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**ON THE NONLINEAR GOURSAT PROBLEM WITH  
A FREE CHARACTERISTIC**

An alternative nonlinear version of the Goursat characteristic problem is considered for the quasilinear mixed type hyperbolic-parabolic equation

$$x\{u_t u_{xx} - (u_x - u_t)u_{xt} - u_x u_{tt}\} + u_x(u_x + u_t) = 0 \quad (1)$$

with order degeneracy on  $x = 0$ .

This problem consists in finding a solution of equation (1) by the characteristic condition

$$u|_{t=x} = \varphi(x), \quad 0 \leq x \leq a, \quad (2)$$

simultaneously with the domain of its definition provided that this solution satisfies the condition

$$\alpha u_x + \beta u_t|_{\gamma} = \nu(x), \quad a \leq x \leq b, \quad (3)$$

on the unknown characteristic curve  $\gamma$ .

Under the conditions  $\varphi \in C^2[0, a]$ ,  $\nu \in C^1[a, b]$  an explicit representation of the desired characteristic curve  $\gamma$  by the relation

$$t = x + (\alpha - \beta) \int_a^x \left\{ \frac{z\nu(z)}{\alpha a \varphi'(a)} - 1 \right\}^{-1} dz \quad (4)$$

is constructed.

Using representation (4), a nonlinear version of the Asgeirssonian mean value property and general representation of solutions for equation (1), we prove the unique solvability of problem (1)–(3).