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ON THE SOLVABILITY OF DIVERGENCE EQUATION IN THE THEORY OF INCOMPRESSIBLE FLUIDS

Let $\Omega \subset R^n$, $n \geq 2$, be a bounded domain with Lipschitz boundary and let $x_0 \in \Omega$.

Let $p : \Omega \to R^1$ be a measurable function satisfying the following conditions:

i) $1 < \underline{p} \leq p(x) \leq \overline{p} < \infty$;

ii) $|p(x) - p(y)| \leq \frac{C}{-\ln|x-y|}$, $|x-y| < \frac{1}{2}$.

Put $p'(x) = \frac{p(x)}{p(x)-1}$.

By $L^{p(\cdot)}_\rho(\Omega)$ we denote a Banach function space $L^{p(\cdot)}_\rho$, i.e., a space of all measurable functions for which

$$\|f\rho\|_{L^{p(\cdot)}} < \infty,$$

where $\rho(x) = |x - x_0|^\alpha$.

For the definition of the norm in $L^{p(\cdot)}$ see, for example, [1].

Define

$$\overset{\circ}{L}{}^{p(\cdot)}_\rho = \left\{ f \in L^{p(\cdot)}_\rho(\Omega) : \int\limits_\Omega f(x)\,dx = 0 \right\}.$$

By $W^{1,p(\cdot)}_\rho$ we denote the weighted Sobolev space.

Theorem. *Let p satisfy the conditions (i) and (ii). Assume that* $\frac{1}{p(x_0)} < \alpha < \frac{1}{p'(x_0)}$.

Then for each $f \in \overset{\circ}{L}{}^{p(\cdot)}_\rho$ the divergence equation

$$\operatorname{div} u = f$$

is solvable in vectorial $W^{1,p(\cdot)}_\rho$ space and the estimate

$$\|\nabla u\|_{L^{p(\cdot)}_\rho} \leq c\|f\|_{L^{p(\cdot)}_\rho}$$

holds.

References

1. L. Diening and M. Ružićka, Calderon–Zygmund operators in generalized Lebesgue spaces $L^{p(\cdot)}$ and problems related to fluid dynamics. *Abert-Ludwig Universität Freiburg, Preprint* Nr. 21/2002–04.07.2002.