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SOLUTION OF A TWO-DIMENSIONAL PROBLEM OF THE FILTRATION THEORY WITH THE PARTIALLY UNKNOWN BOUNDARY CONNECTED WITH UNDERGROUND WATER MOTION TOWARD THE EARTH EMBANKMENT SLOPE

A two-dimensional problem with the partially unknown boundary connected with steady-state motion of underground water toward the earth embankment slope is solved: the underground water motion domain is bounded from below by a horizontal impermeable foundation, while from above it is bounded by the unknown depression curve. In passing to the slope, the moving underground water forms a leakage interval, while a part of the moving liquid is contiguous to the water basin. Thus the boundary of the infinite singly-connected domain occupied by the moving liquid consists of the unknown curve and two segments – straight and semi-straight.

The porous medium is homogeneous, isotropic and nondifferentiable. Liquid motion in a porous mediun obeys the Darcy law. The liquid motion plane is brought into coincidence with a complex plane. The reduced complex potential is introduced, whose real and imaginary parts satisfy the Cauchy–Riemann conditions and two linearly independent boundary conditions.

Consideration is given to the complex planes of liquid motion, a complex potential, complex velocity and an auxiliary complex plane. On these complex planes, the domain occupied by the moving liquid is associated with singly-connected domains; for example, on the complex velocity plane, it is associated with a singly-connected circular pentagon one of whose vertices lies at the point at infinity. Characteristic properties are found at singular points and used to construct differential equations of the Fuchs class and its corresponding nonlinear Schwartz equation. Solutions of these equations are obtained.

To solve the filtration problem by means of solutions of the above-mentioned differential equations, three analytical functions are constructed, which conformally map the half-plane of the auxiliary complex plane on the domains of liquid motion, a complex potential and complex velocity. These functions satisfy the respective boundary conditions.

A system of higher transcendental equations is obtained for determining mechanical and geometrical characteristics of the problem and the abovementioned three domains. A solution algorithm is given for the obtained system of equations. A parametric equation of the depression curve is derived.