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FUCHSIAN SYSTEMS ON RIEMANN SURFACES

Dedicated to the memory of Andrey Bolibruch

The fundamental work of A.Bolibruch gave a new impetus to the study of Fuchsian systems [1]. In particular, A.Bolibruch invented some powerful tools for investigation of the famous Riemann-Hilbert problem and global properties of Fuchsian system in terms of associated holomorphic vector bundles with meromorphic connections. This technique later was successfully applied for solving the Riemann-Hilbert problem for compact Riemann surfaces of higher genus (see [2] and references therein). The results presented in the talk belong to the same direction.

Denote by $\Lambda^1_X(\log S)$ the sheaf of 1-forms holomorphic over $X \setminus S$, where X is a compact Riemann surface and S is a finite subset. We consider admissible pairs over (X, S) consisting of a holomorphic vector bundle $E \to X$ and holomorphic connection $\nabla : \Lambda^0(E) \to \Lambda^0 \otimes \Lambda^1_X(\log S)$. For such pairs one can define the monodromy representation and splitting type [1]. A logarithmic connection $(E, \bigtriangledown)$ is called quasi-Fuchsian if there exists a splitting of E such that $-2g(j-1) \leq k_{j-1} - k_j \leq 2g$, $i = 1, \ldots, n-1$. If $k_1 = 0$, then logarithmic connection (E, ∇) is Fuchsian. The following theorem explicates some previously known results of Fuchsian systems with prescribed monodromy.

Theorem. 1) *If the monodromy matrix $\rho(\gamma_j)$ is diagonalisable for some j, then for representation ρ there exists a Fuchsian system whose monodromy representation coincides with ρ.*

2) *For any two dimensional representation ρ there exists a rank-two Fuchsian system whose monodromy representation coincides with ρ.*

3) *Let (E, ∇) be a stable pair. Then there exists a semi-stable pair (E', ∇') such that $deg(E') = 0$, ∇' has the same singular points as ∇, and the monodromy representations induced by the ∇ and ∇' coincide.*

We also present a similar result for holomorphic principal G-bundles with holomorphic connection and G-system on Riemann surfaces where G is a compact Lie group (see [3]). Such generalization of Fuchsian systems opens wide perspectives for studying certain differential equations of modern mathematical physics, for example, the two-dimensional Yang–Mills equations.

References

1. A. A. Bolibruch, The Riemann–Hibert problem. (Russian) *Russian Math. Surveys* **45**(1990), No. 2, 1–47.

2. A. A. Bolibruch, Stable vector bundles with logarithmic connections and Riemann–Hilbert problem. (Russian) *Dokl. Akad. Nauk* **381**(2001), No. 1, 10–13.

3. G. Giorgadze, Regular systems on Riemann surfaces. *J. Math. Sci.* **118** (2003), No. 5, 5347–5399.