with condition

$$u\,(x,t)\Big|_{t=0} = \varphi\,(x) \tag{2}$$

is obtained. The following theorem holds:

Theorem *Let* $\varphi\,(x) \in W_{1,1}^{2\mu}\,(R_{n+m})$*.Then as* $t \to +\infty$ *for solution of Cauchy problem* (1), (2) *the following asymptotic expansion holds*

$$u\,(x,t) = t^{-\frac{n}{2}} \int\limits_{R_{n+m}} C_0\,(x-\xi)\,(1-\Delta_{n+m})^{\mu}\,\varphi\,(\xi)\,d\xi + t^{-\frac{n}{2}-1} C_1\,(x)\,,$$

where $C_0\,(x)$,$C_1\,(x)$ *are continuous functions and*

$$|C_0\,(x)| \le C, \quad |C_1\,(x)| \le C\,|x|\,,$$

$$2\mu = \begin{cases} n+m+2, & \textit{if}\; n+m \textit{ is odd}, \\ n+m+3, & \textit{if}\; n+m \textit{ is even}, \end{cases}$$

$W^{2\mu}_{1,1}\left(R_{n+m}\right)$ *is Sobolev space with weight* $(1+|x|)$.

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On Representation of a Solution of General Boundary-Value Problem for Two Dimensional Laplace Equation

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In the finite simply connected domain $\Omega\subset R^2$, $\partial\Omega\subset C^1$ we consider the boundary value problem:

The task of RV: Find in Ω a regular solution $u(x)$ of equation

$$Lu-\lambda u\equiv -\Delta u-\lambda u=f(x), \tag{1}$$

satisfying the boundary condition

$$(Q_0u)(x)+(Q_1u)(x)=0. \tag{2}$$

Here,Q_0 and Q_1 are given linear operators defined on functions

$$u_0(x)=u\mid_{\partial\Omega},\quad u_1(x)=\frac{\partial}{\partial n_x}u\mid_{\partial\Omega}, \tag{3}$$

where $\frac{\partial}{\partial n_x}$ stands for the normal derivative to the boundary. Note that all regular boundary-value problems for (1) can be reduced to the boundary-value problem of the form (2) [3]. Problem (1)–(2) is called regular Volterra if for any complex parameter λ equation (1) under the boundary condition (2) for any function $f\in L_2(\Omega)$ has a unique solution $u(x)$ in $W^2_2(\Omega)$ satisfying the following estimate

$$\parallel u\parallel_{L_2(\Omega)}\leq C(\lambda)\parallel f\parallel_{L_2(\Omega)}. \tag{4}$$

The main result of this paper is as follows.

Theorem. *Let problem* (1)–(2) *be regular Volterra. Then there exist functions* $q_0(x,y)\in L_2(\partial\Omega\times\partial\Omega)$ *and* $q_1(x,y)\in L_2(\partial\Omega\times\partial\Omega)$ *such that the boundary* condition (2) becomes

$$\int_{\partial\Omega}\overline{q_0}(x,y)u_0(\lambda,y)+\overline{q_1}(x,y)u_1(\lambda,y)\,dy=0. \tag{5}$$

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On Some Properties of the Root Functions and Eigenvalues

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In this work we study properties of systems of root functions generated by differential operators. Let L be a differential operator with the following ordinary differential expression in the $L_2(0,1)$ space

$$l(y) \equiv y^{(n)}(x) + p_{n-2}(x)y^{(n-2)}(x) + p_{n-3}(x)y^{(n-3)}(x) + \dots + p_0(x)y(x) = f(x) \qquad (1)$$

and with inner boundary conditions

$$y^{(\nu-1)}(0) - \delta_{k\nu}\int_0^1 (l(y(x)))\overline{\sigma(x)}dx = 0, \quad \nu = \overline{1,n}, \qquad (2)$$

where $f(x), \sigma(x) \in L_2(0,1)$, $\delta_{k\nu}$ is the Kronecker delta and $\overline{\sigma(x)}$ denotes complex conjugate of the $\sigma(x)$.

The following theorems are valid.

Theorem 1. *If the limits* $\lim\limits_{\varepsilon\to 1-0}\frac{1}{\varepsilon}\int_\varepsilon^1 \sigma(x)dx = \alpha$, *for* $k \neq n$ $\lim\limits_{\varepsilon\to +0}\frac{1}{\varepsilon}\int_0^\varepsilon \sigma(x)dx = \beta$, *and for* $k = n$ $\lim\limits_{\varepsilon\to +0}\frac{1}{\varepsilon}\int_0^\varepsilon (1-\sigma(x))dx = \gamma$ *do not vanish, then the system of root functions of the operator* L *is complete in* $L_2(0,1)$.

Theorem 2. *Let* $\{\lambda_k\}_{k=1}^\infty$ *eigenvalues of the problem* (1)–(2) *be numbered in the increases order with multiplicities, then for every* $l \in Z_+$ *the following equality is true* $\sum_{k=1}^\infty (\frac{1}{\lambda_k})^{l+1} = \sum_{j+i=l}^\infty < (L)^{-j}\varphi, (K^*)^{-i}\sigma > .$

In particular, for $l=0$ we obtain $\sum_{k=1}^{\infty}\frac{1}{\lambda_k} =< \varphi,\sigma >$, where K^* is the conjugate Cauchy operator, $< \cdot,\cdot >$ is a scalar product and $\varphi(x)$ is the solution of the equation $l(\varphi(x))=0$ with $y^{(\nu-1)}(0)=\delta_{k\nu}, \nu=\overline{1,n}$.

From [2] it follows that for all $\sigma(x)\in L_2(0,1)$ conditions (2) describe correctly solvable problems corresponding to the expression (1).

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The Cauchy Characteristic Problem for One Class of the Second Order Semilinear Hyperbolic Systems in the Light Cone of the Future

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Consider a nonlinear hyperbolic system of the form

$$\frac{\partial^2 u_i}{\partial t^2}-\sum_{i=1}^{n}\frac{\partial^2 u_i}{\partial x_i^2}+f_i(u_1,\dots,u_N)=F_i(x,t), \quad i=1,\dots,N, \tag{1}$$

where $f=(f_1,\dots,f_N)$, $F=(F_1,\dots,F_N)$ are given and $u=(u_1,\dots,u_N)$ is an unknown real vector-functions, $n\geq 2$, $N\geq 2$.

For the system of equations (1) consider the Cauchy characteristic problem on finding a solution $u(x,t)$ in the frustrum of light cone of the future $D_T: |x|<t<T, T=const>0$, by the boundary condition

$$u\big|_{S_T}=g, \tag{2}$$

where $S_T : t = |x|$, $t \leq T$, is the conic surface, characteristic to the system (1), and $g = g(g_1, \dots, g_N)$ is a given vector-function on S_T. For $T = \infty$ we assume that $D_\infty : t > |x|$ and $S_\infty = \partial D_\infty : t = |x|$.

Let $f \in C(R^N)$ and

$$|f(u)| \leq M_1 + M_2|u|^\alpha, \quad \alpha = const \geq 0, \quad u \in R^N \quad (M_i = const \geq 0, \quad i = 1, 2). \tag{3}$$

For $F \in L_{2,loc}(D_\infty)$, $g \in W^1_{2,loc}(S_\infty)$ and $F|_{D_T} \in L_2(D_T)$, $g|_{S_T} \in W^1_2(S_T)$ $\forall t > 0$, and under fulfillment of the condition (3), where $0 \leq \alpha < \frac{n+1}{n-1}$, notion of local and global solvability of the problem (1), (2) in the class W^1_2 are introduced. We prove the local solvability of the problem (1), (2) for $1 \leq \alpha < \frac{n+1}{n-1}$ and the global solvability of this problem in the case when $0 \leq \alpha < 1$. For $1 \leq \alpha < \frac{n+1}{n-1}$ the global solvability of the problem is proved in the case when $f = \nabla G$, where $\nabla = \big(\frac{\partial}{\partial u_1}, \dots, \frac{\partial}{\partial u_N}\big)$, $G = G(u) \in C^1(R^N)$ is a given scalar function satisfying the following condition: $G(u) \geq -M_3|u|^2 - M_4$, $M_i = const \geq 0$, $i = 3, 4$. The class of vector-functions f, for which the problem (1), (2) is not globally solvable in the case when $1 \leq \alpha < \frac{n+1}{n-1}$, is given.

Oscillation Criteria for Higher Order Nonlinear Functional Differential Equations

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Consider the following general type equation

$$u^{(n)}(t) + F(u)(t) = 0,$$

where $n \geq 2$, $F : C(R_+; R) \to L_{loc}(R_+; R)$ is a continuous mapping. Investigation of asymptotic behavior of solutions of the general equation is interesting on its own. In particular, we used to succeed in obtaining such results for this equation which as a rule were new for linear ordinary differential equations as well. It is an interesting fact that investigation of such general equations enabled me to single out new classes ("almost linear" differential equation) of ordinary differential equations which had not been considered earlier. The obtained results make it clear that the considered equations are interesting and constitute a transitory stage between linear and nonlinear equations.

There are considered "almost linear" (essentially nonlinear) differential equation and the sufficient (necessary and sufficient) conditions are established for oscillation of solutions.

Some of the already got results are published in the following papers [1–4].

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Dynamical Equivalence of Impulsive Differential Systems

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With the use of Green's type map, sufficient conditions under which the impulsive differential systems

$$\begin{cases} dx/dt &= A(t)x + f(t,x,y), \\ dy/dt &= B(t)y + g(t,x,y), \\ \Delta x\big|_{t=\tau_i} &= x(\tau_i+0) - x(\tau_i-0) &= C_i x(\tau_i-0) + p_i(x(\tau_i-0), y(\tau_i-0)), \\ \Delta y\big|_{t=\tau_i} &= y(\tau_i+0) - y(\tau_i-0) &= D_i y(\tau_i-0) + q_i(x(\tau_i-0), y(\tau_i-0)), \end{cases}$$

and

$$\begin{cases} dx/dt &= A(t)x, \\ dy/dt &= B(t)y + g(t,u(t,y),y), \\ \Delta x\big|_{t=\tau_i} &= C_i x(\tau_i-0), \\ \Delta y\big|_{t=\tau_i} &= D_i y(\tau_i-0) + q_i(u(\tau_i-0, y(\tau_i-0)), y(\tau_i-0)), \end{cases}$$

in the Banach space $\mathbf{X}\times\mathbf{Y}$ are dynamically equivalent are obtained.

Furthermore, relevant results concerning partial decoupling and simplification of the impulsive differential systems are also given.

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Lower Bounds for the Counting Function of an Integral Operator

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Consider a self-adjoint integral operator K with continuous kernel $\mathscr{K}(x,y)$ acting in the space $L_2(M)$, where M is a compact metric space provided with a finite Borel measure. The operator K is compact and its spectrum consists of positive and negative eigenvalues accumulating to zero. Let $\mathscr{N}(K,t)$ be the number of eigenvalues of K lying below $t \le 0$.

The talk will discuss the seemingly simple question: how can one estimate $\mathscr{N}(K,t)$ from below? It will be shown how this problem arises in the study of the relation between counting functions of the same differential operator with different boundary conditions. The main result is a theorem which gives an explicit estimate for $\mathscr{N}(K,t)$ in terms of the integral kernel $\mathscr{K}$.

Variation Formulas of Solution for a Controlled Functional-Differential Equation, Considering Delay Perturbation and the Discontinuous Initial Condition

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Linear representation of the main part of the increment of a solution of an equation with respect to perturbations is called the variation formula of solution .The variation formula of solution allows one to construct an approximate solution of the perturbed equation in an analytical form on the one hand, and in the theory of optimal control plays the basic role in proving the necessary conditions of optimality, on the other. In this paper the variation formulas of solution are given for the controlled delay functional-differential equation

$$\dot{x}(t) = f(t, x(t), x(t-\tau_0), u_0(t))$$

with the discontinuous initial condition

$$x(t) = \varphi_0(t), \ \ t \in [t_{00} - \tau_0, t_{00}), \quad x(t_{00}) = x_{00}$$

under perturbations of initial moment t_{00}, delay parameter τ_0, initial function $\varphi_0(t)$, initial vector x_{00} and control function $u_0(t)$. Variation formulas for various classes of functional-differential equations without perturbation of delay are given in [1], [2].

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A Partial Differential Equation with a Closed Parabolic Boundary

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The following equation is considered

$$\partial_{\overline{z}}(w + q(z)\overline{w}) = f(z), \quad z \in G, \tag{1}$$

where $w(z)$ is an unknown complex-valued function, $q(z)$ and $f(z)$ are given functions in a bounded simply connected domain G, the boundary of which is denoted by ∂G. It is assumed that $q(z)$, $w(z) \in D_{1,p}(\overline{G})$ ($\overline{G} = G + \partial G$), $p > 2$; $\partial G \in C^1$, and what is especially important

$$|q(z)| = 1 \quad \texttt{as} \quad z \in \partial G.$$

It is stated that the study of equation (1) is reduced to the Riemann–Hilbert boundary value problem

$$Re[i(1+\overline{q(z)})\Phi(z)] = h(z), \quad z \in \partial G,$$

in which $\Phi(z)$ is an analytic function belonging to the class of functions $D_{1,p}(\overline{G})$, $p > 2$, and $h(z)$ is the known function.

Let $æ = Ind_{\partial G}\, q(t)$ be the index of $q(z)$ calculated along the boundary ∂G. Then the following result is valid:

for $æ \geq 0$ *the problem* (1) *has* $æ + 1$- *parametric set of solutions;*

for $æ < 0$ *the problem* (1) *either has no solution or admits only one solution provided that* $f(z)$ *satisfies* $|æ| - 1$ *real relations*. A particular case, when $f(z)$ does not satisfy the solvability relations and the problem (1) does not have a solution is presented in [1].

In the case $q(z) \equiv -1$, $z \in \partial G$, *equation* (1) *has only one-parametric set of solutions*.

It should be noted that the results obtained do not depend on a type of equation (1) in the domain G. Here a defining role belongs to the behavior of $q(z)$ on the boundary ∂G. If $|q(z)| = 1$ on ∂G, then equation (1) becomes parabolic, and this fact together with the value of the index of $q(z)$ has influence on the solvability of equation (1).

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Real and Complex Analysis

On Generalized Analytic Vectors

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Some results connected with the theory of generalized analytic vectors are introduced. The study of special elliptic systems solutions of which are called generalized analytic vectors was initiated by Prof. B. Bojarski. On this basis the boundary value problems for these elliptic systems on the complex plane in some functional classes are investigated.

Compactness Criteria in Weighted Variable Lebesgue Spaces

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Let p be Lebesgue measurable function on R^n such that $1 \leq \underline{p} \leq p(x) \leq \overline{p} \leq \infty$, $\underline{p} = ess \inf_{x \in R^n} p(x)$, $\overline{p} = ess \sup_{x \in R^n} p(x)$, and ω be a weight function on R^n, i.e. ω be non-negative, almost everywhere (a.e.) positive function on R^n.

Definition. *By $L_{p(x),\,\omega}(R^n)$ we denote the set of measurable functions f on R^n such that for some $\lambda_0 > 0$*

$$\int\limits_{R^n} \left(\frac{|f(x)|}{\lambda_0} \right)^{p(x)} \omega(x)\, dx < \infty.$$

Note that the expression

$$\|f\|_{L_{p(x),\,\omega}(R^n)} = \|f\|_{p(\cdot),\,\omega} = \inf \left\{ \lambda > 0 : \int\limits_{R^n} \left(\frac{|f(x)|}{\lambda} \right)^{p(x)} \omega(x)\, dx \leq 1 \right\}$$

defines the norm in the space $L_{p(x),\,\omega}\left(R^n\right)$. The spaces $L_{p(x),\,\omega}\left(R^n\right)$ is a Banach function space with respect to the norm $\|f\|_{p(\cdot),\,\omega}$.

By $\mathscr{P}^{\log}\left(R^n\right)$ we denote the class of variable exponents satisfying the condition:

$$|p(x)-p(y)| \le \frac{C}{-\ln|x-y|},\ 0<|x-y|\le\frac{1}{2} \text{ and } |p(x)-p_\infty| \le \frac{C}{\ln(e+|x|)},\ |x|>2.$$

Let us define the class $A_{p(\cdot)}$ to consist of those weights $w \in L_1^{loc}\left(R^n\right)$ for which

$$\sup_B |B|^{-1}\|w^{1/p(\cdot)}\|_{L_{p(\cdot)}(B)}\|w^{-1/p(\cdot)}\|_{L_{p'(\cdot)}(B)} < \infty,$$

where the supremum is taken over all balls $B \subset R^n$.

Theorem. *Let $p \in \mathscr{P}^{\log}\left(R^n\right)$ and $1 < \underline{p} \le p(x) \le \overline{p} < \infty$. Suppose that $\omega(x)$ is weight function on R^n and $\omega \in A_{p(\cdot)}$. Then a subset $\mathscr{F}$ of $L_{p(x),\,\omega}\left(R^n\right)$ is totally bounded if and only if:*

1) *$\mathscr{F}$ is bounded in $L_{p(x),\,\omega}\left(R^n\right)$, i.e., $\sup\limits_{f\in\mathscr{F}}\|f\|_{p(\cdot),\,\omega} < \infty$;*

2) *for every $\varepsilon>0$, there is some η such that for every $f\in\mathscr{F}$, $\displaystyle\int\limits_{|x|>\eta}|f(x)|^{p(x)}\,\omega(x)\,dx < \varepsilon$;*

3) *$\lim\limits_{h\to0+}\|f_h-f\|_{p(\cdot),\,\omega} = 0$ uniformly for $f\in\mathscr{F}$, where f_h is the Steklov average of the function f.*

A New Trend in Real Analysis Interlacing with Different Branches of Pure and Applied Mathematics

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In this lecture we present a new trend in real analysis interlacing with rather different fields (Hilbert problem 16 in real algebraic geometry, Nevanlinna theory and Gamma-lines theory in complex analysis, integral geometry) and admitting interpretations in many applied topics (hydro-aero dynamics, meteorology, wave processes etc.).

We study the geometry of level sets of real functions: the length, integral curvature of the level sets. Also we study the number of connected components of level sets of real functions which, in particular case of polynomials, was widely studied in the frame of Hilbert problem 16.

The results of this new trend strength and generalize the key result in all above mentioned fields. This development unfolds as follows. The new results: (a) imply the key conclusions in Gamma-lines theory [1] which, in turn, contains so called proximity property, which, in turn, strengths the key results in Nevanlinna theory; (b) imply estimates of the cardinalities of level sets of real functions which, in particular case of polynomials, strength the key result in real algebraic geometry; (c) imply some new formulas in integral geometry which, in turn, generalize the key Crofton's identity in integral geometry.

The geometry of level sets was studied earlier in the frame of Gamma-lines theory dealing with some classes of real functions determined by complex functions. In fact the obtained results constitute a far going generalization of Gamma-lines theory which now is valid for any "reasonably smooth" real function.

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On Riemann Boundary Value Problem in Hardy Classes with Variable Summability Exponent

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In this talk the Riemann boundary value problem with piecewise-continuous coefficient is considered. Under certain conditions on the coefficients, the general solution to the nonhomogeneous problem in the Hardy classes with variable summability exponent is constructed.

Consider the nonhomogeneous problem

$$\left.\begin{array}{l} F^{+}(\tau)-G(\tau)F^{-}(\tau)=f(\tau),\quad \tau\in\partial\omega, \\ F^{+}\in H^{+}_{p(\cdot)},\quad F^{-}\in {}_{m}H^{-}_{p(\cdot)} \end{array}\right\}, \tag{1}$$

where $f\in L_{p(\cdot)}$ and $G(\tau)=e^{2i\alpha(\arg\tau)}$. Obviously, the general solution of problem (1) is of the form $F(z)=F_0(z)+F_1(z)$, where F_0 is a general solution to appropriate homogeneous problem and F_1 is one of the particular solutions of problem (1). As it was already shown, $F_0(z)$ is of the form $F_0(z)=Z(z)P_m(z)$, where $Z(z)$ is a canonical solution of homogeneous problem, $P_m(z)$ is a polynomial of $m_0\leq m$ degree.

We'll presuppose that the function $\alpha(t)$ satisfies the following main assumption:

(a) $\alpha(t)$ is piecewise Hölder on $[-\pi,\pi]$, $\{s_k\}_1^r: \ -\pi = s_0 < s_1 < \cdots < s_r < s_{r+1} = \pi$ are its discontinuity points on $(-\pi,\pi)$. Let $\{h_k\}_1^r: \ h_k = \alpha(s_k+0) - \alpha(s_k-0)$, $k=\overline{1,r}$ be the jumps of the function $\alpha(t)$ at the points s_k and $h_0 = \frac{\alpha(-\pi)-\alpha(\pi)}{\pi}$.

Theorem. *Let $p \in WL_\pi$, $G(e^{it}) = e^{2i\alpha(t)}$, $\alpha(t)$ satisfy the condition* (a) *and inequalities*

$$-\frac{1}{q(s_k)} < \frac{h_k}{\pi} < \frac{1}{p(s_k)}, \quad k=\overline{0,r},$$

be fulfilled. Then, the general solution of Riemann nonhomogeneous problem (1) *in the class $H^+_{p(\cdot)} \times {}_m H^-_{p(\cdot)}$ $(m \ge 0)$ is of the form $F(z) = Z(z)P_{m_0}(z) + F_1(z)$, where*

$$F_1(z) = \frac{Z(z)}{2\pi i} \int\limits_{\partial\omega} \frac{f(\tau)}{Z^+(\tau)} \frac{d\tau}{\tau - z},$$

$Z(z)$ is a canonical solution of appropriate homogeneous problem.

P-Systems of Exponents, Cosines and Sines

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Accept the following standard notation: B-space is a Banach space; $L(X;Y)$ is a B-space of bounded operators acting from X to Y; $L(X) \equiv L(X;X)$; Z are integers; N are natural numbers; $Z_+ = \{0\} \bigcup N$.

We will consider some abstract analogies of bases formed from the exponents, cosines and sines. Let X be some B-space and $P \in L(X)$: $P^2 = I$. Assume $X^\pm \equiv Ker(I \mp P)$. Take $\forall x \in X$. We have $x = \frac{I+P}{2}x + \frac{I-P}{2}x$. It is clear that $x^\pm = \frac{I \pm P}{2} x \in X^\pm$. Take $y \in X^+ \bigcap X^-$. Consequently, $(I \pm P)y = 0 \Rightarrow y = \mp Py \Rightarrow Py = 0 \Rightarrow y = 0$. It is obvious that $X^\pm$ are closed. Thus, X is a direct sum $X = X^+ \dot{+} X^-$.

Let the system $\{x_n\}_{n\in Z}$ form a basis in X. Assume $x_n^\pm = \frac{I\pm P}{2} x_n$, $\forall n \in Z$.

Definition. *The system $\{x_n\}_{n \in Z}$ is said to be P-system if $Px_n = x_{-n}, \forall n \in Z$.*

The following theorems are valid:

Theorem 1. *Let $P^2 = I$ and $X^\pm \equiv Ker(I \mp P)$, $\{x_n\}_{n\in Z} \subset X$ be some P-system. Assume $x_n^\pm = \frac{I\pm P}{2} x_n, \forall n \in Z$. The system $\{x_n\}_{n\in Z}$ forms a basis in X iff $\{x_n^+\}_{n\in Z_+}$, $\{x_n^-\}_{n \in N}$ form bases in X^+ and X^-, respectively.*

Theorem 2. *Let all the conditions of Theorem 1 be fulfilled. Then the system $\{x_n\}_{n\neq 0}$ forms a basis in X iff $\{x_n^+\}_{n\in N}$, $\{x_n^-\}_{n\in N}$ form bases in X^+ and X^-, respectively.*

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Differential Subordinations for Multivalent Functions Associated with an Extended Fractional Differintegral Operator

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Let $\mathscr{A}_k(p)$ denote the class of functions of the form $f(z) = z^p + \sum\limits_{n=k}^{\infty} a_{p+n} z^{p+n}$ $(p, k \in \mathbb{N} = \{1, 2, 3, \ldots\})$ which are analytic in the open unit disk $\mathbb{U} = \{z : \ z \in \mathbb{C} \text{ and } |z| < 1\}$. We write $\mathscr{A}_1(p) = \mathscr{A}(p)$. Very recently, Patel and Mishra [1] defined the extended fractional differintegral operator $\Omega_z^{(\lambda,p)} : \mathscr{A}(p) \to \mathscr{A}(p)$ given for a function $f(z) \in \mathscr{A}(p)$ and for a real number λ $(-\infty < \lambda < p+1)$ by

$$\Omega_z^{(\lambda,p)} f(z) = \frac{\Gamma(p+1-\lambda)}{\Gamma(p+1)}\, z^{\lambda} D_z^{\lambda} f(z),$$

where $D_z^{\lambda} f$ is the *fractional integral* of f of order $-\lambda$, when $-\infty < \lambda < 0$, and the *fractional derivative* of f of order λ if $0 \le \lambda < p+1$, respectively.

Using the above operator we study differential subordinations in the class $\mathscr{A}_k(p)$.

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Bellman Function and the Bilinear Embedding: Recent Examples and Applications

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Bellman functions were introduced in harmonic analysis by Nazarov, Treil and Volberg in 1994. Given the method's tendency to yield sharp (and, when applicable, dimension-free) results, there has been a whole series of subsequent applications of their approach, including sharp estimates of Riesz transforms due to Nazarov, Petermichl, Treil, Volberg and the present author. In the latter works a central rôle is played by the so-called *bilinear embedding theorem*.

Recently, a few additional results have been obtained along the same lines and in various contexts: Schrödinger operators with nonnegative potentials, real divergence-form operators with nonnegative potentials, Laplace–Beltrami operators on complete Riemannian manifolds with Bakry–Emery Ricci curvature bounded from below, and Heisenberg groups. All of them feature a simplified version of a single Bellman function, invented in 1995 by Nazarov and Treil. We discuss the properties of this functions and how they can be applied in the above-mentioned settings in a uniform way.

The talk will be based in part on joint works with Alexander Volberg and Andrea Carbonaro.

A Method to Obtain L^p-Boundedness from Weak Type Conditions

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In this talk we explain a method to obtain L^p-boundedness of some integral operators, for example the Calderón-Zygmund operators. Our results are not new, actually our argument was formed twenty years ago. However, by this method we can get L^p-boundedness results from very slight weak conditions without the assumption of L^2-boundedness.

We hope our method might be able to be generalized by another mathematicians, and it would have some applications for new problems.

Some Aspects of the Stable Partial Indices

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We consider the impact of stability properties of the partial indices on the solvability conditions of two classical problems: 1) the *Riemann boundary value problem*, consisting in finding a piecewise holomorphic matrix function with some boundary condition and 2) the *Riemann monodromy problem*, consisting in the construction of a Fuchs type system of differential equations with given monodromy. Both problems discussed in an unfinished work of Riemann. There, Riemann considered the first problem as an *auxiliary method* for solving the second problem.

Starting from the second half of the 19th century, differential equations with meromorphic coefficients were a subject of intensive research. In particular, the form of the fundamental matrix in a neighborhood of a regular singular point was established Poincaré, influence of the configuration of singular points of a differential equation on monodromy matrices was investigated Shlesinger, canonical form of systems of differential equations, in general case, in the neighborhood of a regular singular point was found Birkhoff. In the first decade of the 20th century investigation of regular systems was to an extent stimulated by the *Hilbert 21st problem* (monodromy problem).

J. Plemelj successfully applied the Fredholm theory of integral equations developed by Hilbert and gave a solution of the monodromy problem for regular systems of differential equations. The works by Plemelj and Birkhoff contain certain defects, which were caused by the noncommutation properties of matrix functions. As it is well known today, not only proofs of theorems contained errors, but the theorems themselves were not true. Later for the boundary value problem Muskhelishvili and Vekua introduced the concept of *partial indices*. These invariants of the boundary problem turned out to be the reason of the above imprecisions. In particular, the case when partial indices are *stable* is "generic" (Bojarski) and in this case the solution of the Riemann monodromy problem and the Birkhoff standard form theorem are both correct. Muskhelishvili has remarked several times about imprecisions in reasonings of Plemelj and give absolutely new proof of the boundary value problem. He moreover noticed that without introduction of partial indices, solution of the problem cannot be considered complete. As later was shown by Bolibruch, the *complete decision of the monodromy problem strictly depends on the partial indices*.

We give detailed analysis of the relationship between the partial indices of the Riemann boundary value and monodromy problems.

On the Problem of Convergence of Multiple Functional Series to ∞

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In 1915 N. Luzin [1] posed the following problem: does there exist the trigonometric series, convergent to ∞ on a set of positive measure? In 1988 S. Konyagin [2] gave the negative answer to this problem. In 1992 (see [3]) the negative answer was given to the similar problem for multiple trigonometric series. Suppose that we have the system of functions

$$\left\{\varphi_{n_j}^{(j)}\left(x_j\right)\right\}_{n_j=0}^{\infty}, \quad x_j \in [0,1], \quad j=1,\ldots,d. \tag{1}$$

Consider the d-multiple series

$$\sum_{n=0}^{\infty} a_n \varphi_n(x), \tag{2}$$

where $x=(x_1,\ldots,x_d)$, $n=(n_1,\ldots,n_d)$

$$\varphi_n\left(x\right)=\prod_{j=1}^{d} \varphi_{n_j}^{(j)}\left(x_j\right).$$

The sufficient conditions are found which should be satisfied by systems (1) so that any series (2) is not convergent in the sense of Pringsheim to ∞ on the set of positive d-dimensional measure.

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On the Convergence of Fourier Series in the Sense of a Metric of $L\varphi(L)$

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Let f be a 2π-periodic measurable function on $[0,2\pi]$. Denote by $S_n(f)$, $n=0,1,\dots,$ the partial sum of the Fourier series of f, and by $\widetilde{S}_n(f)$, $n=0,1,\dots,$ the partial sum of the conjugate series.

Let Φ be a set of continuously differentiable nonnegative nondecreasing functions φ, defined on $[0,\infty)$ and satisfying the conditions, $\varphi\neq 0$, $\varphi(2t)=O(\varphi(t))$ and $t\varphi'(t)=O(\varphi(t))$, $t\to\infty$.

The following statements are true.

Theorem 1. *Let $\varphi\in\Phi$. Then for arbitrary function $f\in L\varphi(L)$, there exists a function $F\in L\varphi(L)$, $|F|=|f|$ such that $\sup\limits_n|S_n(F)|\in L\varphi(L)$ and $\sup\limits_n|\widetilde{S}_n(F)|\in L\varphi(L)$.*

An analogous theorem for conjugate functions was proved by O. Tsereteli (see [1, Theorem 1-a]).

Theorem 2. *Let $\varphi\in\Phi$. Then for arbitrary function $f\in L\varphi(L)$ and $\varepsilon>0$, there exists a function $F\in L\varphi(L)$, $|F|=|f|$ such that $\mu\{x: F(x)\neq f(x)\}<\varepsilon$ and both the Fourier series of a function F and its conjugate series converge in the sense of a metric of $L\varphi(L)$.*

Thus the convergence of a Fourier series and its conjugate series in the sense of a metric of $L\varphi(L)$ does not impose any restrictions on the module of a function f. Theorem 2 is a generalization of the theorem from [2] (under the assumption that $\phi(t)=1$, $t\geq 0$).

This modification of the notion of a "corrected" function on a set of small measure in order to obtain a function with the wanted property belongs to O. Tsereteli [3]: on a set of small measure a function can be changed not arbitrarily as is done in the classical theorems of N. Luzin and D. Menshov, but only by multiplying its value by -1 or by permutation (after such a change of a function, its metric class remains unchanged).

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Elliptic Systems in the Plane

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We establish a sufficiently general result characterizing the behavior of elliptic (regular and singular) systems (see, for example, [1], [2]) in the neighborhood of their singularities.

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On Problem of Gonchar Related Singular Operators

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Let

$$Sf = \frac{1}{\pi i}\int\limits_{\gamma_0} \frac{f(z)}{z-t}\,dz = \widetilde{f}(t) \ \ (\gamma_0 \ \text{ is a unit circle})$$

be the singular operator with Cauchy kernel. The Plemelj–Privalov theorem states: if $f \in H^\alpha(\gamma_0)$ $(0 < \alpha < 1)$, then $\widetilde{f} \in H^\alpha(\gamma_0)$ (H^α is the Hölder class of order α). A. A. Gonchar posed the following problem: what will happen with the singular operator Sf, if f at the point z_0 has a smoothness of order $\alpha + \beta$ $(\beta > 0)$? A number of mathematicians answered to this question, but the final answer was given in our joint work with V. V. Salayev.

Further we have also considered the case $\beta < 0$ and new local class $D_\alpha^\beta(z_0)$ having a high smoothness in some neighborhood of z_0. The mentioned class has many applications, in particular, in approximation problems on open curves in a complex plane. Moreover, notice that the results obtained by us for the most classes of curves in a complex plane, and these results are unimprovable.

Heisenberg Uniqueness Pairs and Partial Differential Equations

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A Heisenberg uniqueness pair (HUP) is a pair (Γ, Λ), where Γ is a curve in the plane and Λ is a set in the plane, with the following property: any bounded Borel measure μ in the plane supported on Γ, which is absolutely continuous with respect to the arc length, and whose Fourier transform $\widehat{\mu}$ vanishes on Λ, must automatically be the zero measure. For instance, when Γ is the hyperbola $x_1x_2 = 1$, and Λ is the lattice-cross

$$(\alpha\mathbb{Z} \times \{0\}) \cup (0 \times \mathbb{Z}),$$

where α and β are positive reals, then (Γ, Λ) is an HUP if and only if $\alpha\beta \leq 1$; in this situation, the Fourier transform $\widehat{\mu}$ of the measure solves the one-dimensional Klein-Gordon equation. Phrased differently, this particular problem is equivalent to the fact that

$$e^{i\pi nt}, \quad e^{i\pi n/t}, \quad n \in Z,$$

span a weak-star dense subspace in $L^1(\mathbb{R})$ if and only if $\alpha\beta \leq 1$. In order to prove this kind of theorems, some elements of linear fractional theory and ergodic theory are needed, such as the Birkhoff Ergodic Theorem. In this connection a number of questions related with the Klein–Gordon equation and other classical partial differential equations arise.

Basicity of Systems of Discontinuous Phase Exponents in Generalized Lebesgue Spaces

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The basicity of a system of exponents with piecewise-continuous phase that is a set of eigen–functions of discontinuous differential operators in Lebesgue spaces with variable summability exponent is established. Consider the following system of exponents

$$\left\{e^{i\lambda_n(t)}\right\}_{n\in Z}, \tag{1}$$

where $\lambda_n(t)$ has the representation

$$\lambda_n(t) = nt - \alpha(t)\, sign\, n + \beta_n(t), \quad n \to \infty. \tag{2}$$

Assume that the following conditions are fulfilled:

(a) $\alpha(t)$ is piecewise Hölder on $[-\pi, \pi]$, $\{s_k\}_1^r : -\pi = s_0 < s_1 < \cdots < s_r < s_{r+1} = \pi$ are its discontinuity points on $(-\pi, \pi)$. Let $\{h_k\}_1^r : \ h_k = \alpha(s_k + 0) - \alpha(s_k - 0)\ ,\ k = \overline{1, r}$ be the jumps of the function $\alpha(t)$ at the points s_k and $h_0 = \frac{\alpha(-\pi)-\alpha(\pi)}{\pi}$;

(b) $\{\frac{h_k}{\pi} - \frac{1}{p(s_k)} : k = \overline{0, r}\} \bigcap Z = \emptyset$;

(c) the functions β_n satisfy the relation

$$\beta_n(t) = O\left(\frac{1}{n^{\gamma_k}}\right), \quad t \in (t_k, t_{k+1}), \quad k = \overline{0, r}; \quad \{\gamma_k\}_1^r \subset (0, +\infty).$$

Theorem. Let the asymptotic formula (2) hold. Here the function $\alpha(t)$ satisfies the condition (a) and for the functions $\beta_n(t)$ condition (c) is valid. Assume that the following relations hold:

$$-\frac{1}{q(\pi)} < \omega_\pi < \frac{1}{p(\pi)}, \quad \gamma > \frac{1}{\tilde{p}},$$

where $\gamma = \min\limits_k \gamma_k$, $\tilde{p} = \min\{p^- \, ; \, 2\}$ and the quantity ω_π is determined from relations (b). Then the following properties in $L_{p(\cdot)}$ for system (1) are equivalent:

1. is complete;
2. is minimal;
3. is ω linearly independent;
4. forms a basis isomorphic to $\{e^{int}\}_{n \in Z}$;
5. $\lambda_i \neq \lambda_j$ for $i \neq j$.

The Hazeman's Problem in Classes of Functions Representable by the Cauchy Type Integral with Density from $L^{p(t)}$

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Let Γ be an oriented rectifiable simple closed curve bounding the domains D^+ and D^-, and let $\alpha = \alpha(t)$ be a preserving orientation homeomorphism of Γ onto itself, $\alpha'(t) \neq 0$, $\alpha' \in H$. We study the Riemann boundary value problem

$$\Phi^+(\alpha(t)) = a(t)\Phi^-(t) + b(t), \quad t \in \Gamma,$$

with shift, (the Hazeman's problem), when the coefficient $a(t)$ is a nonvanishing piecewise-continuous coefficient for which $\inf|a(t)| > 0$.

A natural setting of Hazeman's problem in the class of Cauchy type integrals with density from a weighted Lebesgue spaces with a variable exponent $p(t)$ is proposed. The influence of discontinuous of the function $a(t)$ on a character of solvability of the problem is revealed, solvability conditions are established, and the solutions are constructed.

Besides that the problem is studied in a more general setting compared with the previos study, we cover the case of non-smooth boundary satisfying the chord condition.

A Note on the Partial Sums of Walsh–Fourier Series

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The present talk is devoted to necessary and sufficient conditions on positive numbers n_k, $k = 0, 1, \dots,$ for which

$$\|S_{n_k}(f(x))\|_{H_p} \le c_p\|f(x)\|_{H_p}, \tag{1}$$

$$\|S_{n_k}(f(x))\|_{L_p} \le c_p\|f(x)\|_{H_p}, \tag{2}$$

$$\|S_{n_k}(f(x))\|_{weak-L_p} \le c_p\|f(x)\|_{H_p}, \tag{3}$$

where $0 < p < 1$, S_n denote n-th partial sums of Walsh–Fourier series.

We prove that

$$\|S_n(f(x))\|_{H_p} \le c_p(n\mu(suppD_n))^{1/p-1}\|f(x)\|_{H_p}.$$

Moreover, there exists a matringale $f \in H_p$ such that

$$\frac{\|S_n(f(x))\|_{weak-L_p}}{\|f(x)\|_{H_p}} \ge c_p(n\mu(suppD_n))^{1/p-1}.$$

It follows that inequalities (1)-(3) hold if and only if

$$\sup_k n_k\mu(suppD_{n_k}))^{1/p-1} < c < \infty.$$

On Representation of Multivariate Continuous Functions

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Let N be the set of all natural numbers and

$$N = N_1 \cup N_2 \cup \cdots \cup N_d,$$

where d is a natural number and $d \geq 2$, also $N_i \cap N_j = \emptyset$ if $i \neq j$.

Let $x_j = [0, 1]$ for any $j \in \{1, \ldots, d\}$ and $\{\varphi_n(t)\}_{n=1}^{\infty}$ be a sequence of functions defined on $[0, 1]$. For each natural number n we introduce the following notion: $\varphi_n(\dot{x}) = \varphi_n(x_j)$, where $n \in N_j$. We denote by $S_m(\dot{x})$ partial sums of a series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2\pi n\dot{x} + b_n \sin 2\pi n\dot{x}. \tag{1}$$

We state that the following theorem holds.

Theorem. *For each natural number $d \geq 2$ there exist series (1) and sequences $\{m_k^{(i)}\}_{k=1}^{\infty}$, where $i = 1, 2, \ldots, 2d + 1$, such that the following is true: for any continuous function f : $[0, 1]^d \to R$ there exists a continuous function $g : [0, 1] \to R$ such that*

$$f(x_1, \ldots x_d) = \lim_{k\to\infty} \sum_{i=1}^{2d+1} g\left(S_{m_k^{(i)}}(\dot{x})\right)$$

for any $(x_1, \ldots x_d) \in (0, 1)^d$.

Boundedness of Linear Operators in Grand Morrey Spaces with Infinite Measures

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Introduce the weighted grand Morrey space with infinite measure:

$$M_{\alpha}^{p),\theta,\lambda}(\Omega, \rho) = \left\{ f : \sup_{0<\varepsilon<p-1} \sup_{x\in\Omega, r>0} \left(\frac{\varepsilon^{\theta}}{|B(x,r)|^{\lambda}} \int_{B(x,r)} |f(y)|^{p-\varepsilon} \rho(y) \langle y \rangle^{-\alpha\varepsilon} dy \right)^{\frac{1}{p-\varepsilon}} < \infty \right\},$$

where $1 < p < \infty$, $\theta > 0$, $0 \le \lambda < 1$ and a number α satisfies the condition

$$\sup_{x\in\Omega, r>0} |B(x,r)|^{-\lambda} \int_{B(x,r)} \rho(y)\langle y\rangle^{-\alpha p} dy < \infty, \quad \langle x\rangle := \sqrt{1+|x|^2}.$$

A class $W_p = W_p(\Omega), p \in (1,\infty)$, of weights on Ω will be called *allowable*, if it possesses the following properties:

$$w \in W_p \implies w \in W_{p-\varepsilon} \quad \text{for some } \varepsilon > 0;$$

$$w \in W_p \implies w^{1+\varepsilon} \in W_p \quad \text{for some } \varepsilon > 0;$$

$$w_1, w_2 \in W_p \implies w_1^t w_2^{1-t} \in W_p \quad \text{for every } \ t \in [0,1].$$

The theorem on the boundedness of linear operators in these spaces states: Let $\Omega \subseteq R^n$ be an open set, $1 < p < \infty$ and let $W_p(\Omega)$ be an allowable class of weights. If

$$T : L^p(\Omega, \varrho) \hookrightarrow M^{p,\lambda}(\Omega, \varrho)$$

for every $\varrho \in W_p$,

$$T : L^{p-\varepsilon_0}(\Omega, w) \hookrightarrow M^{p-\varepsilon_0,\lambda}(\Omega, w)$$

for every $w \in W_{p-\varepsilon_0}$ and for some $\varepsilon_0 \in (0, p-1)$, then

$$T : L^{p),\theta}(\Omega, \varrho) \hookrightarrow M^{p),\theta,\lambda}(\Omega, \varrho),$$

where $\varrho \in W_p$.

Stochastic Analysis

Denjoy–Lusin Sequences and Unconditional Convergence in a Banach Space

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Let (α_n) be an infinite sequence of real numbers and

$$f_n(t) = \cos(nt + \alpha_n)\,, \quad n = 1, 2, \ldots, \quad t \in R\,.$$

The well-known Denjoy–Lusin theorem asserts that if for an infinite sequence (x_n) of scalars the series $\sum\limits_n |x_n f_n|$ converges on a set of strictly positive Lebesgue measure, then $\sum\limits_{n=1}^{\infty} |x_n| < \infty$. Motivating from this theorem we call an infinite sequence (ξ_n) of random variables given on a probability space (Ω, P) a *Denjoy–Lusin sequence* if for an infinite sequence (x_n) of scalars the P-a.e. convergence of the series $\sum\limits_n |x_n \xi_n|$ implies $\sum\limits_{n=1}^{\infty} |x_n| < \infty$. We plan to discuss the following statements.

Theorem 1. *If an infinite sequence (ξ_n) of random variables given on a probability space (Ω, P) for some $r > 0$ satisfies the condition*

$$\liminf_{n \in N} P[|\xi_n| > r] > 0\,,$$

then (ξ_n) is a Denjoy–Lusin sequence.

Theorem 2. *Let (ξ_n) be a Denjoy–Lusin sequence given on a probability space (Ω, P) and X be a (real or complex) Banach space. Then for an infinite sequence (x_n) of elements of X the P-a.e.* ***unconditional*** *convergence of the series $\sum\limits_n x_n \xi_n$ implies the* ***unconditional*** *convergence of the series $\sum\limits_n x_n$.*

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Linear Stochastic Differential Equations in a Banach Space

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A linear stochastic differential equation in a separable Banach space is considered. The questions of the existence and uniqueness of the (ordinary and generalized) solution are discussed. For various conditions on coefficients the forms of the solutions are given.

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On Limit Distribution of a Quadratic Deviation for Nonparametric Estimate of the Bernoulli Regression

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Let Y be a random variable with Bernoulli distribution and $p = p(x) = P\{Y = 1/x\}$. Suppose that x_i, $i = \overline{1,n}$ are points of partition of $[0,1]$ which are chosen from relation $\int\limits_0^{x_i} h(x)\,dx = \frac{i-1}{n-1}$, where $h(x)$ is the known positive density of the distribution on the interval $[0,1]$. Furthermore Y_{ij}, $i = \overline{1,n}$, $j = \overline{1,N}$, are independent Bernoulli random variables with $P\{Y_{ij} = 1/x_i\} = p(x)$, $P\{Y_{ij} = 0/x_i\} = 1 - p(x)$, $i = \overline{1,n}$, $j = \overline{1,N}$. Then $Y_i = \sum\limits_{j=1}^{N} Y_{ij}$ has the binomial distribution law $B(N, p_i = p(x_i))$. We observe values $Y_1, \dots, Y_n$. Our aim is to construct estimate for $p(x)$ analogously of the Nadaraya–Watson regression function estimation:

$$\widehat{p}_{nN}(x)=\frac{\varphi_{nN}(x)}{f_n(x)}\,,$$

$$\varphi_{nN}(x)=\frac{1}{nb_n}\sum_{i=1}^{n}K\left(\frac{x-x_i}{b_n}\right)\frac{1}{h(x_i)}Y_i,\quad f_n(x)=\frac{1}{nb_n}\sum_{i=1}^{n}K\left(\frac{x-x_i}{b_n}\right)\frac{1}{h(x_i)}\,,$$

where $b_n>0$, $b_n\to 0$ as $n\to\infty$. Suppose that following properties are fulfilled:

i) $K(x)$ is the distribution density, $\sup\limits_x K(x)<\infty$, $K(-x)=K(x)$, $\operatorname{supp}(K)\subset[-\tau,\tau]$ and has bounded derivative; ii) $p\in C^2[0,1]$; iii) $h(x)\geq\mu>0$ and $h\in C^1[0,1]$.

We study the properties of such estimations.

Denote $U_{nN}=nNb_n\int\limits_{\Omega_n}(\varphi_{nN}(x)-E\varphi_{nN}(x))^2\,dx$, $\Omega_n=[\tau b_n,1-\tau b_n]$, $\Delta_n=EU_{nN}$,

$$\sigma_n^2=4(nb_n)^{-2}\sum_{k=2}^{n}p(x_k)(1-p(x_k))\sum_{i=1}^{k-1}p(x_i)(1-p(x_i))Q_{ik}^2,$$

$$Q_{ij}=\frac{1}{h(x_i)h(x_j)}\int\limits_{\Omega_n}K\left(\frac{x-x_i}{b_n}\right)K\left(\frac{x-x_j}{b_n}\right)\,dx.$$

Theorem. *If $nb_n^2\to\infty$ as $n\to\infty$, then $\Delta_n=\Delta(p)+O(b_n)$, where*

$$\Delta(p)=\int\limits_0^1 p(x)(1-p(x))h^{-1}(x)\,dx\int\limits_{-\tau}^{\tau}K^2(u)\,du,\quad b_n^{-1}\sigma_n^2=\sigma^2(p)+O(b_n)+O(n^{-1}b_n^{-2}),$$

$$\sigma^2(p)=2\int\limits_0^1 p^2(x)(1-p(x))^2h^{-2}(x)\,dx\int\limits_{-2\tau}^{2\tau}K_0^2(u)\,du,\quad K_0=K*K$$

and $\sigma^{-1}b_n^{-1/2}(U_{nN}-\Delta)\Rightarrow N(0,1)$, where the symbol $\Rightarrow$ denotes the weak convergence.

Cramer–Rao Inequalities in a Functional Space

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Let $\{\Omega, \mathscr{F}, \mathscr{P}\}$ be a complete probability space. $X = X(\omega;\theta)$ is a random element, with values in real separable Hilbert space E, $\theta \in \Theta \subset \Xi$, where Ξ is a separable real Banach space. When $X_1, X_2, ..., X_n$ are i.i.d. realizations of X, then we have the sequence of the statistical structures: $\{\aleph, \Re, (P(\theta;\cdot)), \theta \in \Theta\}$, where $\aleph = E^n$ $(n = 1, 2, \dots, \infty)$ is the Hilbert space generated by the sequence $X_1, X_2, \dots, X_n$. Here $\Re$ is the σ-algebra generated by the observable sets and $\{P(\theta;\cdot), \theta \in \Theta\}$ is the system of probability measures generated by the vector $Y = (X_1, X_2, \dots, X_n) : P(\theta; A) = P(Y^{-1}(A))$, $A \in \Re$.

Let $z(x) : \aleph \to \aleph$ be the vector field with $\sup_{x\in\Re} \|z'(x)\| < \infty$. Suppose that $P(\theta;\cdot)$ is the measure differentiated along the $z(x)$ and possesses a logarithmic derivative along the vector field $z(x) : \beta_\theta(x; z)$. For any fixed $A \subset \Re$ and vector $\vartheta \in \Xi$ consider derivative $d_\theta P(\theta; A)\vartheta$ of function $\tau(\theta) = P(\theta; A)$ at the point θ along the ϑ. It is easy to see that $d_\theta P(\theta; A)\vartheta \ll P(\theta;\cdot)$ and by Radon–Nykodym theorem there exists the density $l_\theta(x;\vartheta)$.

Assumption 1. $X(\theta) = X(\theta;\omega) : \Theta \times \Omega \to \aleph$ and the derivative $X^{'}(\theta)$ by θ along $\vartheta \in \Xi_0$ exists, where $\Xi_0 \subset \Xi$. It is a linear mapping $\Xi \to \aleph$ for any $\theta \in \Theta$. Thus for any $\vartheta \in \Xi_0$ and $\theta \in \Theta$ we have $\| X^{'}(\theta)\vartheta \|_\aleph \in L_2(\Omega, P)$.

Assumption 2. $E\{X^{'}(\theta)\vartheta \mid X(\theta) = x\}$ is strongly continuous function of x for each $\vartheta \in \Xi_0, \theta \in \Theta$.

Assumption 3. The family of measures $P(\theta;\cdot), \theta \in \Theta$ possess the logarithmic derivative by parameter along the constant directions from tight in Ξ subspace $\Xi_0 \subset \Xi$ and $l_\theta(x;\vartheta) \in L_2(\aleph, P(\theta)), \vartheta \in \Xi_0, \theta \in \Theta$.

Assumption 4. The family of measures $P(\theta;\cdot), \theta \in \Theta$ possess the logarithmic derivative along a constant directions from tight subspace $\Xi_0 \subset \Xi$ and $\beta_\theta(x; h) \in L_2(\aleph, P(\theta))$, $h \in \Xi_0$, $\theta \in \Theta$.

Assumption 5. For the statistics $T = T(x) : \aleph \to R$ following equality is fair

$$d_\vartheta \int_\aleph T(x)P(\theta; dx) = \int_\aleph T(x)d_\vartheta P(\theta; dx).$$

Theorem (Inequality Cramer–Rao). *Under the conditions* 1–5 *we have*
1. $l_\theta(x;\vartheta) = -\beta_\theta(x; K_{\theta,\vartheta})$, *where* $K_{\theta,\vartheta} = E\{\frac{d}{d\theta}X(\theta)\vartheta \mid X(\theta) = x\}$;
2. $VarT(x) \geq \frac{(g^{'}_\vartheta(\theta))^2}{E_\theta l^2_\theta}(X;\vartheta)$.

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Multiplicators for the Laws of Large Numbers

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It will be discussed the following statement and its generalizations: let (ξ_n) be an infinite sequence of independent identically distributed real random variables such that

$$\sup_{n\in N} nP[|\xi_1| > n] < \infty\,.$$

Then for each infinite null-sequence (t_n) of real numbers the sequence $(t_n\xi_n)$ satisfies the weak law of large numbers.

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Mathematical Physics

Wave in an Elastic Tube of Variable Cross Section with a Two-Phase Viscous Fluid Flows

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The problem of wave dynamics in deformable tubes with flowing fluids is very important. This is due to the prevalence of such situations in fluid transmission systems in technology and living organisms. Here, first of all, we have in mind the elements of the hydraulic systems of an aircraft, various pipelines, and hemodynamics. Presented report is devoted to the formulation of the mathematical basis of the problem of hydroviscosity of a two-phase viscous bubble fluid flow [1], enclosed in an elastic semi-infinite cylindrical tube, when the effect of narrowing is taken into account. One-dimensional linear equation is used, assuming that the tube is rigidly attached to the environment and the shift in the axial direction is absent. The tube radius is taken constant at infinity (conical narrowing of the tube). To describe the pressure, density, fluid flow and displacement, a pulsating pressure boundary condition is set at the end of the tube. After separation of variables, we get a singular boundary value problem of the Sturm–Liouville type:

$$y'' + \lambda^2 y = \lambda^2 q(x) y,$$

$$y(0) = y_0,$$

which in turn reduces to the integral equation of the Volterra type. It is solved by successive approximations. Under the condition of absolute integrability of the potential

$$\int |q(x)|\, dx < +\infty$$

we prove their convergence to the exact solution of the problem. Thus, the constructed series gives a convenient representation of the desired functions.

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Resonances in Effective Field Theory

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Inclusion of resonances in effective field theory is a non-trivial problem. The complex-mass scheme (CMS) is a consistent framework of dealing with unstable states in quantum field theory. This approach has proven successful in various applications. The usage of the CMS leads to complex-valued renormalized parameters of the Lagrangian. Within this scheme one does not change the bare Lagrangian, therefore unitarity is not violated in the complete theory. On the other hand, it is not obvious that the approximate expressions of physical quantities in perturbation theory also satisfy the unitarity conditions. We derive cutting rules for loop diagrams and demonstrate at one-loop level perturbative unitarity of the scattering amplitude within CMS. As an application of the CMS in chiral effective field theory we consider the vector form factor of the pion in the timelike region and the electromagnetic form factors of the Roper resonance.

Dynamical Origin of the Jaffe–Witten Mass Gap in QCD

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Quantum Chromodynamics (QCD) is widely accepted as a realistic quantum field gauge theory of the strong interactions not only at the fundamental (microscopic) quark-gluon level but at the hadronic (macroscopic) level as well. It is a $SU(3)$ color gauge invariant theory. One of the important challenges of QCD is that its Lagrangian does not contain a mass scale parameter which could have a physical meaning even after the corresponding renormalization program is performed. Thus the only place where it may appear explicitly is the gluon Schwinger–Dyson equation of motion for the full gluon propagator. Precisely this problem has been addressed and solved in our papers.

The general scale parameter, having the dimensions of mass squared, is dynamically generated in the QCD gluon sector. It is introduced through the difference between the regularized full gluon self-energy and its value at some finite point. It violates transversality of the full gluon self-energy. The Slavnov–Taylor identity for the full gluon propagator, when it is given by the corresponding equation of motion, is also violated by it. So in order to maintain both transversality and the identity it should be disregarded from the very beginning, i.e., put formally zero everywhere. However, we have shown how to preserve the Slavnov–Taylor identity at non-zero mass squared parameter. This allows one to establish the structure of the full gluon propagator when it is explicitly present. Its contribution does not survive in the perturbation theory regime, when the gluon momentum goes to infinity. At the same time, its contribution dominates the structure of the full gluon propagator when the gluon momentum goes to zero. We have also proposed a method how to restore transversality of the relevant gluon propagator in a gauge invariant way, while keeping the mass squared parameter "alive". In this case, the two independent general types of formal solutions for the full gluon propagator as a function of the regularized mass gap have been found. The nonlinear iteration solution at which the gluons remain massless is explicitly present. It is nothing else but the Laurent expansion in inverse powers of the gluon momentum squared and multiplied by the corresponding powers of the regularized mass gap. The existence of the solution with an effective gluon mass is also demonstrated.

Four-Fermion Interaction Model with Dimensional Regularization

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Because of the strong QCD coupling we can not avoid to consider non-perturbative effects in the confinement hadronic phase. A four-fermion interaction model is introduced to particle physics by Y. Nambu and G. Jona-Lasinio to study low energy phenomena of the strong interactions. In the model the chiral symmetry is spontaneously broken by non-vanishing expectation value for a composite operator constructed by the quark and anti-quark fields. The model is useful to evaluate many low-energy phenomena in the hadronic phase of QCD.

Since the four-fermion interaction is non-renormalizable in four space-time dimensions, we have to regularize divergent fermion loop integrals to obtain a finite result. The dimensional regularization is one of analytic regularizations. We analytically continue the space-time dimensions to a non-integer value and calculate the fermion loop integrals in the space-time dimensions less than four [1]. It preserves most of the symmetries of the theory, including the general covariance.

In this talk we apply the dimensional regularization to the four-fermion interaction model. The space-time dimension can be fixed to reproduce some physical observables. We discuss physical results in the model using the dimensional regularization.

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The Equations for Multi-Particle Relativistic Systems in the Framework of Multi-Local Source Formalism

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In this report we will discuss the description of multi-particle bound-states (baryons and mesons) in the field-theoretical multilocal fermion formalism. We present some our results on multi-quark equations of the four-fermion interacting field theoretical model with $SU(2) \times SU(2$ chiral symmetry in the mean-field expansion (MFE). To formulate the MFE, we use an iterative solution of the Schwinger–Dayson equations with the fermion bilocal source [1]. We apply the method to the Nambu–Jona-Lasinio (NJL) model [2] – one of the most successful effective field theory models of quantum chromodynamics. We have analyzed the structure of equations for the Green functions of the NJL model up to next-to-next-to-next-to-leading order (NNNLO) [3]. To calculate the high-order corrections to the mean-field approximation, we propose to use the Legendre transformation with respect to the bilocal source, which allows effectively to take into account the symmetry constraints related with the chiral Ward identity [4].

We have considered, also, the generalization of MFE, which includes other types of multiquark sources except of bilocal source. Such generalization can be useful for the description of baryons in the framework of MFE. We discuss the problem of calculating the multiquark functions in the framework of MFE with the three-fermion sources [5].

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Singular Value Decomposition for Data Unfolding in High Energy Physics

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An overview of the data unfolding problem in high energy particle physics will be given, followed by a presentation of the algorithm based on the Singular Value Decomposition of the detector response matrix. The ways of regularizing the inversion procedure will be considered, and various examples will be given, together with recommendations and advice for practical uses of the method.

Euclidean Relativistic Quantum Mechanics

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We discuss a formulation of Poincaré invariant quantum mechanics where the dynamical input is a Euclidean generating functional or a collection of Euclidean Green functions. The purpose of this approach is to construct well-defined mathematical models whose structure is motivated by quantum field theories. The generating functional must satisfy reality, cluster

properties, Euclidean invariance, and reflection positivity. With the exception of reflection positivity, all of these properties are easily satisfied. We discuss the following topics: (1.)The construction of the physical Hilbert space, (2.)The construction of model generating functionals, (3.)The construction of the Poincaré Lie algebra, (4.)The construction of single particle states, (5.)The construction of finite Poincaré transformations of single particle states, (6.)The construction of scattering amplitudes, (7.)The construction of finite Poincaré transformations of scattering states, (8.)Test calculations of scattering amplitudes using matrix elements of $e^{-\beta H}$ in normalizable states.

Model generating functionals can be represented as Schur products of exponentials of model connected multi-point Green functions. The construction of the Lie algebra is based on the observation that the real Euclidean group is a subgroup of the complex Lorentz group. Elements of the Lie Algebra of the physical Hilbert space are generators of unitary one-parameter groups, contractive semigroups, or local symmetric groups. One-particle states are point-spectrum eigenvectors of the mass Casimir operator for the Poincaré group. Scattering states are constructed using time-dependent methods. A version of Cook's theorem relates the existence of wave operators to properties of connected Green functions. The Kato–Birman invariance principle is used to replace the Hamiltonian by $-e^{-\beta H}$, which has a compact spectrum. This is used to reduce the calculation of transition matrix elements to the accurate evaluation of matrix elements of polynomials in $e^{-\beta H}$ in normalizable states. All of the calculations can be done using Euclidean test functions and Euclidean generating functionals without any analytic continuation. Tests on an exactly solvable non-relativistic model suggest that accurate calculations are possible over a wide range of energies.

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Light Bending by a Coulomb Field and the Aichelburg–Sexl Ultraboost

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We use light deflection by a Coulomb field, due to non-linear quantum electrodynamics effects, as an opportunity for a discussion of the electrodynamical analog of the Aichelburg–Sexl ultraboost.

Relativistic Quantum Mechanics on the Light-Front Consistent with Quantum Field Theory

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A relativistic quantum mechanics (RQM) on the light-front (LF) is derived from quantum field theory (QFT) through the use of a unitary transformation. Cluster separability of the derived RQM is proved by solving the so-called decoupling equation for the disconnected parts of the unitary transformation relating the Poincar'e generators of RQM to the ones of QFT. We show that the two theories, RQM and QFT, are equivalent.

On the History of the Strong Interaction

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I intend to review the history of the conceptual developments which led from the discovery of the neutron to our present understanding of strong interaction physics.

On Hydrodynamics and Thermodynamics of the Barochronic Flow of Ideal Fluid (Gas)

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In the articles [1–3] a minimal full set of polynomial invariants for a linear operator and its matrix was constructed in Euclidean space. The developed approach has turned out to be effective while considering some physical problems, in particular, in studying of the so called barochronic flow of an ideal fluid (gas). Such a type of flow is possible in the infinite homogeneous space only. Therefore it is interesting to study such a motion not only from the mathematical point of view, but also due to the cosmological aspect since it gives a possibility to distinguish between the effects caused by gravitation (space curvature) and the kinematic ones in a flat infinite homogeneous space.

In this article we have shown that a smooth vector field, which describes the hydrodynamic speed of barochronic flow of an ideal fluid (gas), is either potential or a solenoidal type. Both types of barochronic flow are studied in detail in the paper. We have found the non-trivial (non-static) solution of the Euler hydrodynamic equations for the hydrodynamic speed of an ideal fluid (gas) potential flow. It is interesting that this solution formally coincides with the well-known Hubble law in a non-relativistic form.

The mathematical approach developed is also applied to the study of the thermodynamics of fluid (gas) barochronic flow.

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The New Properties of Dirac Delta and Euler Beta Functions

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As it is known, up to now the mathematical formalism of quantum mechanical problem of two charged particles of continuous spectrum has not a perfect form. We have tried to correct partially that deficiency, which has required some investigations in the theory of special and generalized functions. In this paper we present some main results of our study.

Theorem 1: *For any real x the following integral representation is valid*:

$$\delta\left(x\right)=\frac{1}{2\pi}\int_{0}^{1}dt\,t^{ix-1}(1-t)^{-ix-1}.$$

Many interesting novelties follow from this Theorem. Namely, one can obtain:

$$\delta\left(x\right)=\frac{1}{\pi}\int_{-1}^{+1}dt\left(t+1\right)^{ix-1}\left(1-t\right)^{-ix-1}=\frac{1}{2\pi}\int_{0}^{\infty}dy\,y^{ix-1}.$$

Besides, we have shown that the Euler beta function defined by the formula

$$B\left(\alpha,\,\beta\right)=\int_{0}^{1}t^{\alpha-1}(1-t)^{\beta-1}dt,\quad \mathrm{Re}\left(\alpha\right)>0,\ \mathrm{Re}\left(\beta\right)>0,$$

has the following property:

Theorem 2: *When* $\mathrm{Re}\,\alpha=\mathrm{Re}\,\beta=0,\ \ \mathrm{Im}\,\alpha=-\,\mathrm{Im}\,\beta=x$, *the following equalities are correct*:

$$B(i\,x,\,-i\,x)=\lim_{\varepsilon\to 0+}B(\varepsilon+ix,\,\varepsilon-ix)=2\,\pi\,\delta\left(x\right).$$

Theorem 2 represents a definition of the B function of imaginary arguments. It is consistent with the Euler formula for the B function and gives a possibility to extend the domain of its definition in a sense of generalized functions. Namely, Theorem 2 gives us the following representation:

$$\delta\left(x\right)=\frac{1}{\pi}\lim_{\varepsilon\to 0}\frac{\Gamma\left(\varepsilon+ix\right)\Gamma\left(\varepsilon-ix\right)}{\Gamma\left(\varepsilon\right)}.$$

These theorems have interesting applications in quantum mechanics. Detailed analysis of the above mentioned theorems will be published elsewhere.

Unitary Approximate Solution of the Many Particle Integral Equation

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An iterative method is very often used to solve Faddeev type simultaneous integral equations, since to find the exact solution is not always possible. In general case, many-particle iteration series is not convergent and even if this series is convergent, its cut-off breaks the unitarity of the quantum mechanical system under consideration. Using Faddeev's and Heitler's formalism for three-particle scattering, J. Mebonia [1] has grouped the iteration series in such a way, that on each stage of cut-of (which has clear physical meaning) the amplitude preserves the property of being unitary. In the present paper the result of [1] is generalized as a theorem for N-particle simultaneous equations.

As it is known, the collision T-matrix of N-particle quantum mechanical system may be represented as:

$$T = \sum_{\alpha} T^{\alpha}. \tag{1}$$

T^{α} represents an auxiliary operator in N-particle space and satisfies the Faddev type simultaneous integral equations:

$$T^{\alpha} = T_{\alpha} + T_{\alpha} G_0 \sum_{\beta \neq \alpha} T^{\beta}, \tag{2}$$

where T_{α} is a pair-interaction operator in N-particle space, G_0 is N-particle Green's function for free particles, and G_1 is its non-Hermitian part. We have proved the following

Theorem: *when the relation*:

$$\| G_1 T_{\alpha} G_1 T_{\beta} \| \ll 1, \tag{3}$$

is implemented, the T-matrix represented with equation (1) *will take the following form*:

$$T \approx \sum_{\alpha} T_{\alpha} (1 + \sum_{\beta \neq \alpha} G_1 T_{\beta}) , \quad \alpha \, , \, \beta \equiv (m \, n) \, , \quad m > n = 1 \, , \; N. \tag{4}$$

(3) represents a single collision test. Its implementation quality depends on the form of interaction and colliding particles kinetic energy.

Expression (4) satisfies N-particle unitarity condition with the accuracy (3), it does not contain many-particle divergences. Formula (4) can be applied for the investigation of both nuclear and atomic single collisions. The proof of the above mentioned theorem and the details connected with it will be published in the next publications.

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The Exact Solution of the Dynamic Mixed Three-Dimensional Elasticity Problem for a Wedge-Shaped Thick Plate

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The finite elastic wedge-shaped thick plate, one side of which is situated on the absolutely rigid smooth basis, and another one is under the influence of the dynamic oscillating loading through the absolutely rigid cover linked to it, is considered in the report. On the lateral sides of the plate the conditions of the smooth contact are fulfilled, on a plate end face the stress is given. The solution is based on the special linear transformation of the Lame's equations with the subsequent application of the vector integral transformation method. The offered approach leads to the one-dimensional vector boundary problem in the transformation's space. The elements of the unknown vector are the transformations of the displacements. The exact solution of the obtained problem is constructed with the help of the matrix differential equations theory. The application of the inverse integral transformations finished the construction of the problem's exact solution. The convergence of the obtained series is investigated and the weak convergent parts are extracted and summarized. The analysis of the self-resonant frequencies was done, and the frequencies of the boundary resonance were revealed.

The similar problem for a case when on the end face of the wedge-shaped thick plate the conditions of the smooth contact are given was considered also and the exact solution for this case was constructed. With the aim to estimate the possibility of exfoliation of the lower bases, the comparison of the stress on the lower bases with the stress on it for the similar problem with the dead weight of the plate was done.

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Some New Aspects and New Quantum Kinetic Equations of Degenerate Fermi Gas

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Answers to some salient questions, which arise in quantum plasmas, are given. Starting from the Schrodinger equation for a single particle it is demonstrated how the Wigner–Moyal equation can be derived. It is shown that the Wigner–Moyal type of equation also exists in the classical field theory. As an example, from the Maxwell equations the Wigner–Moyal type of equation is obtained for a dense photon gas, which is classical, concluding that the Wigner–Moyal type of equation can be derived for any system, classical of quantum. A new types of quantum kinetic equations are presented. these novel kinetic equations allows to obtain a set of quantum hydrodynamic equations, which is impossible to derive by the Wigner–Moyal equation. The propagation of small perturbation and instabilities of these perturbations are then discussed, presenting new modes of quantum plasma waves. In the case of low frequency oscillations with ions, a new Bogolyubov type of spectrum is found. furthermore, the Korteweg–de Vries (KdV) equations is derived and the contribution of the Madelung term in the formation of the KdV solitions is discussed.

Low Energy Behavior of 1D Superconductor at Magnetic Field Induced Quantum Phase Transition

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Exact solution of one-dimensional (1D) Hubbard model is combined with effective field theory description beyond the linearized hydrodynamic approximation to determine correlation functions and low temperature thermodynamic properties of one-dimensional superconductor in external magnetic field equal to the spin gap. In 1D system of the lattice electrons away of half filling, when discrete particle-hole symmetry is broken, spin and charge degrees of freedom are

coupled by the curvature of the bare particle dispersion at the Fermi points. In many cases this coupling is irrelevant, and thus negligible in infrared limit, however in certain cases to describe even static low energy properties of the 1D electron system away of half filling one can not use linearized hydrodynamic approximation (which is known as bosonization procedure and results in spin-charge separation), but has to account for the non-linear dispersion of the bare electron spectrum. In particular, due to the finite curvature, at the magnetic field induced commensurate-incommensurate quantum phase transition point the magnetic susceptibility stays finite and specific heat instead of the square root shows a linear behavior with temperature albeit with logarithmic corrections.

Numerical Analysis and Mathematical Modelling

On One Nonclassical Two-Dimensional Model of Thermoelastic Shells

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In the present paper we study initial-boundary value problems for thermoelastic shells within the framework of Lord–Shulman model. We investigate the three-dimensional initial-boundary value problem with mixed boundary conditions corresponding to Lord–Shulman non-classical model depending on one relaxation time for thermoelastic bodies in suitable Sobolev spaces applying variational approach. On basis of Lord–Shulman three-dimensional model we construct a hierarchy of two-dimensional models of thermoelastic shells in curvilinear coordinates. We investigate the constructed dynamical two-dimensional models in suitable function spaces and study relationship between the original three-dimensional problem and the hierarchy of reduced two-dimensional ones. We prove the convergence in the corresponding spaces of the sequence of vector-functions of three space variables, restored from the solutions of the two-dimensional initial-boundary value problems, to the solution of the original three-dimensional problem and estimate the rate of convergence.

Continuous Non-Linear Mathematical Model of Information Warfare

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The presented work provides a non-linear, continuous mathematical model of Information warfare between two antagonistic states (or two states of the Union, or the two powerful economic institutions - a consortium), taking into account the fact that there is a third, the peace-keeping side. The model includes as an equal, as well as significantly different associations with the power of controversy. We believe that the information warfare against each other, providing the first and second side, and the third party to consider the international organizations. At the moment of time $t \in [0, \infty)$ the quantity of the information spread by each of the sides we will accordingly designate by $N_1(t)$, $N_2(t)$, $N_3(t)$. Built continuous non-linear mathematical model of information warfare have the form:

$$\frac{dN_1(t)}{dt} = \alpha N_1(t) - \beta N_1(t) N_3(t),$$

$$\frac{dN_2(t)}{dt} = \alpha N_2(t) - \beta N_2(t) N_3(t),$$

$$\frac{dN_3(t)}{dt} = \gamma(N_1(t) + N_2(t)),$$

with initial conditions $N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $N_3(0) = N_{30}$ where,$\alpha, \beta, \gamma > 0$ -are constant factors.

Exact analytical solutions of the offered non-linear mathematical model of information warfare found. It is established that two antagonistic sides, under influence the third parties, in not dependences on character of influence of the peace-making party (preventive or not preventive), reduce information attacks, asymptotic to zero, i.e. practically stop information warfare.

On Modelling of Liquid-Phase Formation in Gas Pipelines for Non-Stationary Flow

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At present, pipelines have become the most popular means of natural gas transportation, but while natural gas is transported by pipelines, pressure and temperature changes cause partial condensation of the gas and formation of a liquid phase-hydrates. There are several methods for avoiding of gas hydrates origination and intensification. Among them are: injection of thermodynamic inhibitors, use kinetic hydrate inhibitors, maintain pipeline normal operating conditions from outside by insulation, heating and controlling of pressure. However, to use the above techniques it is necessary to know the hydrate origination zone. From existing methods of definition hydrates origin zone the mathematical modelling with hydrodynamic method is more acceptable as it is very cheap and reliable. Nowadays there are existing mathematical models which can define possible section of hydrates formation in the main pipeline but for stationary flow of gas. But as we know in reality gas flow in pipelines is non-stationary. In the present paper the problem of prediction of possible points of hydrates origin in the main pipelines taking into consideration gas non-stationary flow and heat exchange with medium is studied. For solving the problem of possible generation point of condensate in the pipeline under the conditions of non-stationary flow in main gas pipe-line the system of partial differential equations is investigated. For learning the affectivity of the method a quite general test is created. Numerical calculations have shown efficiency of the suggested method.

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Stability of Approximation Methods for Muskhelishvili and Sherman–Lauricella Equations

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Let Γ be a simple closed piecewise smooth contour in the complex plane, $\tau_j, j = 1, \dots, k$ be the corner points of Γ, and let $f \in L_2(\Gamma)$. In the present talk, we discuss the stability of various spline approximation methods for integral equations

$$\omega(t) + \frac{1}{2\pi i}\int\limits_{\Gamma} \omega(\tau)\, d\ln\left(\frac{\tau - t}{\overline{\tau} - \overline{t}}\right) - \frac{1}{2\pi i}\int\limits_{\Gamma} \overline{\omega(\tau)}\, d\left(\frac{\tau - t}{\overline{\tau} - \overline{t}}\right) = f(t), \quad t \in \Gamma. \tag{1}$$

$$-k\overline{\varphi(t)} - \frac{k}{2\pi i}\int\limits_{\Gamma} \overline{\varphi(\tau)} d\log\frac{\overline{\tau} - \overline{t}}{\tau - t} - \frac{1}{2\pi i}\int\limits_{\Gamma} \varphi(\tau) d\frac{\overline{\tau} - \overline{t}}{\tau - t} = f_0(t), \quad t \in \Gamma, \tag{2}$$

where the bar denotes the complex conjugation, k is a constant specified and

$$f_0(t) = -\frac{1}{2}\overline{f(t)} + \frac{1}{2\pi i}\int\limits_{\Gamma} \frac{\overline{f(\tau)}d\tau}{\tau - t}.$$

In particular, Galerkin, collocation, and Nyström methods are studied. It is shown, that the method under consideration is stable if and only if certain operators B_{τ_j}, $j = 1, \dots, k$ associated with the corner points τ_j and with the corresponding approximation method, are invertible. It is worth noting that the operators B_{τ_j} have a complicated structure. However, they belong to an algebra of Toeplitz operators, hence some of their properties can be investigated.

Numerical examples show the excellent convergence of the methods mentioned.

The talk is based on joint works with J. Helsing and B. Silbermann.

Numerical Modelling of Some Ecometeorologically Actual Processes

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Some ecometeorologically actual processes have been numerically simulated with the help us of the developed two-dimensional nonstationary model of a mesometeorological boundary layer of atmosphere (MBLA):

a) Fog and low stratus clouds. Temperature-inversion layers formation are closely connected with them (these layers are formed at a fog- and cloud generation because of release of the latent warmth at condensation of water vapour). At humidity and inversion processes there is an accumulation of polluting substances of atmosphere.

b) Distribution of an aerosol from a point instant source against the thermohydrodynamics of MBLA.

It is received space-time distribution of the fields defining MBLA.

Influence of various physical parameters is studied on the investigated processes.

At comparison of theoretical results with known meteorological data it has appeared, that the numerical model, developed by us, qualitatively well describes investigated processes.

On Some Methods of Decomposition for Approximate Solution of Problems of Mathematical Physics

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The development of the methods of parallel realization of problems on the computers is a complex problem, which requires joint the efforts of specialists of mathematical modeling, computational technology (computational mathematics), computer technology and programming. In the direction of creation of parallel computing technology especially should note the importance and role of methods of decomposition: the decomposition of the basic area of definition of the initial problem and decomposition of the basic operator of the initial problem. The methods of decomposition offer a natural way to parallelize the process of implementation on computers: they are based on the reduction of the solution of a initial problem to the solution of some more ”simple” subproblems, which opens up great possibilities for designing algorithms for parallel implementation of these problems and creation on this basis program product for computers. In the talk some methods of constructing computational algorithms are considered called the additive averaged schemes (AAS) of parallel calculation. For parabolic and hyperbolic problems the construction of such AAS is based on the decomposition of the operator of the initial problem; simultaneously with this are proposed and investigated AAS for solving the specific problems of thermoelasticity, shell theory, problems of distribution of pollution in water substances and etc. In the talk we also consider some versions of the method of summary approximation (MSA) for the multidimensional equations of parabolic and hyperbolic types; the questions of convergence of the solutions of models MSA to generalized and classical solution of initial problem are investigated.

Investigation of the Influence of Preliminary Buckling of Cylindrical Shell Reinforced by a Cross System of Ribs and Filled with Medium on Critical Stresses of General Stability Loss

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Cylindrical shells reinforced by a regular cross system of ribs are important structural elements of rockets, submarines, motor vehicles and etc. Investigation of the behavior of such structures with regard to external factors is of special importance in the field of contact problems in the theory of ribbed shells. In papers [1–3], the stability under longitudinal compression was considered without taking into account the preliminary buckling of ribbed cylindrical shells filled with medium. The Influence of preliminary buckling of a shell reinforced by a regular cross system of ribs and filled with medium on critical load parameters of general stability loss is investigated. The investigation is based on the problem statement using the mixed energy method and nonlinear equations of combined deformations.

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Investigation and Numerical Resolution of Initial-Boundary Value Problem with Mixed Boundary Conditions for a Nonlinear Integro-Differential System

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In the domain $[0,1]\times[0,T]$ the following initial-boundary value problem is considered:

$$\frac{\partial U}{\partial t}=\left\{1+\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dx\,d\tau\right\}\frac{\partial^2 U}{\partial x^2},$$

$$\frac{\partial V}{\partial t}=\left\{1+\int_0^t\int_0^1\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2\right]dx\,d\tau\right\}\frac{\partial^2 V}{\partial x^2}, \tag{1}$$

$$U(0,t)=V(0,t)=0,\quad \left.\frac{\partial U(x,t)}{\partial x}\right|_{x=1}=\left.\frac{\partial V(x,t)}{\partial x}\right|_{x=1}=0,\quad t\in[0,T],$$

$$U(x,0)=U_0(x),\quad V(x,0)=V_0(x),\quad x\in[0,1],$$

where $U_0=U_0(x)$ and $V_0=V_0(x)$ are given functions.

The existence and uniqueness as well as asymptotic behavior of the solution of problem (1) are studied. The corresponding difference scheme and realization algorithms are constructed and investigated. Several numerical experiments are given too. The numerical results are compared to the theoretical ones.

Cubature of Volume Potentials over High-Dimensional Half-Spaces

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We present high order cubature formulas for the computation of harmonic potentials over the n-dimensional half-space within the framework of approximate approximations (see [1]). The cubature of the potentials is reduced to the quadrature of one-dimensional integrals over the half-line. We derive a tensor product representation of the integral operator which admits efficient cubature procedures for densities with separated approximation in very high space dimensions. Numerical experiments for the half-space up to dimension $n = 10^6$ confirm the predicted approximation errors. This is a joint work with V. Maz'ya (University of Liverpool and Linköping University) and G. Schmidt (Weierstrass Institute for Applied Analysis and Stochastics, Berlin).

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On the Approximate Solution of One Class of Integro-Differential Equation

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Let us consider the following integro-differential equation:

$$\begin{cases} u''(x) - \varepsilon \int\limits_0^1 K(x,t)u(t)\,dt = -f(x), \\ u(0) = u(1) = 0, \end{cases} \tag{1}$$

$$f(x) \in L_2[0,1], \;\; |K(x,t)| \leq M < \infty, \;\; K(x,t) \in C([0,1] \times |[0,1]|).$$

Problem (1) has a unique solution in the class $W^2_{2,0}(0;1)$. It can be found by the successive approximation method. Approximate solving of some linear nonhomogeneous boundary problems are presented [1], [2]. In our case the basic operator is $Lu = u''$ and the operator, describing the perturbation degree, is $Mu = \int\limits_0^1 K(x,t)u(t)\,dt$.

For approximately solving the boundary value problem the complex of programs and many numerical experiments are carried out.

The author express hearty thanks to Prof. T. Vashakmadze and Prof. A. Papukashvili for his active help in problem statement and solving.

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Optimum Search in a Discrete Space for Sequential Probability Ratio Test Planning

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In an earlier work devoted to sequential testing of binomial processes, or of processes reducible to ones ([1]–[3]) it was established that the error probabilities of the first and the second kind are incapable of analytical formulation but have a discrete nature. Hence, choosing the optimal testing parameters requires a search for extremes over discrete sets.

In our current work we propose a procedure, based on application of the theory of continued fractions, which makes it possible to choose appropriate inter-step intervals in the search for the optimal test boundaries.

These intervals have to differ for the accept and reject boundaries, and to depend on the truncation level or on the distribution of the sample number up to the decision.

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An Approximate Solution of a One-Dimensional Nonlinear Timoshenko System

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The one-dimensional variant of a static system of equations describing the stress-deformed state of a flexible plate

$$\begin{aligned} N' &= 0, \\ Q' + (Nw')' + f &= 0, \\ M' - Q &= 0, \end{aligned} \tag{1}$$

is considered, where $N = \frac{Eh}{1-\nu^2}(u' + \frac{1}{2}(w')^2)$, $Q = k_0^2 \frac{Eh}{2(1+\nu)}(\psi + w')$, $M = D\psi'$. $u = u(x)$, $w = w(x)$, $\psi = \psi(x)$ are the sought functions, $f = f(x)$ is the known function, $x \in [0,1]$, ν, E, h, D and k_0 are given positive values, $D = Eh^3/12(1-\nu^2)$, $0 < \nu < 0,5$.

The boundary conditions are as follows

$$u(0) = u(1) = 1, \quad w(0) = w(1) = 0, \quad \psi(0) = \psi(1) = 0. \tag{2}$$

From problem (1), (2) we obtain the problem for the function w [1]

$$\Phi(w) = 0, \tag{3}$$

$$w(0) = w(1) = 0, \tag{4}$$

where Φ is a nonlinear integro-differential operator. The other sought functions u and ψ of problem (1), (2) are expressed through the functions w.

The existence of a generalized solution of problem (3), (4) is proved. An approximate solution is constructed by using the Bubnov–Galerkin method and the question of its convergence is studied.

The one-dimensional problem for the dynamic Timoshenko system is also considered in [2].

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On the Approximate Solution of 3D Mixed Boundary Value Problem of the Elasticity Theory by Means of the Finite-Difference Method and Some of Its Applications to Nanostructures

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This work deals with 3D mixed boundary value problem of the elasticity theory for the orthotropic beam. It is supposed that static forces act at the beam and the displacement vector is given at the upper and lower boundaries, also at the lateral surface the components of external stress tensor are given. We consider the equations of equilibrium in terms of displacements.

For this system two different methods of the approximations are proposed:

1. Variational-Difference method.This method is based on I. Vekua Method by means of which 3D problem is reduced to 2D and then the approximate solutions are obtained by means of the finite-difference schemes [1].

2. In the second case 3D problems are approximately solved directly by means of the finite-difference schemes [2]. This method could be applied to the large beams in the problems of mining mechanics and to nanoststructures with the size over 10nm as well [3], [4].

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An Approximate Solution of One System of the Singular Integral Equations by Collocation Method

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Let us consider the system of singular integral equations containing an immovable singularity (see [1])

$$\begin{aligned} &\int_0^1 \left[\frac{1}{t-x} - \frac{1}{t+x}\right] \rho_1(t)\,dt + b_1 \int_{-1}^0 \frac{\rho_2(t)dt}{t-x} = 2\pi f_1(x), \quad x \in (0;1), \\ &b_2 \int_0^1 \frac{\rho_1(t)}{t-x}\,dt + \int_{-1}^0 \left[\frac{1}{t-x} - \frac{a_2}{t+x}\right] \rho_2(t)\,dt = 2\pi f_2(x), \quad x \in (-1;0), \end{aligned} \tag{1}$$

where $\rho_k(x)$, $f_k(x)$ are unknown and given real functions, respectively, a_k, b_k are constants, $f_k(x) \in H$, $\rho_k(x) \in H^*$, $k = 1, 2$.

The system (1) of the singular integral equations is solved by a collocation method, in particular, a method of discrete singular (see [2]) in both cases uniform, and non-uniformly located knots.

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An Approximate Algorithm for a Timoshenko Beam Equation

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Let us consider the initial boundary value problem

$$
\begin{gathered}
u_{tt}(x,t) + \delta u_t(x,t) + \gamma u_{xxxxt}(x,t) + \alpha u_{xxxx}(x,t) - \left(\beta + \rho \int_0^L u_x^2(x,t)\,dx\right) u_{xx}(x,t) \\
-\sigma \left(\int_0^L u_x(x,t) u_{xt}(x,t)\,dx \right) u_{xx}(x,t) = 0, \quad 0 < x < L, \quad 0 < t \le T,
\end{gathered} \tag{1}
$$

$$
u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x),
$$

$$
u(0,t) = u(L,t) = 0, \quad u_{xx}(0,t) = u_{xx}(L,t) = 0,
$$

where α, γ, ρ, σ, β and δ are the given constants among which the first four are positive numbers, while $u^0(x)$ and $u^1(x)$ are the given functions.

The equation (1) obtained by J. Ball [1] using the Timoshenko theory describes the vibration of a beam. The problem of construction of an approximate solution for this equation is dealt with in [2], [3]. Applying the Galerkin method [4] and a symmetric difference scheme, we can approximate the solution with respect to a spatial and a time variables. Thus the problem is reduced to a system of nonlinear discrete equations which is solved by the iteration method. The question of algorithm accuracy is discussed.

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On Approximation Methods for Singular Integrals and Their Applications

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Approximate schemes for Cauchy type singular integrals are indicated. Questions of some of their applications are considered.

A Numerical Solution of the Kinetic Collection Equation Using High Spectral Grid Resolution: A Proposed Reference

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The multi-moments method of S. Tzivion, G. Feingold, and Z. Levin was applied to the original kinetic collection equation in order to obtain a set of equations with respect to moments in spectral bins. For solving this set of equations an accurate and efficient method is proposed. The method conserves total mass independently of the number of bins, time step, initial conditions, or kernel of interaction. In the present paper the number of bins was varied from 36, 72, 108, and 144 in order to study the behavior of the solutions. Different kernels and initial conditions were tested. In all cases the results show that when the number of bins increases from 36 to 144 the numerical solution of the KCE gradually converges. Increasing the number of bins from 108 to 144 produces only a small difference in the numerical solution, indicating that the solution obtained for 144 bins approaches the "real" solution of the KCE. The use of this solution for evaluating the accuracy of other numerical methods that solve the KCE is suggested.

New Technologies for Approximate Solution of Ordinary Differential Equations

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We consider the problem of approximate solution of two-point boundary value problems (BVP) for ordinary differential equations (DEs) using multipoint finite-difference method and new technology for Cauchy (initial) problems named for brevity as Gauss-Hermite processes. In this report we will construct and investigate high accuracy schemes for initial and BV problems. Particularly such types of statements are true:

Stmt. 1. The order of arithmetic operations for calculation of approximate solution and its derivative of BVP for nonlinear second order DE of normal form with Sturm–Liouville conditions is $O(nlnn)$ *Horner unit*. The convergence of the approximate solution and its derivative has $(p-1)$-th order with respect to mesh width $h = 1/n$ if $y(x)$ has $(p+1)$th order continuously differentiable derivative. If the order is less than p, the remainder member of corresponding scheme has best constant in A. Sard's sense.

Stmt. 2. There are created new schemes and corresponding programs by which are possible to calculate the classical Orthogonal Polynomials (Legendre, Laguerre, Hermite, Chebyshev, all other Ultraspherical ones) when the order of degrees is not less than 50.000 and an accuracy about 200 decimal points.

Parametric Oscillations of a Nonlinear Visco-Elastic Inhomogeneous over the Thickness of the Light Damage-Filled Environment of a Cylindrical Shell Under External Pressure

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The problem of creating optimal designs lead to the need to better integrate features of the properties of materials and links, imposed on the design and components from external contact protection. One of the most significant features of the deformation characteristics is a reduction in their levels during the operation, associated with the process of accumulation and development in the amount of material of various kinds of defects. Allowance for damage allows us to refine the working resource.

Reliable calculation of a cylindrical shell in contact with the medium to long-term strength implies taking into account generation and accumulation of defects and the influence of an ambient media. We treat the case when the periodicity of the voltage is complicated in nature, associated with the so-called process of healing the defect. We have studied the parametric vibrations of nonlinear and non-uniform thickness of a visco-elastic cylindrical shell with filler without the process of healing the defect.

In this report we investigate parametric vibrations of a thin-walled nonlinear - visco-elastic inhomogeneous thickness using the variational principle and taking into account of damage to the cylindrical shell in contact with the medium and is subjected to internal pressure in a geometrically nonlinear formulation.

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A Method of Conformal Mapping for Solving the Generalized Dirichlet Problem of Laplace's Equation

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In this report we investigate the question how the method of conformal mapping can be applied for approximate solving of the generalized Dirichlet boundary problem for harmonic function. Under the generalized problem is meant the case when a boundary function has a finite number of first kind break points. The problem is considered for finite and infinite simply connected domains. It is shown that the method of fundamental solutions is ineffective for solving of the considered problem from the point of view of the accuracy. We propose an efficient algorithm for approximate solving the generalized problem, which is based on the method of conformal mapping. Examples of application of the proposed algorithm and the results of numerical experiments are given.

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