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ON NORMS OF SOME INTEGRAL OPERATORS

Theorem 1. *Let $f \in L_1(\mathbb{R})$, $f(x) \geq 0$ a.e. Then the norm of the convolution operator*

$$(W_f\varphi)(t) = \int_{-\infty}^{+\infty} f(t-\tau)\varphi(\tau)\,d\tau$$

in Lebesgue space $L_p(\mathbb{R})$, $p \geq 1$, is equal to $\widehat{f}(0)$, where

$$\widehat{f}(t) = \int_{-\infty}^{+\infty} e^{it\tau}\varphi(\tau)\,d\tau.$$

Let $p \geq 1$, $\alpha, r \in \mathbb{R}$. Denote by $L_{p,\alpha}$ the Banach space with the norm

$$\|\varphi\|_{p,\alpha} = \left(\int_0^1 |x^\alpha \varphi(x)|^p\right)^{1/p}$$

and by H_r denote the Hardy operator

$$(H_r\varphi)(x) = \int_0^x x^{r-1}y^{-r}\varphi(y)\,dy.$$

Theorem 2. *The operator H_r is bounded in the space $L_{p,\alpha}$ if and only if $r < 1-\alpha-1/p$. In this case*

$$\|H_r\|_{p,\alpha} = \frac{1}{1-\alpha-1/p-r}\,.$$

Let $r > 0$. Denote by R_r the Riemann-Liouville operator

$$(R_r\varphi)(t) = \int_0^x (x-y)^{r-1}x^{-r}\varphi(y)\,dy.$$

Theorem 3. *The operator R_r is bounded in the space $L_{p,\alpha}$ if and only if $\alpha + 1/p < 1$. In this case*

$$\|R_r\|_{p,\alpha} = B\big(1-\alpha-1/p, r\big),$$

where $B(a,b)$ is the Beta-function.