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ON SOME NON-LOCAL PROBLEMS FOR QUASI-LINEAR HYPERBOLIC EQUATIONS WITH RECTILINEAR CHARACTERISTICS

On the x, y plane the non-local characteristic problem is considered for the quasi-linear hyperbolic second order equation

$$u_{xx} + (1 + u_x + u_y)u_{xy} + (u_x + u_y)u_{yy} = 0 \tag{1}$$

with admissible parabolic degeneracy, depending on the behavior of the first order derivatives of an unknown solution. In terms of characteristic invariants $\xi = y - x$, $\eta = y - (u_x + u_y)x$ equation (1) has the explicit representation of solutions

$$u(\xi, \eta) = f(\xi) - g(\eta) + (\xi - \eta)\big[1 - g'(\eta) + e^{g'(\eta)}\big] \tag{2}$$

with arbitrary functions f, g.

For equation (1) the following non-local characteristic problem is posed and investigated: define the regular solution $u(x, y)$ of equation (1) simultaneously with its domain of definition under the conditions

$$(u_x + u_y)\big|_{y=0} = \alpha(x), \quad u(x, 0) + \gamma(x)u\Big(\frac{\alpha(x)x}{\alpha(x) - 1}, \frac{\alpha(x)x}{\alpha(x) - 1}\Big) = \beta(x), \tag{3}$$

where $\alpha \in C^1[0, b]$, $\beta, \gamma \in C^2[0, a]$, $a = \frac{\alpha(b)b}{\alpha(b)-1}$ are the given functions. The following theorem is proved.

Theorem. *If*

$$\alpha(x) > 1, \ \ x \in [0, b], \quad \alpha(x) + (\log \alpha(x))'(y - x) \neq 1, \ \ x, y \in [0, a], \tag{4}$$

then problem (1), (3) *has the unique solution defined in the triangle, bounded by supports of the problem data and passing through its endpoints characteristic* $y = \alpha(b)(x - b)$. *In the case under consideration the parabolic degeneracy is excluded by conditions* (4).