On One Boundary Value Problem for Semilinear Equation with the Iterated Multidimensional Wave Operator in the Principal Part

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In the Euclidian space $\mathbb{R}^{n+1}$ of the variables $x_1, \dots, x_n$, t we consider the semilinear equation of the type

$$L_\lambda u := \square^2 u + \lambda |u|^\alpha \operatorname{sgn} u = F, \tag{1}$$

where $\lambda \neq 0$ and $\alpha > 0$ are given real numbers, F is a given and u is an unknown real functions,

$$\square := \frac{\partial^2}{\partial t^2} - \sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}\,, \ \ n \geq 2.$$

For the equation (1) we consider the boundary value problem: find in the cylindrical domain $D_T = \Omega \times (0, T)$, where Ω is an open Lipschitz domain in $\mathbb{R}^n$, a solution $u(x_1, \dots, x_n, t)$ of that equation according to the boundary conditions

$$u\big|_{\partial D_T} = 0, \quad \frac{\partial u}{\partial \nu}\Big|_{\partial D_T} = 0, \tag{2}$$

where $\nu = (\nu_1, \dots, \nu_n, \nu_{n+1})$ is the unit vector of the outer normal to ∂D_T.

Let

$$\overset{\circ}{C}{}^k(D_T, \partial D_T) := \Big\{ u \in C^k(\overline{D}_T) : \ u\big|_{\partial D_T} = \frac{\partial u}{\partial \nu}\Big|_{\partial D_T} = 0 \Big\}, \ \ k \geq 2.$$

Introduce the Hilbert space $\overset{\circ}{W}{}^1_{2,\square}(D_T)$ as the completion with respect to the norm

$$\|u\|^2_{\overset{\circ}{W}{}^1_{2,\square}(D_T)} = \int\limits_{D_T} \Big[u^2 + \Big(\frac{\partial u}{\partial t}\Big)^2 + \sum_{i=1}^{n} \Big(\frac{\partial u}{\partial x_i}\Big)^2 + (\square u)^2 \Big]\, dx\, dt$$

of the classical space $\overset{\circ}{C}{}^2(\overline{D}_T, \partial D_T)$.

Definition. Let $\alpha < \frac{n+1}{n-1}$ and $F \in L_2(D_T)$. The function $u \in \overset{\circ}{W}{}^1_{2,\square}(D_T)$ is said to be a weak generalized solution of the problem (1), (2) if the integral equality

$$\int\limits_{D_T} \square u \square \varphi \, dx\, dt + \lambda \int\limits_{D_T} |u|^\alpha \operatorname{sgn} u \varphi \, dx\, dt = \int\limits_{D_T} F\varphi \, dx\, dt$$

is valid for any $\varphi \in \overset{\circ}{W}{}^1_{2,\square}(D_T)$.

It is not difficult to verify that if a weak generalized solution u of the problem (1), (2) belongs to the class $\overset{\circ}{C}{}^4(D_T, \partial D_T)$, then it will also be a classical solution of that problem.

Theorem. *Let $\lambda > 0$, $\alpha < \frac{n+1}{n-1}$. Then for any $F \in L_2(D_T)$ the problem (1), (2) has a unique weak generalized solution in the space $\overset{\circ}{W}{}^1_{2,\square}(D_T)$.*

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