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PROJECTIVE METHODS FOR THE ELLIPTIC BOUNDARY VALUE PROBLEMS

Projective methods originate in Ritz's (1908) and Galerkin's (1915) works. These methods have been developed in the Friedrichs, Kantorovich, Michlin, Vainikho, Marchuk, Agoshkov, Siarle, Strang, Fix and others works.

In 1963, we obtained the general estimate for the equation $Au = f$, $u \in \mathcal{D}(A)$, $f \in H$,

$$\|u - u_n\|_{L_2(\Omega)} \leq C\lambda_{n+1}^{-1}\|Au_n - f\|,$$

A is a self-adjoint positive definite operator in the Hilbert space $H \equiv L_2$, $0 < \lambda_1 \leq \cdots \leq \lambda_n \to \infty$, $n \to \infty$, $B\varphi_k = \lambda_k\varphi_k$, $k = 1, 2, \dots$, $\mathcal{D}(B) = \mathcal{D}(A)$, $\|Au_n - f\| \to 0$, $n \to \infty$, φ_k are the basic functions.

The basic estimate of F.E.M. (Strang, Fix, Siarle, etc.) is

$$\|u - u_h\|_{H_A} = O(h^{k-m}),$$

where h is a mesh size, $2m$ is the order of differential operator A, $u \in W_2^k(\Omega)$, $k-1$ is the order of finite polynomials, H_A is the energetic space. By us was obtained the estimate (DAN USSR, 1980) $O(h^{k-s})$, $m < s \leq q$, when $u_h \in W_2^q(\Omega)$.

In 1993, Porter and Stirling proposed projective-iterative scheme (cyclic) for the second kind equation $u + Tu = f$, $u, f \in H$.

This scheme for the equation $Au + Ku = f$, $u \in \mathcal{D}(A)$, $f \in H$, is generalized by us, when the form of inverse operator A is known. The estimates

$$\|u - u_{h,l}\|_{L_2(\Omega)} = O(h^{k+m(l+1)}),$$

$$\|u - u_{h,l}\|_{C(\Omega)} = O(h^{k+m(l+1)-\frac{p}{2}}), \quad u_m > p, \quad k \geq 2m,$$

where $u \in W_2^k(\Omega)$, l is a number of cycle, p is a dimension of the domain Ω, are obtained.