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ON INITIAL-BOUNDARY VALUE PROBLEMS IN AN INFINITE STRIP FOR A NONLINEAR HYPERBOLIC EQUATION OF THIRD ORDER

Let $I \subset \mathbb{R}$ be a compact interval containing zero. For the nonlinear equation

$$u^{(2,1)} = f\big(x, t, u, u^{(1,0)}, u^{(2,0)}, u^{(0,1)}\big) \tag{1}$$

consider the initial–boundary value problems

$$\begin{gathered} u(x,0) = \varphi(x) \ \text{ for } \ x \in [0, +\infty), \\ u(0,t) = \psi(t), \quad \sup\{|u(x,t)| : \ x \in [0,+\infty)\} < +\infty \ \text{ for } \ t \in I; \end{gathered} \tag{2_1}$$

$$u(x,0) = \varphi(x) \ \text{ for } \ x \in \mathbb{R}, \quad \sup\{|u(x,t)| : \ x \in \mathbb{R}\} < +\infty \ \text{ for } \ t \in I. \tag{2_2}$$

Here $u^{(j,k)}(x,y) = \frac{\partial^{j+k} u(x,y)}{\partial x^j \partial y^k}$, $f : \mathbb{R} \times I \times \mathbb{R}^4 \to \mathbb{R}$ is a continuous function, $\varphi : \mathbb{R} \to \mathbb{R}$ is a twice continuously differentiable function, and $\psi : I \to \mathbb{R}$ is a continuously differentiable function such that $\varphi(0) = \psi(0)$.

The linear case of equation (1) arises in study of nonsteady simple shearing flow of second order fluids (c.f. [1]) and also in the theory of seepage of homogeneous fluids through fissured rocks ([2]).

We assume that f satisfies the following three conditions: (i) there exists a continuous function $\delta : I \to \mathbb{R}$ such that $(f(x,t,u,v,w,z_1) - f(x,t,u,v,w,z_2))\operatorname{sgn}(z_1 - z_2) \geq \delta(t)|z_1 - z_2|$; (ii) f is locally Lipschitz continuous with respect to w; (iii) there exist continuous functions l and $g : I \to \mathbb{R}_+$ such that $|f(x,t,u,v,w,z)| \leq l(t)(|u| + |v| + |w| + |z|) + g(t)$.

Theorem. *Let $\delta(t) > 0$ for $t \in I$. Then problem $(1), (2_k)$ $(k = 1, 2)$ is solvable. Moreover an arbitrary solution u_k of problem $(1), (2_k)$ $(k = 1, 2)$ admits the estimate*

$$\|u_1\|_{C^{(2,1)}(\mathbb{R}_+ \times I)} \leq M\Big(\|\varphi\|_{C^2(\mathbb{R}_+)} + \|\psi\|_{C^1(I)} + \|g\|_{C(I)}\Big) \ \textit{for} \ k = 1,$$

$$\|u_2\|_{C^{(2,1)}(\mathbb{R} \times I)} \leq M\Big(\|\varphi\|_{C^2(\mathbb{R})} + \|g\|_{C(I)}\Big) \ \textit{for} \ k = 2,$$

where $M > 0$ is a constant independent of φ, ψ and g. Furthermore if f is locally Lipschitz continuous with respect to u and v, then problem $(1), (2_k)$ $(k = 1, 2)$ is uniquely solvable.

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References

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