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ON A SYSTEM OF NONLINEAR INTEGRO-DIFFERENTIAL EQUATIONS

In the cylinder $Q = (0,1)\times(0,\infty)$, we consider the initial boundary value problem

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x}\Big[a(S)\,\frac{\partial U}{\partial x}\Big], \quad \frac{\partial V}{\partial t} = \frac{\partial}{\partial x}\Big[a(S)\,\frac{\partial V}{\partial x}\Big], \quad (x,t)\in Q, \tag{1}$$

$$U(0,t) = U(1,t) = V(0,t) = V(1,t) = 0, \quad t \ge 0, \tag{2}$$

$$U(x,0) = U_0(x), \quad V(x,0) = V_0(x), \quad x \in [0,1], \tag{3}$$

where

$$S(t) = \int_0^t \int_0^1 \Big[\Big(\frac{\partial U}{\partial x}\Big)^2 + \Big(\frac{\partial V}{\partial x}\Big)^2\Big]\,dx\,d\tau, \tag{4}$$

or

$$S(x,t) = \int_0^t \Big[\Big(\frac{\partial U}{\partial x}\Big)^2 + \Big(\frac{\partial V}{\partial x}\Big)^2\Big]\,d\tau. \tag{5}$$

Theorem 1. *If* $a(s) \ge a_0 = \text{const} > 0$, $U_0, V_0 \in \overset{\circ}{W}{}_2^1(0,1)$, *then for problem* (1)–(4) *the following estimate is true*

$$\|U\|_{W_2^1} + \|V\|_{W_2^1} \le Ce^{-\frac{a_0}{2}t}.$$

Consider the boundary conditions

$$U(0,t) = V(0,t) = 0, \quad U(1,t) = \psi_1, \quad V(1,t) = \psi_2, \quad t \ge 0. \tag{6}$$

Then the following statement is true.

Theorem 2. *If* $a(S) = (1+S)^p$, $p \in (-1/2, 0)$ *or* $p \in (0,1]$, $U_0(0) = V_0(0) = 0$, $U_0(1) = \psi_1$, $V_0(1) = \psi_2$, $\psi_1^2 + \psi_2^2 \ne 0$, $U_0, V_0 \in W_2^2(0,1)$, *then for problem* (1)–(4) *and* (1), (3), (5), (6) *the following estimates are true:*

$$\frac{\partial U(x,t)}{\partial x} = \psi_1 + O(t^{-1-p}), \quad \frac{\partial V(x,t)}{\partial x} = \psi_2 + O(t^{-1-p}),$$

$$\frac{\partial U(x,t)}{\partial t} = O(t^{-1}), \quad \frac{\partial V(x,t)}{\partial t} = O(t^{-1})$$

as $t \to \infty$, *uniformly in* x *on* $[0,1]$.

The numerical solutions of problems (1)–(4) and (1), (3), (5), (6) are also obtained. The numerical experiments are agreed with the theoretical conclusions.