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ON NONLINEAR HYPERBOLIC EQUATIONS WITH GENERAL SOLUTIONS REPRESENTABLE BY SUPERPOSITION OF ARBITRARY FUNCTIONS

On a plane of variables x, t we consider a class of quasi-linear non-strictly hyperbolic equations of the second order. It is assumed that differential relations of characteristic directions are both completely integrable and define a pair of smooth invariants $\mu = \mu(x,t)$, $\lambda = \lambda(x,t,u)$. The invariant λ, which depends on an unknown solution $u(x,t)$, has its inverse $u = \Lambda(x,t,\lambda)$. Depending on the right side, a function $h \in C^2(R^3)$ in a number of cases is defined, allowing one to obtain an explicit representation of the general integral $\lambda(x,t,u) = f[h(x,t,g(\mu))]$ of the equation under consideration. Under our assumptions, this results in the explicit general solution

$$u(x,t) = \Lambda\big\{f\big[h(x,t,g(\mu))\big]\big\} \tag{1}$$

in the form of a complicated superposition of arbitrary functions f, $g \in C^2(R^1)$. As an illustration, we have considered a number of examples, the simples of which is of the form

$$(1+u_t)u_{xx} + (1-u_x+u_t)u_{xt} - u_x u_{tt} = 0 \tag{2}$$

with the characteristic invariants $\lambda = u + y$, $\mu = x - y$ and with the general solution

$$u(x,t) = f[x + g(x-y)] - y \tag{3}$$

for arbitrary functions f, g; the smoothness of these functions defines that of the solution. As is seen, equation (2) belongs to a class whose general integrals are linear with respect to arbitrary functions of the invariants λ, μ. In the presence of the general solution (1), the D'Alembert method is used for the investigation of nonlinear hyperbolic problems and, as a result, not only solutions, but also the domains of their definition are established.