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CRITERIA FOR OSCILLATION OF SOLUTIONS OF TWO–DIMENSIONAL DIFFERENTIAL SYSTEMS WITH DEVIATING ARGUMENTS

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Consider the system of differential equations

$$\begin{aligned} u_1'(t) &= f_1\Big(t, u_1(\tau(t)), u_2(\sigma(t))\Big), \\ u_2'(t) &= f_2\Big(t, u_1(\tau(t)), u_2(\sigma(t))\Big), \end{aligned} \tag{1}$$

where $f_i : R_+ \times R^2 \to R$ $(i = 1, 2)$ satisfy the local Carathéodory conditions, $\tau, \sigma : R_+ \to R$ are nondecreasing continuous functions and $\sigma(t) \le t$, $\sigma(\tau(t)) \le t$ for $t \in R_+$, $\lim\limits_{t\to+\infty} \tau(t) = +\infty$, $\lim\limits_{t\to+\infty} \sigma(t) = +\infty$.

In this paper, we establish sufficient conditions for oscillation of so-called proper solutions of (1), i.e., of nontrivial solutions defined in some neighbourhood of $+\infty$. Analogous problems for higher order functional differential equations have been considered in [1].

In the sequel, we assume that the inequalities

$$\begin{aligned} &f_1(t, x, y) \operatorname{sign} y \ge p(t)|y|, \\ &f_2(t, x, y) \operatorname{sign} x \le -q(t)|x| \quad \text{for} \quad t \in R_+, \quad (x, y) \in R^2 \end{aligned}$$

are fulfilled, where $p, q : R_+ \to R_+$ are locally summable functions.

Theorem 1. *Let the function*

$$h(t) = \int_0^t p(s)\, ds \tag{2}$$

be such that

$$h(+\infty) = +\infty, \tag{3}$$

$$\varliminf_{t\to+\infty} \int_{\sigma(\tau(t))}^t q(s) h(\tau(s))\, ds > \frac{1}{e}. \tag{4}$$

Then every proper solution of (1) *is oscillatory.*

Remark. Note that the inequality (4) cannot be replaced by

$$\varliminf_{t\to+\infty} \int_{\sigma(\tau(t))}^t q(s) h(\tau(s))\, ds > \frac{1}{e} - \varepsilon \tag{5}$$

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for no $\varepsilon \in]0, \frac{1}{e}[$.

Indeed, let $\varepsilon \in]0, \frac{1}{e}[$ and $\mu \in [0,1[$. Let us choose $\lambda \in]0, 1-\mu[$ such that $\lambda > (1-e\varepsilon)(1-\mu)$. Obviously the system

$$u_1'(t) = \frac{c_1}{t^\mu} u_2(\alpha t), \quad u_2'(t) = \frac{c_2}{t^{2-\mu}} u_1(\beta t) \quad \text{for} \quad t \geq 1,$$

where $\alpha \in]0,1]$, $\beta \in]0,+\infty[$, $\alpha\beta = e^{\frac{1}{\lambda+\mu-1}}$, $c_1 = \lambda\alpha^{1-\lambda-\mu}$ and $c_2 = (\lambda+\mu-1)\beta^{-\lambda}$, has the nonoscillatory solution $(t^\lambda, t^{\lambda+\mu-1})$ despite the fact that (5) is fulfilled.

Theorem 2. *Let* (3) *be fulfilled. Let, moreover, there exist* $\alpha \in]0,+\infty[$, $\beta \in [1,+\infty[$ *such that*

$$\varliminf_{t\to+\infty} \frac{h(\sigma(\tau(t)))}{h(t)} \geq \alpha, \quad \varliminf_{t\to+\infty} \frac{h(t)}{h(\sigma(t))} \geq \beta,$$

and for some $\lambda \in]0,1]$,

$$\varliminf_{t\to+\infty} h^\lambda(t) \int\limits_t^{+\infty} h^{1-\lambda}(s) q(s)\, ds > \frac{1}{\lambda^2} \max\left\{\alpha^{x-1}\beta^{-x}(1-x)x \,:\, x \in [0,1]\right\},$$

where the function h is defined by (2). *Then every proper solution of* (1) *is oscillatory.*

Theorem 3. *Let condition* (3) *be fulfilled. If, moreover,*

$$\varliminf_{t\to+\infty} \frac{h(\sigma(\tau(t)))}{h^\mu(t)} > 0,$$

$$\lim_{\lambda\to 0} \left(\varliminf_{t\to+\infty} h^\lambda(t) \int\limits_t^{+\infty} h^{\mu-\lambda}(s) q(s)\, ds \right) > 0,$$

where $\mu \in]0,1[$ *and the function h is defined by* (2), *then every proper solution of* (1) *is oscillatory.*

References

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