# On a Mixed Nonlinear Hyperbolic Problem

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In the plane of independent variables $x$ and $t$, in the domain $D_T : 0 < x < l,\ 0 < t < T$, consider a mixed problem of finding a solution $u(x,t)$ for semilinear wave equation of the form

$$u_{tt} - u_{xx} + g(u) = f(x,t), \ \ (x,t) \in D_T, \tag{1}$$

satisfying the following initial

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x), \ \ 0 \le x \le l, \tag{2}$$

and boundary value conditions

$$u_x(0,t) = F[u_t(0,t)], \quad u(l,t) = 0, \ \ 0 \le t \le T, \tag{3}$$

where $f$, $\varphi$, $\psi$, $g$ and $F$ are given, while $u$ is unknown real functions.

Let the following conditions of smoothness

$$f \in C^1(\overline{D}_T), \ \ g, F \in C^1(\mathbb{R}), \ \ \varphi \in C^2([0,l]), \ \ \psi \in C^1([0,l]) \tag{4}$$

be fulfilled. We assume that at points $(0,0)$ and $(l,0)$ the following conditions of agreement

$$\begin{gathered} \varphi'(0) = F[\psi(0)], \ \ \psi'(0) = F'[\psi(0)]\big\{\varphi''(0) - g[\varphi(0)] + f(0,0)\big\}, \\ \varphi(l) = 0, \quad \psi(l) = 0, \quad g(0) - \varphi''(l) = f(l,0) \end{gathered} \tag{5}$$

are also fulfilled. Let

$$\int\limits_0^s g(s_1)\,ds_1 \ge -M_1 s^2 - M_2, \ \ sF(s) \ge -M_3 \ \ \forall\, s \in \mathbb{R}, \tag{6}$$

where $M_i := const \ge 0$, $1 \le i \le 3$. When conditions (4)–(6) are fulfilled, for the solution $u \in C^2(\overline{D}_T)$ of the problem (1)–(3) it is valid the following a priori estimate

$$\|u\|_{C(\overline{D}_T)} \le c_1 \|f\|_{C(\overline{D}_T)} + c_2 \|\varphi\|_{C^1([0,l])} + c_3 \|\psi\|_{C([0,l])} + c_4 \|g\|_{C([-\|\varphi\|_{C([0,l])}, \|\varphi\|_{C([0,l])}])} + c_5 \tag{7}$$

with positive constants $c_i = c_i(M_1, M_2, M_3, l, T)$, $1 \le i \le 5$, independent on functions $u$, $f$, $\varphi$ and $\psi$.

The problem (1)–(3) can be reduced to the system of Volterra type nonlinear integral equations, which has a continuous solution due to a priori estimate (7), Leray–Schauder's theorem and additional condition $F'(s) \ne -1$, $s \in \mathbb{R}$. In view of the conditions of smoothness (4) and agreement (5) this solution is a classical solution of the original problem. It is proved that the solution is unique.

**Remark.** When the conditions (6) are violated, the problem (1)–(3) may turn out to be insolvable even locally, or locally solvable with a blow-up solution. For example, when $g = 0$, $f = 0$, $F(s) = \arctan s - s$, $s \in \mathbb{R}$, and $|\varphi'(t) + \psi(t)| > \frac{\pi}{2}$, $t \in [0,l]$, then the problem (1)–(3) is insolvable even locally. Besides, if $|\varphi'(t) + \psi(t)| < \frac{\pi}{2}$ for $0 \le t < t_0 \le l$ and $|\varphi'(t_0) + \psi(t_0)| = \frac{\pi}{2}$, then a solution of this problem exists in $[0, t_0)$, and

$$\lim_{t \to t_0 - 0} \|u\|_{C^1(\overline{D}_t)} = \infty.$$

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