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NONEXISTENCE OF POSITIVE SOLUTIONS FOR SOME CLASSES OF NONLINEAR ELLIPTIC INEQUALITES IN $\mathbb{R}^N$

The report deals with the investigation of the nonexistence of positive solutions for a class of nonlinear elliptic inequalites

$$-\operatorname{div}\big(A(x,u,Du)Du\big) \geq a(x)u^q, \quad u \geq 0, \quad u \not\equiv 0 \ \text{ in } \ \mathbb{R}^{\mathbb{N}}, \tag{1}$$

where $\operatorname{div} := \sum\limits_{i=1}^{N} \frac{\partial}{\partial x_i}$, $D := \big(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_N}\big)$; $\mathbb{R} :=]-\infty,\infty[$, $\mathbb{R}^{\mathbb{N}} := \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{\mathbb{N}-\text{times}}$; $D_i u$, $i = 1,\dots,N$ are understood in the sense of distributions.

Bellow we consider two particular cases of the function A:

(i) $A(x,u,Du) := |x|^{\alpha_1}|u|^{q_1}|Du|^{p-2}$, where $|\cdot|$ denotes the Euclidean norm in the space $\mathbb{R}^{\mathbb{N}}$;

(ii) $A(x,u,Du) := A(|Du|)$ and the continuous function $A : \mathbb{R}_+ \to \mathbb{R}_+$ satysfies the condition of uniform boundedness from above: $0 < A(t) \leq C$, $t \in \mathbb{R}_+ := [\not\vdash,\infty[$, $C :=$ const. The latter case includes two important examples:

(a) $A(t) := \frac{1}{\sqrt{1+t^2}}$, $t \in \mathbb{R}_+$ (the operator of mean curvature),

(b) $A(t) := \frac{1}{(1+|t|^k)^s}$, $t \in \mathbb{R}_+$, $k,\ s > 0$ (the generalised operator of mean curvature).

The continuous function $a : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}_+$ is such that

$$a(x) \geq C_0|x|^{\gamma}, \ \ \forall x : \ |x| \geq R_0 > 0, \ \ \gamma \in \mathbb{R}, \ \ \mathbb{R}_{\not\vdash}, \mathbb{C}_{\not\vdash} := \text{const}\,. \tag{2}$$

There is hold the following

Theorem. *Let* $N + \alpha_1 > p > 1 - q_1$ $(N > 2)$. *It is also assumed that the functions A and a satisfy the conditions* (i) ((ii)) *and* (2), *respectively. If*

$$q_1 + p - 1 < q \leq \frac{(N+\gamma)(q_1+p-1)}{N+\alpha_1-p} \quad \Big(1 < q \leq \frac{N+\gamma}{N-2}\Big),$$

then the problem (1) *has no solution in the functional class* $W^{1,p,q,q_1}_{a,\alpha,\alpha_1,loc}(\mathbb{R}^{\mathbb{N}}) :=$ $\{\approxeq : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}_+,\ \eth(\curvearrowleft)\approxeq^{''+\alpha},\ |\curvearrowleft|^{\alpha_{\not\models}}|\mathbb{D}\approxeq|^{'}\approxeq^{''_{\not\models}+\alpha-\not\models} \in \mathbb{L}^{\not\models}_{\lessdot\rtimes}(\mathbb{R}^{\mathbb{N}})\}$ $(W^{1,2,q,0}_{a,\alpha,0,loc}(\mathbb{R}^{\mathbb{N}}))$ *for sufficiently small $\alpha < 0$, respectively.*

These problems were posed in the paper [1] by E. Mitidieri and S. I. Pohozaev.

References

1. E. Mitidieri, S. I. Pohozaev, Nonexistence of positive solutions for quasilinear elliptic problems on $\mathbb{R}^{\mathbb{N}}$. *Proc. Steklov Math. Inst.* **227**(1999), 192–222.