ABSTRACTS

DIFFERENTIAL EQUATIONS
AND
MATHEMATICAL PHYSICS

Symposium Dedicated to the 110th Birthday
Anniversary of Academician N. Muskhelishvili

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All abstracts except the plenary talks are assorted to the following sections:

- Differential Equations;
- Mathematical Physics.

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PLENARY TALKS

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APPLICATION OF THE METHODS OF THE THEORY OF ANALYTIC FUNCTIONS IN THE THEORY OF ELASTICITY

The paper gives a survey of the results obtained for the last decade by the Georgian scientists on the problems of elasticity with the use of the theory of analytic functions. A description of basic stages of the development of this area is presented.

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A SURVEY OF RECENT RESULTS ON THE BOUNDARY VALUE PROBLEMS FOR HOLOMORPHIC FUNCTIONS

In our lecture we give a survey of the results obtained for the recent years by the participants of seminars held at A. Razmadze Mathematical Institute on boundary value problems for holomorphic and harmonic functions and on singular integral equations. The original source of these investigations turned out to be the richest scientific hertigate of N. Muskhelishvili.

SECTION I

DIFFERENTIAL EQUATIONS

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ON SOME FAMILIES OF MINIMAL SURFACES IN $\mathbb{R}^3$

Let $g : D^2 \to \mathbb{R}^{\nVdash}$ be a conformal representation of a generalized minimal surface S and π be its orthogonal projection onto the chosen coordinate plane. The surface will be called "a good surface" if the mapping $\pi \circ g$ is an interior mapping. Let B be a family of closed broken lines in $\mathbb{R}^{\nVdash}$ with co-directed segments and let G denote the family of "good minimal surfaces" bounded by lines belonging to B. The restriction of gaussian mapping of surfaces $S \in G$ to the boundary generates another family of planar closed curves of specific kind and thus the condition of the existence of interior extension of these curves is one of the necessary conditions for $G \neq \varnothing$. The second necessary condition we obtain by the calculation of orders of zeros and poles of some quadratic differential assosiated with conformal representation of surfaces of the family G. Using the solution of Riemann–Hilbert boundary value problem we show that these two necessary conditions jointly are sufficient conditions as well.

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ON THE NECESSARY AND SUFFICIENT CONDITIONS FOR THE STABILITY OF LINEAR SYSTEMS OF GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

Consider the linear system of generalized ordinary differential equations

$$dx(t) = dA(t) \cdot x(t), \tag{1}$$

where $A : [0, +\infty[\to \mathbb{R}^{n \times n}$ is a real matrix function with locally bounded variation components.

By a solution of system (1) we understand a vector function $x : [0, +\infty[\to \mathbb{R}^n$ with locally bounded variation such that

$$x(t) = x(s) + \int\limits_s^t dA(\tau) \cdot x(\tau) \quad (0 \leq s < t < +\infty),$$

where $\int\limits_s^t dA(\tau) \cdot x(\tau)$ is the Lebesgue–Stieltjes integral.

The theory of generalized ordinary differential equations enables one to investigate ordinary differential, difference and impulsive equations from the unified standpoint.

The necessary and sufficient conditions are found for the stability (uniform stability, asymptotic stability or the so-called ξ-exponential asymptotic stability) of system (1).

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N. MUSKHELISHVILI ON THE REVERSE PROBLEM OF THE THERMAL RADIATION THEORY

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N. MUSKHELISHVILI ON ONE METHOD OF INTEGRATION OF HAMILTON–JAKOB'S MOTION EQUATIONS

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ON ONE PROPERTY OF A 2-RANK HOLOMORPHIC VECTOR BUNDLE ON THE RIEMANN SPHERE

Let $S = \{s_1, s_2, \ldots, s_m\}$ be a set of marked points on $\mathbf{CP}^1$ and

$$df = \Big(\sum_{i=1}^{m} \frac{A_i}{z - s_i}\, dz\Big) f \tag{1}$$

be the system of ODE's which is induced by the representation

$$\rho : \pi_1(\mathbf{CP^1} \backslash \mathbf{S}, \mathbf{z_0}) \to \mathbf{GL(n, C)},$$

where the matrices A_i satisfy the condition $\sum\limits_{i=1}^{m} A_i = 0$.

For every Fuchsian equation there exists a Fuchsian system (1), which has the same singular points and monodromy. In particular, for the hypergeometric equation

$$y'' + \frac{\gamma + (\alpha + \beta + 1)}{z(z-1)} y' - \frac{\alpha\beta y}{z(z-1)} = 0$$

the aforementioned system will be:

$$df = \left(\begin{pmatrix} 0 & 0 \\ -\alpha\beta & -\gamma \end{pmatrix} \frac{dz}{z} + \begin{pmatrix} 0 & 1 \\ 0 & \gamma - (\alpha + \beta) \end{pmatrix} \frac{dz}{z-1} \right) f \tag{2}$$

Fix a new representation ρ for any Riemann data $(\mathbf{CP^1}, \mathbf{S}, \rho)$ and denote by Ω_ρ the set of Fuchsian systems corresponding to this data. The number $\gamma_\rho = \min_{\Omega_\rho} \gamma_\omega$ is called the Fuchsian weight for the representation ρ.

Let $\mathbf{E} \to \mathbf{CP^1}$ be the holomorphic vector bundle induced by the representation ρ and (k_1, k_2) be its splitting type. Then $\gamma_\rho = k_1 - k_2$.

Every rank two holomorphic bundle on $\mathbf{C}P^1$ is holomorphically equivalent to any bundle $\mathbf{F} \to \mathbf{CP^1}$, which is obtained by the extension of the bundle induced by an irreducible representation. So, for every rank 2 holomorphic bundle there exists an irreducible connexion, which is holomorphic except for the finite number n_ω of points, where it has simple poles. Denote by Ω^{irr} the space of irreducible Fuchsian connexions. Let $p = \min_{\omega \in \Omega^{\mathrm{irr}}} n_\omega$. Then the identity $p = k_1 - k_2 + 2$ is satisfied.

From this follows a criterion of stability of such bundles.

Proposition. *A rank two vector bundle* $\mathbf{F} \to \mathbf{CP^1}$ *is stable if and only if it is induced by the system* (2).

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ON NONLINEAR HYPERBOLIC EQUATIONS WITH GENERAL SOLUTIONS REPRESENTABLE BY SUPERPOSITION OF ARBITRARY FUNCTIONS

On a plane of variables x, t we consider a class of quasi-linear non-strictly hyperbolic equations of the second order. It is assumed that differential relations of characteristic directions are both completely integrable and define a pair of smooth invariants $\mu = \mu(x,t)$, $\lambda = \lambda(x,t,u)$. The invariant λ, which depends on an unknown solution $u(x,t)$, has its inverse $u = \Lambda(x,t,\lambda)$. Depending on the right side, a function $h \in C^2(R^3)$ in a number of cases is defined, allowing one to obtain an explicit representation of the general integral $\lambda(x,t,u) = f[h(x,t,g(\mu))]$ of the equation under consideration. Under our assumptions, this results in the explicit general solution

$$u(x,t) = \Lambda\{f\left[h(x,t,g(\mu))\right]\} \tag{1}$$

in the form of a complicated superposition of arbitrary functions f, $g \in C^2(R^1)$. As an illustration, we have considered a number of examples, the simples of which is of the form

$$(1+u_t)u_{xx} + (1-u_x+u_t)u_{xt} - u_x u_{tt} = 0 \tag{2}$$

with the characteristic invariants $\lambda = u + y$, $\mu = x - y$ and with the general solution

$$u(x,t) = f[x + g(x-y)] - y \tag{3}$$

for arbitrary functions f, g; the smoothness of these functions defines that of the solution. As is seen, equation (2) belongs to a class whose general integrals are linear with respect to arbitrary functions of the invariants λ, μ. In the presence of the general solution (1), the D'Alembert method is used for the investigation of nonlinear hyperbolic problems and, as a result, not only solutions, but also the domains of their definition are established.

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NONEXISTENCE OF POSITIVE SOLUTIONS FOR SOME CLASSES OF NONLINEAR ELLIPTIC INEQUALITES IN $\mathbb{R}^{\mathbb{N}}$

The report deals with the investigation of the nonexistence of positive solutions for a class of nonlinear elliptic inequalites

$$-\operatorname{div}\big(A(x,u,Du)Du\big) \geq a(x)u^q, \quad u \geq 0, \quad u \not\equiv 0 \ \text{ in } \ \mathbb{R}^{\mathbb{N}}, \tag{1}$$

where $\operatorname{div} := \sum\limits_{i=1}^{N} \frac{\partial}{\partial x_i}$, $D := \big(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}, \dots, \frac{\partial}{\partial x_N}\big)$; $\mathbb{R} :=]-\infty,\infty[$, $\mathbb{R}^{\mathbb{N}} := \underbrace{\mathbb{R}\times\cdots\times\mathbb{R}}_{\mathbb{N}-\text{times}}$; $D_i u$, $i=1,\dots,N$ are understood in the sense of distributions.

Bellow we consider two particular cases of the function A:

(i) $A(x,u,Du) := |x|^{\alpha_1}|u|^{q_1}|Du|^{p-2}$, where $|\cdot|$ denotes the Euclidean norm in the space $\mathbb{R}^{\mathbb{N}}$;

(ii) $A(x,u,Du) := A(|Du|)$ and the continuous function $A : \mathbb{R}_+ \to \mathbb{R}_+$ satysfies the condition of uniform boundedness from above: $0 < A(t) \leq C$, $t \in \mathbb{R}_+ := [0,\infty[$, $C :=$ const. The latter case includes two important examples:

(a) $A(t) := \frac{1}{\sqrt{1+t^2}}$, $t \in \mathbb{R}_+$ (the operator of mean curvature),

(b) $A(t) := \frac{1}{(1+|t|^k)^s}$, $t \in \mathbb{R}_+$, k, $s > 0$ (the generalised operator of mean curvature).

The continuous function $a : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}_+$ is such that

$$a(x) \geq C_0|x|^\gamma, \ \ \forall x: \ |x| \geq R_0 > 0, \ \ \gamma \in \mathbb{R}, \ \ R_0, C_0 := \text{const}. \tag{2}$$

There is hold the following

Theorem. *Let* $N + \alpha_1 > p > 1 - q_1$ $(N > 2)$. *It is also assumed that the functions* A *and* a *satisfy the conditions* (i) ((ii)) *and* (2), *respectively. If*

$$q_1 + p - 1 < q \leq \frac{(N+\gamma)(q_1+p-1)}{N+\alpha_1-p} \ \ \Big(1 < q \leq \frac{N+\gamma}{N-2}\Big),$$

then the problem (1) *has no solution in the functional class* $W^{1,p,q,q_1}_{a,\alpha,\alpha_1,loc}(\mathbb{R}^{\mathbb{N}}) := \{u : \mathbb{R}^{\mathbb{N}} \to \mathbb{R}_+,\ a(x)u^{q+\alpha},\ |x|^{\alpha_1}|Du|^p u^{q_1+\alpha-1} \in L^1_{loc}(\mathbb{R}^{\mathbb{N}})\}$ $(W^{1,2,q,0}_{a,\alpha,0,loc}(\mathbb{R}^{\mathbb{N}}))$ *for sufficiently small* $\alpha < 0$, *respectively.*

These problems were posed in the paper [1] by E. Mitidieri and S. I. Pohozaev.

References

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FORMULAE FOR THE GENERAL SOLUTIONS FOR SOME CLASSES OF LINEAR NONHOMOGENEOUS DIFFERENTIAL EQUATIONS OF SECOND ORDER WITH VARIABLE COEFFICIENTS

Let us consider the equation

$$\alpha(t)x'' + \beta(t)x' + \gamma(t)x = \delta(t), \;\; t_1 < t < t_2, \tag{1}$$

where α, β, γ, δ are given continuous functions.

The function is called possible if the above operations are correct.

Theorem 1. *If the following conditions are satisfied*

$$\alpha(t) \equiv 1, \quad \beta(t) \equiv 0, \quad \gamma(t) = -\frac{\eta''(t)}{\eta(t)}, \;\; t_1 < t < t_2, \tag{2}$$

where η is any possible function, then the general solution of the equation (1) *has the following form*

$$x = \eta\Big(c_1 + \int \frac{1}{\eta^2}\Big(c_2 + \int \eta\delta\, dt\Big)\, dt\Big), \;\; t_1 < t < t_2. \tag{3}$$

Let

$$\eta = e^{\int \xi(t)\, dt}, \;\; t_1 < t < t_2, \tag{4}$$

where ξ is any possible function, then from the Theorem 1 follows

Theorem 2. *If the following conditions are satisfied $\alpha(t) \equiv 1$, $\beta(t) \equiv 0$, $\gamma(t) = -(\xi'(t)+\xi^2(t))$, $t_1 < t < t_2$, then the general solution of the equation* (1) *is given by* (3)*, where the function η is determined by* (4)*.*

There also is much general result.

Theorem 3. *If the following conditions are satisfied*

$$\begin{cases} ab = \alpha(t) \\ ab' + qb + ah = \beta(t) \\ ah' + qh = \gamma(t), \quad t_1 < t < t_2 \end{cases},$$

where $a = a(t)$, $b = b(t)$, $q = q(t)$, $h = h(t)$ are any possible functions, then the general solution of the equation (1) *is represented in finite terms by* (3)*.*

Finally, we note that Theorem 2 generalizes the results of M. Elshin (*Bull. Acad. Sci. USSR*, **XVIII**(1938)) and has important applications. For example if $\xi(t) = Ae^{\omega t} + B$, where A, B, ω are constants, then from the equation (1), assuming $\alpha(t) \equiv 1$ and $\beta(t) \equiv 0$, one gets generalized Hille's type equation.

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CHARACTERISTIC PROBLEMS FOR THE FIRST ORDER SYMMETRIC HYPERBOLIC SYSTEMS IN THE CONIC DOMAINS

It is known that Goursat problem, i.e., the characteristic problem for the second order hyperbolic equation admits different formulations by passing from the two-dimensional case to multi-dimensional. For example, the characteristic problem for the multi-dimensional wave equation can be set as in a conic domain whose boundary is the characteristic conoid as well in a dihedral angle when both sides are characteristic planes. These problems are well-posed if we consider the Dirichlet boundary conditions. In the sense of the correct formulation the situation becomes essentially complicated when in the multidimensional case we are passing from a single equation to hyperbolic systems. For example, though for a second order hyperbolic system with a split main part the Goursat problem with Dirichlet data on the characteristic conoid is well-posed, A. V. Bitzadze constructed an example of second order hyperbolic system with a non-split main part for which the corresponding homogeneous characteristic problem has an infinite number of linearly independent solutions. The difficulties arise even for a strictly hyperbolic system with a non-split main part whose cone of normals consists of infinitely smooth cavities. In this case the corresponding cones of cavities of rays may have strong singularities. Therefore the difficulties arise already in the formulation of a characteristic problem in wich the support of the boundary data should be pointed out.

We propose an approach permiting to formulate the correct characteristic problems for a class of the first order symmetric hyperbolic systems in conic domains. For example, the well-known Maxwell, Dirac and crystal-optics systems of differential equations of mathematical physics belong to that class.

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A NOTE ON A CONFORMAL MAPPING OF SIMPLY CONNECTED DOMAINS WITH JORDAN BOUNDARIES

Let Γ_0 be a Jordan piecewise smooth curve bounding a finite domain D_0, and Γ_1 be the same curve having one common with Γ point; all the rest points lie in D_0. Denote by D a domain with the boundary $\Gamma = \Gamma_0 \cup \Gamma_1$.

The report states that for the case under consideration the analogue of Warschawski's result on properties of a derivative, mapping conformally the unit circle onto D [1] (established for domains with Jordan piecewise Ljapunov curves), is valid.

References

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ON SINGULAR BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Suppose m and n are natural numbers, $-\infty < a < b < +\infty$, $\alpha : [0, n-1]$, $\beta \in [0, n-1]$,

$$\alpha_i = \frac{\alpha + i - n + |\alpha + i - n|}{2}, \quad \beta_i = \frac{\beta + i - n + |\beta + i - n|}{2} \ \ (i = 1, \dots, n);$$

$\mathbb{R}^{\geqslant}$ is the m-dimensional real Euclidean space with norm $\|\cdot\|_{\mathbb{R}^{\geqslant}}$;

$C^{n-1}_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\geqslant})$ is the Banach space of $(n-1)$–times continuously differentiable vector functions $x :]a, b[\to \mathbb{R}^{\geqslant}$ having the limits

$$\lim_{t \to a} (t-a)^{\alpha_i} x^{(i-1)}(t), \quad \lim_{t \to b} (b-t)^{\beta_i} x^{(i-1)}(t) \ \ (i = 1, \dots, n),$$

where the norm is defined by the equality

$$\|x\|_{C^{n-1}_{\alpha,\beta}} = \sup \Big\{ \sum_{k=1}^{n} (t-a)^{\alpha_i} (b-t)^{\beta_i} \|x^{(i-1)}(t)\|_{\mathbb{R}^{\geqslant}} : \ \ a < t < b \Big\};$$

$L_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\geqslant})$ is the Banach space of summable with weight $(t-a)^{\alpha} \times (b-t)^{\beta}$ vector functions $y :]a, b[\to \mathbb{R}^{\geqslant}$ with the norm

$$\|y\|_{L_{\alpha,\beta}} = \int_a^b (t-a)^{\alpha} (b-t)^{\beta} \|y(t)\|_{\mathbb{R}^{\geqslant}} \, dt.$$

Sufficient conditions for the solvability and unique solvability are established for the singular boundary value problem

$$x^{(n)}(t) = f(x)(t), \quad h_i(x) = 0 \ \ (i = 1, \dots, n), \tag{5}$$

where $f : C^{n-1}_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\geqslant}) \to \mathbb{L}_{\alpha,\beta}(]\eth, [\,; \mathbb{R}^{\geqslant})$ and $h_i : C^{n-1}_{\alpha,\beta}(]a, b[\,; \mathbb{R}^{\geqslant}) \to \mathbb{R}^{\geqslant}$ $(i = 1, \dots, n)$ are continuous operators.

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ON FOURIER MULTIPLIERS AND THE SOLVABILITY OF CERTAIN SUPERLINEAR PDE

The main goal of this report is to present two-weighted estimates for the multipliers of Fourier transforms and their application to the multidimensional Ricatti equation.

Let $S(R^n)$ be Schwartz space of rapidly decreasing functions. Denote by $F(\varphi)$ the Fourier transform of $\varphi \in S(R^n)$. Let the operator K defined by the Fourier transform equation

$$F(K\varphi) = mF(\varphi), \quad \varphi \in S.$$

The following statement is true:

Theorem. *Let the function v_j, $j = 1, 2$, satisfy the conditions*
(i) $v_j \in L^1(R^1)$, $j = 1, 2$.
(ii) $\displaystyle\int\limits_{R^1} \frac{1}{v_2(x)|x|^{(1-\alpha)2}}\, dx < \infty.$

Moreover, assume that

$$m(\lambda) = \int\limits_{-\infty}^{\lambda} d\mu,$$

where μ is a finite measure on R^1. Then the following inequality

$$\int\limits_{R^1} |Kf(x)|^2 \frac{1}{w(x)}\, dx \leq$$

$$\leq \Bigg(\int\limits_{R^1} \frac{1}{v_2(x)|x|^{(1-\alpha)2}}\, dx \Bigg) \Bigg(\int\limits_{R^1} |f(x)|^2 \frac{1}{v_1(x)}\, dx \Bigg)$$

holds, where $w \stackrel{def}{=} v_1 * v_2$.

We intend the application to the solvability problem for equation

$$-\Delta u = |\nabla u|^p + f, \quad 1 < p < \infty,$$

to discuss.

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ON A TRACE INEQUALITY FOR ONE-SIDED POTENTIALS AND APPLICATIONS TO THE SOLVABILITY OF NONLINEAR INTEGRAL EQUATIONS

In the present note criteria governing the trace inequality

$$\int_0^\infty |R_\alpha f(x)|^p v(x)\,dx \le c_0 \int_0^\infty |f(x)|^p\,dx, \quad f \in L^p(R_+), \tag{1}$$

for the Riemann–Liouville integral operator

$$R_\alpha f(x) = \int_0^x f(y)(x-y)^{\alpha-1}\,dy, \quad x>0,$$

are establish, where $0<\alpha<1/p$. Some applications to the solvability of a certain nonlinear integral equation are presented. The following statement holds:

Theorem 1. *Let $1<p<\infty$ and let $0<\alpha<1/p$. Then inequality* (1) *holds if and only if $W_\alpha v \in L^{p'}_{\mathrm{loc}}(R_+)$ and $W_\alpha[W_\alpha v]^{p'}(x) \le cW_\alpha v(x)$ a.e. where $p'=p/(p-1)$ and*

$$W_\alpha f(x) = \int_x^\infty f(y)(y-x)^{\alpha-1}\,dy.$$

An analogous problem for the Volterra integral operator

$$Kf(x) = \int_0^x f(y)k(x,y)\,d\mu(y)$$

is discussed.

Theorem 2. *Let $1<p<\infty$, $0<\alpha<1$, and $A_p=(p'-1)(p')^{-p}$.*

(i) *If $W_\alpha v \in L^p_{\mathrm{loc}}(R_+)$ and the inequality $W_\alpha[W_\alpha v]^p(x) \le A_p W_\alpha v(x)$ a.e. holds, then the integral equation*

$$\varphi(x) = \int_x^\infty \frac{\varphi^p(t)}{(t-x)^{1-\alpha}}\,dt + \int_x^\infty \frac{v(t)}{(t-x)^{1-\alpha}}\,dt, \quad 0<\alpha<1 \tag{2}$$

has a non-negative solution $\varphi \in L^p_{\mathrm{loc}}(R_+)$. Moreover, $(W_\alpha v)(x) \le \varphi(x) \le p'(W_\alpha v)(x)$.

(ii) *If $0<\alpha<1/p'$ and* (2) *has a non-negative solution in $L^p_{\mathrm{loc}}(R_+)$, then $W_\alpha v \in L^p_{\mathrm{loc}}(R_+)$ and $W_\alpha[(W_\alpha v)^p](x) \le cW_\alpha v(x)$ a.e. for some constant $c>0$.*

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COMPARISON THEOREMS FOR DEVIATED DIFFERENTIAL EQUATIONS

Consider the equations

$$u^{(n)}(t) + \sum_{i=1}^{m} p_i(t)u(\tau_i(t)) = 0, \tag{1}$$

$$v^{(n)}(t) + \sum_{i=1}^{k} q_i(t)v(\sigma_i(t)) = 0, \tag{2}$$

where $m,\ k \in N$, $n \geq 2$, $p_i \in L_{loc}(R_+;R)$ $(i = 1,\dots,m)$, $q_i \in L_{loc}(R_+;R)$ $(i = 1,\dots,k)$, $\tau_i \in C(R_+;R_+)$, $\lim\limits_{t\to+\infty} \tau_i(t) = +\infty$ $(i = 1,\dots,m)$, $\sigma_i \in C(R_+;R_+)$, $\lim\limits_{t\to+\infty} \sigma_i(t) = +\infty$ $(i = 1,\dots,k)$. Comparison theorems of new type are established for the equations (1), (2) which enable one to obtain sufficient conditions for the equations (1) to have properties A and B.

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THE ROBBINS–MONRO TYPE SDE. ASYMPTOTIC PROPERTIES OF SOLUTION AND AVERAGING PROCEDURES

We consider the Robbins–Monro family (RM) type SDE introduced in [1], that is,

$$z_t = z_0 + \int_0^t H_s(z_{s-})\,dK_s + \int_0^t M(ds, z_{s-}),$$

where the random field $\{H_t(\omega,u),\, t \geq 0,\, u \in R^1\}$ is such that $(H_t(u))_{t\geq 0}$ is a predictable process for each $u \in R^1$ and

$$H_t(0) = 0,$$
$$uH_t(u) \leq 0 \ \text{ if } \ u \neq 0,$$

for each $t \geq 0$, P-a.s., $K = (K_t)_{t\geq 0}$ is a predictable increasing process, $M(u) = (M(t,u))_{t\geq 0} \in \mathcal{M}^2_{loc}$ for each $u \in R^1$, $\int_0^t M(ds, z_{s-})$ is the stochastic line integral w.r.t. the family $\{M(t,u),\ t \geq 0,\ u \in R^1\}$ along the predictable curve $u = (u_t)_{t\geq 0}$.

Assuming that there exists a unique strong solution $z = (z_t)_{t\geq 0}$ of this equation, the general results concerning the asymptotic behavior of $z = (z_t)_{t\geq 0}$ as $t \to \infty$ are presented. In particular, the rate of convergence $z_t \to 0$ P-a.s. is established. Moreover, it is shown that z admits an asymptotic expansion which enables one to obtain the asymptotic distribution of the randomly normed solution from a martingale limit theorems.

Further, we consider the Polyak weighted averaging procedure

$$\overline{z}_t = \mathcal{E}(-g \circ K) \int_0^t z_s\, d\mathcal{E}_s^{-1}(-g \circ K),$$

where $\mathcal{E}(-g\circ K)$ is the Dolean exponential of the process $g\circ K_t = -\int_0^t g_s\,dK_s$, $g_t \geq 0$.

The asymptotic properties of $\overline{z} = (\overline{z}_t)_{t\geq 0}$ as $t \to \infty$ are investigated.

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ON SOME NON-LOCAL PROBLEMS FOR QUASI-LINEAR HYPERBOLIC EQUATIONS WITH RECTILINEAR CHARACTERISTICS

On the x, y plane the non-local characteristic problem is considered for the quasi-linear hyperbolic second order equation

$$u_{xx} + (1 + u_x + u_y)u_{xy} + (u_x + u_y)u_{yy} = 0 \tag{1}$$

with admissible parabolic degeneracy, depending on the behavior of the first order derivatives of an unknown solution. In terms of characteristic invariants $\xi = y - x$, $\eta = y - (u_x + u_y)x$ equation (1) has the explicit representation of solutions

$$u(\xi, \eta) = f(\xi) - g(\eta) + (\xi - \eta)\big[1 - g'(\eta) + e^{g'(\eta)}\big] \tag{2}$$

with arbitrary functions f, g.

For equation (1) the following non-local characteristic problem is posed and investigated: define the regular solution $u(x, y)$ of equation (1) simultaneously with its domain of definition under the conditions

$$(u_x + u_y)\big|_{y=0} = \alpha(x), \quad u(x, 0) + \gamma(x)u\Big(\frac{\alpha(x)x}{\alpha(x) - 1}, \frac{\alpha(x)x}{\alpha(x) - 1}\Big) = \beta(x), \tag{3}$$

where $\alpha \in C^1[0, b]$, β, $\gamma \in C^2[0, a]$, $a = \frac{\alpha(b)b}{\alpha(b)-1}$ are the given functions. The following theorem is proved.

Theorem. *If*

$$\alpha(x) > 1, \;\; x \in [0, b], \quad \alpha(x) + (\log \alpha(x))'(y - x) \neq 1, \;\; x, y \in [0, a], \tag{4}$$

then problem (1), (3) *has the unique solution defined in the triangle, bounded by supports of the problem data and passing through its endpoints characteristic* $y = \alpha(b)(x - b)$. *In the case under consideration the parabolic degeneracy is excluded by conditions* (4).

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ON A PERIODIC BOUNDARY VALUE PROBLEM FOR THE TWO-DIMENSIONAL LINEAR DIFFERENTIAL SYSTEM

The differential system

$$u_i'(t) = p_{ii}(t)u_i(t) + p_{i,3-i}u_{3-i}(t) + q_i(t) \ \ (i = 1, 2) \tag{1}$$

is considered with the periodic boundary conditions

$$u_i(w) = u_i(0) \ \ (i = 1, 2), \tag{2}$$

where $p_{ik} : [0, w] \to R$ $(i, k = 1, 2)$ are Lebesgue integrable functions, $w > 0$.

Let

$$p_i(t) = p_{i,3-i}(t) \exp\left(\int_0^t (p_{3-i,3-i}(s) - p_{ii}(s))\, ds\right) \ \text{ for } \ 0 \leq t \leq w,$$

$$\lambda_i = \exp\left(-\int_0^w p_{ii}(s) ds\right) \ \ (i = 1, 2), \quad l = \int_0^w |p_1(s)|\, ds \int_0^w |p_2(s)|\, ds,$$

$$\alpha_1 = \min\{1, \lambda_1\lambda_2\}, \quad \alpha_2 = \max\{1, \lambda_1\lambda_2\},$$

$$\beta_1 = \min\{\lambda_1, \lambda_2\}, \quad \beta_2 = \max\{\lambda_1, \lambda_2\}.$$

Then the following statemants are valid.

Theorem 1. *Let*

$$l > 0, \quad l^{-1}(\lambda_1 - 1)(\lambda_2 - 1) \notin]\alpha_1, \alpha_2[$$

and $\sigma \in \{-1, 1\}$ *be such that*

$$\sigma p_{12}(t) \geq 0, \quad \sigma p_{21}(t) \leq 0 \ \text{ for } \ 0 \leq t \leq w.$$

Then the problem (1), (2) *has one and only one solution.*

Theorem 2. *Let* $\sigma \in \{-1, 1\}$ *be such that*

$$\sigma p_1(t) \geq 0, \quad \sigma\left(\int_t^w p_2(s)\, ds + \frac{\lambda_2}{\lambda_1}\int_0^t p_2(s)\, ds\right) < 0 \ \text{ for } \ 0 \leq t \leq w$$

and

$$0 < \int_0^w |p_1(s)|\, ds \int_0^w [\sigma p_2(s)]_-\, ds \leq 16\, \frac{\beta_1}{\beta_2}\,.$$

Let, moreover,

$$\int_0^w (p_{22}(s) - p_{11}(s))\, ds \int_0^w p_{11}(s)\, ds \geq 0.$$

Then the problem (1), (2) *has one and only one solution.*

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PLURIREGULAR, PLURIGENERALIZED REGULAR EQUATIONS IN CLIFFORD ANALYSIS

The Dirac operator is a powerful tool for the Clifford analysis. It is well-known that the classical equations of mathematical physics, electromagnetic fields, relativistic quantum mechanics and their generalizations are obtained by using the Dirac operator without application of any physical laws. In our paper new higher order equations are obtained by using the Clifford analysis. They have some mathematical beauty and can have important applications.

The pluriregular, plurigeneralized regular and pluri-Beltrami equations

$$\overline{\partial}^m u(x) = 0, \tag{1}$$

$$P^m u(x) = 0, \quad Pu = \overline{\partial} u + \widetilde{u} h, \tag{2}$$

$$(\overline{\partial} + q\partial)^m u = 0, \ \ |q| \neq 1, \ \ m \geq 2, \ \ x(x_0, \dots, x_n) \tag{3}$$

are considered, which are elliptic in $R(n)$, hyperbolic in $R(n, n-1)$ and pluriparabolic in $R^0(n)$.

If $u(x)$ is the solution of (1) or (2), then it will be the solution of a poly-harmonic or poly-Helmholtz equation in $R(n)$ and poly-wave or poly-Klein-Gordon equation in $R(n, n-1)$, supposing $h = \text{const}$, $\Delta^m u = 0$, $(\Delta - |h|^2)^m u = 0$,

$$\Big(\Delta - \frac{\partial^2}{\partial t^2}\Big)^m u(x) = 0, \quad \Big(\Delta - |h|^2 - \frac{\partial^2}{\partial t^2}\Big)^m u(x) = 0, \quad x_n \equiv t.$$

In a parabolic case we obtain the poly-heat equation

$$\Big(\Delta - \frac{\partial}{\partial t}\Big)^m u(x) = 0.$$

Moreover, in the Clifford analysis the higher order equations of new type are considered; they are connected with the following equations:

$$\Delta\Big(\Delta - \frac{\partial}{\partial t}\Big) u(x,t) = 0, \ \ x(x_0, \dots, x_{n-1}), \quad x_n \equiv t > 0,$$

$$\Delta\Big(\Delta - \frac{\partial^2}{\partial t^2}\Big) u(x,t) = 0,$$

$$\Big(\Delta - \frac{\partial^2}{\partial t^2}\Big)\Big(\Delta - \frac{\partial}{\partial t}\Big) u(x,\tau,t) = 0, \ \ x(x_0, \dots, x_{n-2}), \quad x_{n-1} \equiv \tau,$$

$$\Delta\Big(\Delta - \frac{\partial^2}{\partial t^2}\Big)\Big(\Delta - \frac{\partial}{\partial t}\Big) u(x,\tau,t) = 0.$$

The above equations are called harmonic-heat, harmonic-wave, wave-heat and harmonic-wave-heat equations, respectively.

For these equations the boundary, initial and boundary-initial value problems are considered, some of them are solved in quadratures.

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BOUNDARY VALUE PROBLEMS FOR TWO-DIMENSIONAL DELAYED DIFFERENTIAL SYSTEMS

The question on the solvability of the boundary value problem

$$\frac{du_i(t)}{dt} = f_i\big(t, u_1(\tau_1(t)), u_2(\tau_2(t))\big) \ \ (i=1,2), \tag{1}$$

$$u_1(a) = \alpha u_2(a), \quad \beta_1 u_1(b) + \beta_2 u_2(b) = \gamma \tag{2}$$

is investigated. Here $f_i : [a,b] \times \mathbb{R}_+^2 \to \mathbb{R}_+$ and $\tau_i : [a,b] \to [a,b]$ $(i=1,2)$ are continuous functions, $\mathbb{R}_+ = [0, +\infty[$, $\alpha \in \mathbb{R}_+$, $\beta_i \in \mathbb{R}_+$ $(i=1,2)$, $\beta_1 + \beta_2 > 0$, $\gamma > 0$, and

$$\tau_i(t) \le t, \quad f_i(t,0,0) = 0 \ \text{ for } \ a \le t \le b \ \ (i=1,2).$$

In particular, the following theorem is proved.

Theorem. *For the solvability of problem* (1), (2) *it is sufficient one of the following five conditions to be fulfilled:*

(i) $\beta_1 > 0$, $\beta_2 > 0$;

(ii) $\beta_1 = 0$, $\beta_2 = 1$, *and there exists a positive constant* ℓ *such that*

$$f_1(t,x,y) \le \ell\big[1 + f_2(t,x,y)\big] \ \ \textit{for} \ \ a \le t \le b, \ \ x \ge 0, \ \ 0 \le y \le \gamma;$$

(iii) $\beta_1 = 0$, $\beta_2 = 1$, *and*

$$\tau_1(t) < t \ \ \textit{for} \ \ a < t \le b;$$

(iv) $\alpha > 0$, $\beta_1 = 0$, $\beta_2 = 0$, *and there exists a positive constant* ℓ *such that*

$$f_2(t,x,y) \le \ell\big[1 + f_1(t,x,y)\big] \ \ \textit{for} \ \ a \le t \le b, \ \ 0 \le x \le \gamma, \ \ y \ge 0;$$

(v) $\alpha > 0$, $\beta_1 = 1$, $\beta_2 = 0$, *and*

$$\tau_2(t) < 0 \ \ \textit{for} \ \ a < t \le b.$$

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ON THE STRUCTURE OF THE SET OF SOLUTIONS OF THE WEIGHTED CAUCHY PROBLEM FOR HIGH ORDER EVOLUTION SINGULAR FUNCTIONAL DIFFERENTIAL EQUATIONS

In the paper there is investigated the structure of the set of solutions of the Cauchy problem for the weighted initial condition

$$u^{(n)}(t) = f(u)(t),$$

$$\lim_{t\to a} \frac{u^{(k)}(t)}{h^{(k)}(t)} = 0 \quad (k = 0, \dots, n-1),$$

where $f : C^{n-1}([a,b];\mathbb{R}^{>}) \to \mathbb{L}_{\lessdot\rtimes}(]\eth,];\mathbb{R}^{>})$ is a continuous Volterra operator and $h : [a,b] \to [0,+\infty[$ is an $(n-1)$-times continuously differentiable function such that

$$h^{(k)}(a) = 0 \quad (k = 0, \dots, n-2), \quad h^{(n-1)}(t) > 0 \ \text{ for } \ a < t \le b.$$

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VARIATION FORMULAS OF SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH DEVIATING ARGUMENTS AND THEIR APPLICATION TO OPTIMAL CONTROL PROBLEMS

For various classes of differential equations with deviating arguments with a continuous and discontinuous initial conditions the variation formulas of solutions are obtained. The continuous (discontinuous) initial condition means that values of an initial function and a trajectory always coinside (generally speaking, do not coinside) at an initial moment.

For optimal problems with non-fixed initial moment and with continuous and discontinuous initial conditions the necessary conditions of optimality are proved.

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OPTIMIZATION OF LINEAR CONTROLLED SYSTEMS WITH RESTRICTIONS OF "NARROW PLACE" TYPE

Consider an industrial process (see [1]), which is described with the linear system of differential equations

$$\dot{x}_1 = u_1, \quad \dot{x}_2 = u_2, \quad \dot{x}_3 = u_3 - b_1 u_1 - b_2 u_2 - b_4 u_4, \quad \dot{x}_4 = u_4 \tag{1}$$

under the initial conditions

$$x_1(0) = c_1, \quad x_2(0) = c_2 > 0, \quad x_3(0) = c_3, \quad x_4(0) = c_4 > 0, \tag{2}$$

where $x_1(t)$ is the reserve of motor-car in moment t, $x_2(t)$ is the industrial power of motor-car branch in t, $x_3(t)$ is the reserve of steel in moment t, $x_4(t)$ is the industrial power of steelfoundry branch in moment t, b_i, c_i are given constants and $u_i = u_i(t)$, $i = \overline{1,4}$, are the control functions.

We consider the optimal control problem for this process in the form:

$$x_1(T) \to \operatorname{Sup} \tag{3}$$

under the restrictions

$$u_1 - x_2 \le 0, \quad u_3 - x_4 \le 0, \quad u_{j-2} \le 0, \;\; j = 3,4,5,6, \quad x_3 > 0 \tag{4}$$

and conjunctions (1), (2). Here T is fixed, u_1 is a speed of motor-car production, u_2 is a speed of industrial power rising of motor-car branch, u_3 is a speed of steelproduction, u_4 is a speed of industrial power rising of steelfoundry branch. In different from general problems, which are considered in common theory from [2], the given problem with restrictions of "narrow place" type is characteristed.

Let $x_i(t)$, $i = \overline{1,4}$, be absolutely continuous, $u_i(t)$, $i = \overline{1,4}$, be integrable functions on $[0,T]$, and the conditions (1), (4) be fulfilled almost everywhere on $[0,T]$, then using the Theorem 1 from [3], we obtain the necessary conditions of optimality for problem (1)–(4).

Detailed analysis of the behaviour of system (1)–(4) when satisfying the necessary conditions of optimality enables one to optimize this system for different corelations between parameters b_1, b_2, b_4 and T. For example, if $T < b_4$, then the optimal is such a behaviour of the system at which the industrial power of both branches remain constant and are used maximally.

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SECTION II

MATHEMATICAL PHYSICS

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INTERFACE SCATTERING PROBLEMS

We consider two-dimensional transmission problems for the Helmholtz equations in non-locally perturbed half-planes Ω_1 and Ω_2 having a common boundary which is a one-dimensional, infinite, smooth, rough interface line. These type of mathematical problems model time-harmonic electromagnetic and acoustic scattering by a penetrable unbounded obstacle in an inhomogeneous (piecewise homogeneous) medium. In both domains we look for scattered waves corresponding to different wave numbers and satisfying certain transmission conditions on the interface. In addition, the scattered waves satisfy the so-called *upward* and *downward propagating radiation conditions* along with some growth conditions in the x_2 direction, suggested by Chandler-Wilde & Zhang [CWZ1], which generalize both the *Sommerfeld radiation condition* and the *Rayleigh expansion condition* for diffraction gratings.

In this paper we first prove the uniqueness theorem for the interface problem provided that an obstacle Ω_1 represents a lossy medium, which means that the corresponding wave number is complex. Afterwards we apply the potential method to reduce the non-homogeneous interface problem to the corresponding system of integral equations and establish the existence results on the basis of the theory developed in [Ar1] for a class of systems of second kind integral equations on unbounded domains.

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THE CONTACT PROBLEM FOR AN ANISOTROPIC WEDGE-SHAPED PLATE WITH AN ELASTIC STRENGTHENING OF VARIABLE RIGIDITY

We consider plane-strassed state of an infinite elastic anisotropic wedge, strengthened over one boundary by an elastic bar whose rigidity varies by the law: x^α (x is the distance from the wedge vertex, α is any real number). Tangential load of intensity $\tau_0(x)$ is applied along the bar.

The problem is reduced to the Carleman type problem for a strip with the boundary condition

$$G(t)\Psi(t) - H\Psi(t - i(\alpha - 1)) = F(t), \quad -\infty < t < \infty,$$

where $G(t)$ and $F(t)$ are the given meramorphic functions and H is a constant.

The problem is solved in a closed form, the contact stress along the line of strengthening in the vicinity of the angle vertex admits the estimate

$$\tau(x) - \tau_0(x) = \begin{cases} O(x^{\alpha-2}), & \alpha > 2, \\ O(x^{\tau_0 - 1}), & 1 < \alpha \leq 2, \\ O(x^{\tau_1 - 1}), & \alpha \leq 1, \end{cases}$$

where $\tau_0 = \operatorname{Im} z_0$ and $\tau_1 = \operatorname{Im} z_1$, z_0 is the nearest to the real axis pole of the function $G(z)$ in the lower half-plane, z_1 is the nearest to the real axis zero of the function $G(z)$ (for $\alpha = 1$, of the function $G(z) - H$) in the lower half-plane.

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ON DYNAMIC PROBLEMS OF GENERALIZED ELECTROELASTICITY

The talk deals with the dynamic boundary value problems for a homogeneous anisotropic piezoelectric body when the vector of polarization is taken into account (R. Toupin's model of piezoelctric medium). Two types of boundary conditions are considered: when on bounday vector of displacement and electric potential or the vector of mechanical stresses and boundary charges are given.

Dynamic problems are reduced to the associated with them problems of pseudo-oscillations by mean the Laplace transform. Using the potential theory and the theory of pseudodifferential equations these problems are investigated, the existence and uniqueness theorems are proved and the asymptotic properties of solutions are established.

Finally, employing the inverse Laplace transform we prove the existence and uniqueness of solutions of dynamic problems.

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ON THE ASYMPTOTIC METHOD OF SOLUTION OF ONE CLASS OF NONLINEAR PROBLEMS OF MATHEMATICAL PHYSICS

This work proposes an asymptotic method of solution for a system of nonlinear nonhomogeneous equations of one class of mixed problems with an unknown external boundary in the domain. The system of equations describes an adiabatic spherical and symmetrical motion of a gravitating gas, while a moving detonation wave (a spherical surface, where the solution undergoes the first kind discontinuity) is the external boundary of the domain.

In particular, the nonautomodel problem of a central explosion followed by a thermonuclear detonation of a nonhomogeneous, bounded with vacuum, gas sphere which is balanced in its own gravitating field, is discussed as the test problem.

The asymptotic method of a thin shock layer is used for the solution of the mixed problem for a system of nonlinear nonhomogeneous equations. Analytical formulas of the first two approximations for the motion law and the thermodynamic characteristics of the medium are calculated. For the zero approximation of the detonating wave motion layer the Cauchy problem is solved exactly in particular case and numerically in the general case. Interpolation formulas and the corresponding asymptotics are found.

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CYLINDRICAL BENDING OF SHELLS WITH CUSPED EDGES

The main purpose of this paper is to study the Initial Boundary Value Problems (IBVPs) for an elastic cusped prismatic shell (see [1], [2]), with the projection $\Omega = \{(x_1, x_2): \ -\infty < x_1 < \infty, \ 0 \le x_2 \le \ell\}$.

The equation of the cylindrical bending of a prismatic shell has the form

$$\big(D(x_2) w,_{22}(x_2,t)\big)_{22} = q(x_2,t) - 2\rho h(x_2)\,\frac{\partial^2 w(x_2,t)}{\partial t^2}\,, \quad 0 \le x_2 \le \ell, \tag{1}$$

where $w(x_2,t)$ is a deflection of the shell, $q(x_2,t)$ is an intensivity of a load, ρ is a density, $2h(x_2) = h_0 x_2^{\alpha/3}(\ell - x_2)^{\beta/3}$, $h_0, \alpha\,\beta = \mathrm{const} > 0$, is the thickness of the shell, $D(x_2)$ is a flexural-rigidity (see [3]).

In the case under consideration, bending moment and interecting force have the following forms $M_2(x_2,t) := -D(x_2)w,_{22}(x_2,t)$, $Q_2(x_2,t) := M_{2,2}(x_2,t)$.

At the points 0 and ℓ all above quantities are defined as the corresponding limits when $x_2 \to 0_+$ and $x_2 \to \ell_-$. On the cusped edge $x_2 = 0$ ($x_2 = \ell$) admissible are only four different pairs of the boundary data (see [2]).

Let $w(\cdot,t) \in C^4(]0,\ell[)$, $w(x_2,\cdot) \in C^1(t \ge 0) \cap C^2(t > 0)$, $w(x_2,t) \in C(t \ge 0,\ 0 \le x_2 \le \ell)$.

$$\begin{matrix} w(\cdot,t) \\ (w,_2(\cdot,t)) \end{matrix} \quad \begin{cases} \in C([0,\ell]) \ \text{ when } \ 0 \le \alpha,\beta < 2, \ \ (0 \le \alpha,\beta < 1) \\ \begin{cases} \in C([0,\ell[), & \text{when } \ 0 \le \alpha < 2, \ \ \beta \ge 2, \\ = O(1), \ \ x_2 \to \ell_-, & (0 \le \alpha < 1, \ \ \beta \ge 1) \end{cases} \\ \begin{cases} \in C(]0,\ell]), & \text{when } \ \alpha \ge 2, \ \ 0 \le \beta < 2, \\ = O(1), \ \ x_2 \to 0_+, & (\alpha \ge 1, \ \ 0 \le \beta < 1) \end{cases} \end{cases} \tag{2}$$

We assume that initial datas are satisfying same boundary conditions, which we have for w, and they are from the class (2).

Equation (1) under usual initial and above mentioned admissible boundary conditions has a unique solution. The IBVPs can be reduced to the integro-differential equations with symmetric kernel, which we solve by the Fourier method. The convergence of appropriate series are proved.

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SOLVABILITY AND ASYMPTOTICS OF SOLUTIONS OF CRACK-TYPE BOUNDARY-CONTACT DYNAMIC PROBLEMS OF ELASTICITY THEORY

The present paper is devoted to the investigation of solvability and complete asymptotics of solutions of boundary-contact dynamic problems with cracks of elasticity theory for homogeneous anisotropic media.

Theorems of the existence and uniqueness of solutions of these boundary-contact problems are obtained by using the potential theory, Laplace transform and the general theory of pseudodifferential equations on manifold with boundary.

The complete asymptotics are studied in near the contact boundaries. The properties of exponents of the first terms of asymptotic expansion of solutions of the above-mentioned problems are established and effective formulas for their calculation are found. In particular, we have found a class of bodies when the oscillation in the expansion vanishes and hence the asymptotic expansion of solutions describes a real physical process.

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STABILITY OF THE PROJECTION-ITERATIVE METHOD FOR ONE CLASS OF SINGULAR INTEGRAL EQUATIONS

The singular integral equation

$$(a + bS + K)\varphi = f \tag{1}$$

is considered, where S is the singular integral operator

$$S\varphi \equiv \frac{1}{n}\int\limits_{-1}^{1} \frac{\varphi(t)}{t - x}\, dt, \quad -1 < x < 1,$$

K is the Fredholm integral operator, a and b are the real constants, $a^2 + b^2 = 1$.

The problem on an approximate solution of equation (1) has been considered by many authors from Georgia, from the Republics of the Former Soviet Union, as well as from the USA, Germany and Greece.

Equation (1) is studied in the weight spaces; we consider three values of the index $\varkappa = 1, -1, 0$.

For $\varkappa = 1$, the additional condition

$$\int\limits_{-1}^{1} \varphi(t)\, dt = p \tag{2}$$

is prescribed, where p is the given real number.

For $\varkappa = -1$, the scalar product in the corresponding weight space $L_2\rho[-1, 1]$ is equal to zero

$$[K\varphi - f, 1] = 0, \tag{3}$$

where φ is the solution of equation (1).

For $\varkappa = 0$, the uniform equations $(a + bS)u = 0$ and $(a + bS)^*u = 0$ have only trivial solution $u = 0$ in the weight space.

D. Porter and D. Stirling proposed (IMA J. Numer. Analysis 13, 1993) the projection-iterative scheme for the second kind equation

$$(I + T)u = f,$$

where T is the completely continuous operator.

In my report the results of investigation of stability of the projection-iterative scheme for equation (1) will be stated.

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A MAXIMUM PRINCIPLE IN THE VARIATIONAL INEQUALITIES

In the present paper a week maximum principle for a bilinear, elliptic, nonsymmetric and coercive form, with the L^∞ coeficients is given. The result is applied to prove the maximum principle for the variational inequalities.

Let $f \in L_2(\Omega)$, $h \in H^1(\Omega)$, $H^1(\Omega)$ be the real Sobolev space, $K_1 = \{ v \in H^1(\Omega) : \ v \geq h\}$, and $a(u,v)$ be the above-mentioned bilinear form. The problem is to find $u \in K_1$, such that

$$a(u, v-u) \geq \int_\Omega f \cdot (v-u)dx, \ \ \forall v \in K_1. \tag{1}$$

In some restrictions there is proved the equality es sup$u$=es sup$h$ for this inequality. This fact give us an opportunity to locate the set of coincidence ($u = h$) for some concrete cases. Inequality (1) is also considered on the set $K_2 = \{v \in H^1(\Omega) : \ \phi \geq v \geq \psi\}$, with ϕ, $\psi \in H^1(\Omega)$. Based on the above week maximum principle, for some cases there is proved the equalities es sup$u$=es supψ and es inf$u$=es infϕ.

With the help of the maximum principle, written for the variational inequality in the set K_1, we prove the following result

$$a(u-u_\lambda, \Psi) \geq 0, \ \ \forall \Psi \in H^1(\Omega), \ \ \Psi \geq 0. \tag{2}$$

Here $\lambda \in R^+$, u and u_λ are the solutions of the inequality (1) in the sets K_1 and $K_1^\lambda = \{v \in H^1(\Omega) : \ v \geq h - \lambda\}$.

Finally, inequality (1) is considered with the implicit obstacle, i.e., in the set $K_3 = \{v \in H^1(\Omega) : \ v \geq h - a(u, \Phi)\}$, with $\Phi \in H^1(\Omega)$, $\Phi \geq 0$. With the help of (2), the unique solvability of this problem is proved.

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A GENERAL REPRESENTATION FORMULA FOR A SOLUTION OF THE SYSTEM OF STATICS OF ELASTIC MIXTURES

The system of differential equations of statics of the linear theory of elastic mixtures (in the case of two-dimensions) reads as follows

$$\begin{aligned} &a_1\Delta u' + b_1 \operatorname{grad}\operatorname{div} u' + c\Delta u'' + d \operatorname{grad}\operatorname{div} u'' = 0,\\ &c\Delta u' + d \operatorname{grad}\operatorname{div} u' + a_2\Delta u'' + b_2 \operatorname{grad}\operatorname{div} u'' = 0, \end{aligned} \tag{1}$$

where $u' = (u_1', u_2')$ and $u'' = (u_1'', u_2'')$ are the partial displacement vectors,

$$a_1 = \mu_1 - \lambda_5, \ \ a_2 = \mu_2 - \lambda_5, \ \ c = \mu_3 + \lambda_5, \ \ b_1 = \mu_1 + \lambda_5 + \lambda_1 - \frac{\rho_2}{\rho}\alpha_2',$$

$$b_2 = \mu_2 + \lambda_2 + \lambda_5 + \frac{\rho_1}{\rho}\alpha_2', \ \ d = \mu_3 + \lambda_3 - \lambda_5 - \frac{\rho_1}{\rho}\alpha_2', \ \ \alpha_2' = \lambda_3 - \lambda_4,$$

$\rho = \rho_1 + \rho_2$, ρ_1 and ρ_2 are the partial densities, $\lambda_1, \lambda_2, \dots, \lambda_5$, μ_1, μ_2, μ_3 are elastic constants.

The following result is proved: a solution to the system (1) can be represented by means of four harmonic functions

$$\begin{aligned} u'(x) &= \operatorname{grad}\Phi_1(x) + \\ &+ r^2 \operatorname{grad}\Big\{\Big[\Big(\alpha_1 + \frac{1}{2}\Big)\Big(r\frac{\partial}{\partial r} + 2\Big) - 1\Big]\Phi_2(x) + \beta_1\Big(r\frac{\partial}{\partial r} + 2\Big)\Phi_3(x)\Big\} - \\ &- xr\frac{\partial}{\partial r}\big[(2\alpha_1 - 1)\Phi_2(x) + 2\beta_1\Phi_3(x)\big] + \chi'(x),\\ u''(x) &= \operatorname{grad}\Phi_4(x) + \\ &+ r^2 \operatorname{grad}\Big\{\alpha_2\Big(r\frac{\partial}{\partial r} + 2\Big)\Phi_2(x) + \Big[\Big(\beta_2 + \frac{1}{2}\Big)\Big(r\frac{\partial}{\partial r} + 2\Big) - 1\Big]\Phi_3(x)\Big\} - \\ &- xr\frac{\partial}{\partial r}\big[2\alpha_2\Phi_2(x) + (2\beta_2 - 1)\Phi_3(x)\big] + \chi''(x), \end{aligned}$$

where $\Delta\Phi_j(x) = 0$, $j = 1, 2, 3, 4$, $r\frac{\partial}{\partial r} = (x \cdot \operatorname{grad})$,

$$\chi'(x) = \begin{cases} A_0x + B_0\widetilde{x}, & x \in \Omega^+,\\ 1/r^2(A_0x + B_0\widetilde{x}), & x \in \Omega^-, \end{cases} \quad \chi''(x) = \begin{cases} C_0x + D_0\widetilde{x}, & x \in \Omega^+,\\ 1/r^2(C_0x + D_0\widetilde{x}), & x \in \Omega^-, \end{cases}$$

$x = (x_1, x_2)$, $\widetilde{x} = (-x_2, x_1)$, A_0, B_0, C_0, and D_0 are arbitrary constants, Ω^+ is a bounded domain, $\Omega^- = \mathbb{R}^2 \setminus \overline{\Omega^+}$,

$$\alpha_1 = \frac{1}{2\Delta_1}(cd - b_1a_2 - \Delta_1), \ \ \beta_1 = \frac{1}{2\Delta_1}(cb_2 - a_2d),$$

$$\alpha_2 = \frac{1}{2\Delta_1}(cb_1 - a_1d), \ \ \beta_2 = \frac{1}{2\Delta_1}(cd - a_1b_2 - \Delta_1), \ \ \Delta_1 = a_1a_2 - c^2.$$

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ON THE PROBLEM OF LINEAR CONJUGATION IN THE CASE OF NON-SMOOTH LINES

1. Let S_Γ be a singular operator, $S_\Gamma\varphi = (\pi i)^{-1} \int\limits_\Gamma \varphi(t)(t-\tau)^{-1}\,dt$. If S_Γ is bounded in $L_p(\Gamma)$, then we write $\Gamma \in R$. If the operator ρS_ρ is also bounded in $L_p(\Gamma)$, where $\rho(t)$ is a positive measurable function, then we write $\rho \in W_p(\Gamma)$. Denote by Γ_t a continuous on Γ arc (which open if t is an exterior point, and closed if Γ is unclosed, and t is one of the end points), such that $t \in \Gamma_t$.

Theorem. *If Γ is the Jordan line, $\Gamma \in R$, and for every $t \in \overline{\Gamma}$ there exists Γ_t such that $\rho \in W_p(\Gamma_t)$, then $\rho \in W_p(\Gamma)$.*

2. Consider the boundary value problem of linear conjugation in the Hardy–Smirnov class $E_p(D_\Gamma^\pm)$, $p > 1$. Suppose that $\Gamma \in R$, Γ is the Jordan line and $\Gamma = \bigcup\limits_{k=1}^{n} \Gamma_{a_k a_{k+1}}$, $a_{n+1} = a_1$ (Γ_{ab} denotes the arc with the ends a and b which is continuous from a to b). Assume that $\Gamma_{a_k a_{k+1}}$ can be supplimented up to the closed contour $\Gamma_k^0 \in R$ for which the problem under Simonenko's conditions with the zero index has the unique solution. Contours of the type Γ_k^0 have been considered by many authors, however, the existence of cusps was exclued everywhere. It is evident that the cusps and more complicated cases may occur for the lines $\Gamma = \bigcup\limits_{k=1}^{n} \Gamma_{a_k a_{k+1}}$ at the points a_k. We introduce obvious enough additional restrictions on $G(t)$ in the neighborhood of a_k which allow one to obtain the classical result for the above-mentioned lines. As an example, we can name both the piecewise smooth lines and the Radon lines.

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ON I. VEKUA DIMENSIONAL-REDUCTION METHOD OF REDUCING THE MULTI-DIMENSIONAL PROBLEMS OF MATHEMATICAL PHYSICS ON LOWER-DIMENSIONAL ONES

The present work is devoted to the application and investigation of a dimensional-reduction method for some problems of linearized elastodynamics. I. Vekua's method of reduction of three-dimensional problems to two-dimensional ones is used.

Vekua's method has rather general nature and it can be applied not only to dynamical problems of elasticity theory. It may be considered as one of the ways of reduction the multi-dimensional dynamical problems of mathematical physics to dynamical problems of lower dimension.

We have considered these problems in Sobolev spaces; we have also considered the problems of convergence and estimate accuracy of the obtained two-dimensional dynamical model.

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BOUNDARY VALUE PROBLEMS FOR ELASTIC BODIES WITH SINGULAR BOUNDARIES

The paper deals with elastic prismatic shell or beam type three-dimensional bodies. The boundary of the projection of the shell is a Lipschitz curve while, in general, a boundary of the body is not a Lipschitz surface. Boundary value problems (BVPs) of three-dimentional theory of elastic bodies with a Lipschitz boundary are studied completely (see e.g. [1], [2]). The main purpose of this paper is to study BVPs for bodies under consideration (especially for the so called cusped prismatic shells and beams [3], [4]) by means of approximate two- and one-dimensional hierarchic models which was constructed by I. Vekua in case of shells [5] and then developed by many authors (for references see [4]). The presented results are mostly obtained within the framework of NATO SCIENCE PROGRAMME for 2000 and 2001 (NATO PST. CLG. 976426/5437, Principal investigators: W. L. Wendland, G. Jaiani, D. Natroshvili, S. Kharibegashvili). The existence and uniqueness theorems for approximate models, in appropriate weighted Sobolev spaces are proved for the admissible Dirichlet, Keldysh's type, and weighted BVPs. The convergence of the approximate solutions of some BVPs to the solution of the original three-dimentional problem is established in the case of a Lipschitz boundary.

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ON ONE PROBLEM OF THE BENDING OF A PLATE FOR A FINITE DOUBLY-CONNECTED DOMAIN WITH A PARTIALLY UNKNOWN BOUNDARY

A problem of the bending of an isotropic elastic plate is studied for a finite doubly-connected domain bounded by two rectangles of common center whose vertices lie on the same beam drawn from the center. The vertices of the internal rectangle are cut out by convex smooth arc (the union of these arcs is shown for the unknown part of the boundary). It is assumed that each rectilinear segment of the boundary contours is supported by a rigid bar and the plate is bent by normal moments applied to the bars so that the rotation angles of the plate middle surface take piecewise-constant values.

The problem consists in determining the bending of the plate middle surface, as well as the analytic form of the unknown part of the boundary, where it is assumed that the tangential normal moment has a constant value.

Using the method of complex analysis, the solution of the posed problem is constructed effectively (in analytic form).

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SAINT-VENANT'S PROBLEMS FOR COMPOSED BODIES BOUNDED BY ELLIPTIC SURFACES

In this paper the problems indicated in the titlle for homogeneous, two-layered isotropic elliptic tube and for composed isotropic elliptic tube with anisotropic kernel are solved with help of Faber's polynomials.

Although analytical solutions of Saint-Venants (S-V) problems for cylindrical (prysmatic) bodies with several crossections already exist, it will be interesting to consider (S-V) problems for the elliptic tubes, from a solution of which, as particular case, a crack's problem may be investigated for a body by an elliptic crossection with a split along a focal length. It should be noted that, although Faber's polynomials are determined for simply connected domains (in particular for an elliptic domain) they turned out to be applicable also to an elliptic ring.

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SOME PROBLEMS OF OSCILLATION AND STABILITY OF ROTATION SHELLS CLOSE TO CYLINDRICAL ONES

Free oscillations and stability of rotation shells whose shape is close to that of cylindrical ones are considered. It is assumed that preliminarily the shell is subjected to the action of normal pressure. The considered shells are of medium length. The shape of the generatrix of the shell middle surface is described by a sufficiently smooth function of constant sign. Low frequencies, which are practically the most important ones, are considered. The respective equations and boundary conditions are investigated and written in a simpler form. For the case, in which the generatrix of the surface middle part changes by the parabolic law, the given equation, as different from the known one, contains an additional term that may have the same order as the other terms taken into consideration. The considered shells are both of positive and negative Gaussian curvature. Cases are investigated, in which both edges of the shell are freely supported, rigidly supported, and one edge is freely supported, while the other is rigidly supported. Formulas and graphs are given for determining minimal frequencies and wave formation forms depending on a type of boundary conditions, prestress and an amplitude of deviation of the shell from the cylinder (of thickness order). Formulas are given for determining critical load. Calculations are performed. A different degree of the influence of boundary conditions on a minimal frequency and critical load is shown for shells of positive and negative Gaussian curvature.

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ON RIEMANN–HILBERT PROBLEM FOR PLANE WITH CUTS

An explicit solution to Riemann-Hilbert problem for plane with two cuts is given.

Two cases are considered: the case of two straight cuts and the case with an arc-wise and straight cuts with a certain symmetry with respect to each other.

The solutions are constructed in Cauchy type integrals.

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ON ONE I. N. VEKUA METHOD OF CONSTRUCTING THE NON-LINEAR THEORY OF NON-SHALLOW SHELLS

In his studies I. N. Vekua, by means of the method of reduction of three-dimensional problems of elasticity to two-dimensional ones, several versions of the refined linear theory of thin and shallow shells, containing the regular process are constructed.

Under thin ans shallow shells I. Vekua meant three-dimensional shell-type elastic bodies satisfying the conditions

$$a_\alpha^\beta - x_3 b_\alpha^\beta \cong a_\alpha^\beta, \quad -h(x_1,x_2) \leq x_3 \leq h(x_1,x_2) \ \ (\alpha,\, \beta = 1,2), \qquad (*)$$

where a_α^β and b_α^β are mixed components of the metric tensor and tensor of midsurface curvature of the shell, x_3 is the thickness coordinate and h is the semi-thickness, depending on curvilinear coordinates x_1, x_2.

The assuption of the type $(*)$ means that the interior geometry of the shell does not vary in thickness and therefore such kind of shells are usually called the shells with non-varying geometry.

In the sequel, under non-shallow shells will be meant elastic bodies free from the assumption of the type $(*)$, or more exactly the bodies with the conditions

$$a_\alpha^\beta - x_3 b_\alpha^\beta \neq a_\alpha^\beta, \quad |x_3 b_\alpha^\beta| < 1.$$

Such kind of shells are called shells with varying in thickness geometry, or non-shallow shells.

In the present paper by means of the method of Vekua using the results obtained by Muskhelishvili and his disciples the author has obtained the system of differential equations for the non-linear theory of non-shallow shells.

One important class of three-dimensional shell-like types of spherical bodies are considered when the vector displacement doesn't depend on the thickness coordinate. In this case general representations of the components of vector displacement are expressed by means of three holomorphic functions of z and some practical probelms of stress concentration are solved.

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SOME PROBLEMS OF THE PLANE THEORY OF ELASTICITY FOR DOMAINS WITH PARTIALLY UNKNOWN BOUNDARIES

Let a homogeneous isotropic elastic plate occupy on the plane $z = x + iy$ a domain S whose boundary is a set of linear segments and unknown smooth arcs.

It is required to determine the stressed state and an equirigid part of the boundary assuming that, on each linear segment, normal displacement takes a constant value, tangential stress is equal to zero all along the boundary, and constant normal stress is given on the unknown contours. In the case of an infinite domain, a constant stress field is assumed to be acting at infinity.

Exact solutions of the problems are obtained in the following cases:

(a) S is the exteriority of simple closed contours lying along ox and oy. It is assumed that linear segments are parallel to the coordinate axes, while the domain is symmetrical with respect to these axes;
(b) S is the exteriority of one contour in the case of cyclic symmetry;
(c) S is a simply connected finite domain in the case of cyclic symmetry.

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ON NORMS OF SOME INTEGRAL OPERATORS

Theorem 1. *Let $f \in L_1(\mathbb{R})$, $f(x) \geq 0$ a.e. Then the norm of the convolution operator*

$$(W_f\varphi)(t) = \int_{-\infty}^{+\infty} f(t-\tau)\varphi(\tau)\,d\tau$$

in Lebesgue space $L_p(\mathbb{R})$, $p \geq 1$, is equal to $\widehat{f}(0)$, where

$$\widehat{f}(t) = \int_{-\infty}^{+\infty} e^{it\tau}\varphi(\tau)\,d\tau.$$

Let $p \geq 1$, $\alpha, r \in \mathbb{R}$. Denote by $L_{p,\alpha}$ the Banach space with the norm

$$\|\varphi\|_{p,\alpha} = \Big(\int_0^1 |x^\alpha\varphi(x)|^p\Big)^{1/p}$$

and by H_r denote the Hardy operator

$$(H_r\varphi)(x) = \int_0^x x^{r-1}y^{-r}\varphi(y)\,dy.$$

Theorem 2. *The operator H_r is bounded in the space $L_{p,\alpha}$ if and only if $r < 1-\alpha-1/p$. In this case*

$$\|H_r\|_{p,\alpha} = \frac{1}{1-\alpha-1/p-r}\,.$$

Let $r > 0$. Denote by R_r the Riemann-Liouville operator

$$(R_r\varphi)(t) = \int_0^x (x-y)^{r-1}x^{-r}\varphi(y)\,dy.$$

Theorem 3. *The operator R_r is bounded in the space $L_{p,\alpha}$ if and only if $\alpha + 1/p < 1$. In this case*

$$\|R_r\|_{p,\alpha} = B\big(1-\alpha-1/p, r\big),$$

where $B(a,b)$ is the Beta-function.

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ON APPLICATION OF SINGULAR INTEGRAL EQUATIONS TO A NUMERICAL SOLUTION OF SOME BOUNDARY PROBLEMS

Numerical solution of Dirichlet modified problem by singular integral equations is represented. Using the obtained computational schemes some concrete problems of conformal mapping are solved. The results of numerical experiments go well with theory.

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ON BIFURCATION OF SECONDARY FLOWS OF VISCOUS INCOMPRESSIBLE FLUID BETWEEN TWO POROUS ROTATING CYLINDERS

In the present report we consider the flow of viscous incompressible liquid between two counter-rotating porous cylinders. The oscillatory modes and bifurcation appearing in the vicinity of the intersection point of neutral curves, corresponding to rotating symmetrical and oscillatory instability is investigated.

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SOLUTIONS OF BOUNDARY LAYER CONDUCTING FLUID PROBLEMS IN THE STRONG MAGNETIC FIELD

When external magnetic field is strong, the problem of solution of the boundary layer equation of conducting fluid is reduced to the problem of solution of integro-differential equations with parameter $\frac{1}{N}$, where N is magnetohydrodynamical interaction factor and it is very large. We have considered four cases.

1. The motion is steady and not similar.

The velocity of external flow is a function of x, and the external magnetic field is constant.

2. The motion is steady and similar.

The velocity of external flow is proportional to x^m and the external magnetic field is proportional to $x^{\frac{m-1}{2}}$, where m is constant.

3. The motion is unsteady and not similar.

The velocity of external flow is a function of x and t, and the external magnetic field is constant.

4. The motion is unsteady and similar.

To determine the velocity of external flow, we considered four cases, when solutions exist. The magnetic field is selected accordingly. Solutions of the examine problems are sought in terms of a series with degrees $\frac{1}{N}$. For every problem the Green's functions are constructed, the expressions of three approximations are written out and all physical boundary layer characteristics are found.

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SOME PROBLEMS OF THE THEORY OF ELASTICITY FOR A NONHOMOGENEOUS PLANE WITH RECTILINEAR CRACKS

An elastic medium occupying the entire infinite plane of the variable $z = x + iy$ and weakened by rectilinear cracks is considered in the conditions of plane stressed state. The nonhomogeneous medium is characterized by the variable Poisson coefficient (the shift modulus is a constant value): $\sigma(y) = \sum\limits_{k=0}^{m} (A_k \cos k\alpha y + B_k \sin k\alpha y)$, where A_k $(k = 0, 1, 2, \dots, n)$, α are real-valued constants. We proceed from the complex representation formulas [1]:

$$X_x + Y_y = 2\big[\Phi(z) + \overline{\Phi(z)}\big] \sum_{k=0}^{n} \big[A_k \cos k(\overline{a}_0 z + a_0 \overline{z}) + B_k \sin k(\overline{a}_0 z + a_0 \overline{z})\big],$$

$$Y_y - X_x - 2iX_y = 2\Big\{A_0 z\overline{\Phi'(z)} + \overline{\Psi(z)} - \frac{\overline{\Phi'(z)}}{a_0} \sum_{k=1}^{n} \frac{1}{k} \big[A_k \sin k(\overline{a}_0 z + a_0 \overline{z}) -$$

$$-B_k \cos k(\overline{a}_0 z + a_0 \overline{z})\big] - \overline{\Phi(z)} \sum_{k=1}^{n} \big[A_k \cos k(\overline{a}_0 z + a_0 \overline{z}) + B_k \sin k(\overline{a}_0 z + a_0 \overline{z})\big] - \qquad (a)$$

$$-a_0 \sum_{k=1}^{n} k G_k^{*'}(z) \big[A_k \sin k(\overline{a}_0 z + a_0 \overline{z}) - B_k \cos k(\overline{a}_0 z + a_0 \overline{z})\big] -$$

$$-a_0^2 \sum_{k=1}^{n} k^2 G_k^{*}(z) \big[A_k \cos k(\overline{a}_0 z + a_0 \overline{z}) + B_k \sin k(\overline{a}_0 z + a_0 \overline{z})\big]\Big\},$$

where $a_0 = \frac{\alpha}{2} i$, $\Phi(z) = \prod\limits_{j=1}^{k} \big(\frac{d^2}{dz^2} + j^2 a_0^2\big) G_k(z)$, $G_k^*(z) = \prod\limits_{j=2}^{k} \big[\big(\frac{d^2}{dz^2} + (j-1)^2 a_0^2\big] G_k(z)$, $G_1^*(z) = G_1(z)$, $G_p(z) = G_{p+1}(z) + (p+1)^2 a_0^2 G_{p+1}(z)$ $(k, p = 1, 2, \dots, n)$; X_x, Y_y, X_y are stress components, $C_k(z)$ $(k = 1, 2, \dots, n)$, $\Psi(z)$ are complex potentials on the plane z.

Let us assume that the considered cracks lie along the ox-axis of the unbounded plane z and consider the case, where load is applied only to the crack surfaces, since other cases of load application can be reduced to this case by superposition. By formulas (a) and the method used in [2, §120], [3, Ch. 3, Section III, A, B], the problem is reduced to the Riemann–Hilbert boundary value problem, an explicit solution is found, and the stress intensity coefficient is determined near the crack ends.

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SOLUTION OF ONE LINEAR INTEGRAL EQUATION OF THE THIRD KIND

In the class of Hölder functions, the nonhomogeneous linear integral equation with coefficient $\cos x$ having zeros in the interval of variation of an independent variable is considered. Under certain assumptions on the kernel of the equations the necessary and sufficient condition for its solvability is given. The solution is constructed analytically via the Fredholm theory and the theory of Singular integral equations.

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ON THE INVESTIGATION OF FUNDAMENTAL PLANE CONTACT VALUE PROBLEM OF STATICS OF THE THEORY OF ELASTIC MIXTURES

1. Prof. M. Basheleishvili proved the analogues of the well-known Kolosov–Muskhelishvili formulas of general representations for non-homogeneous equations of statics in the theory of elastic mixtures. It is shown that in this theory the displacement and stress vector components, as well as the stress tensor components, are represented through four arbitrary analytic functions.

2. Applying the complex-valued representation of components of the displacement and stress vector, the two-dimensional fundamental contact value problem of statics in the theory of elastic mixtures is investigated in the paper. Using the potential method and the theory of Fredholm type equations, the existence and uniqueness theorems are proved.

3. The paper deals with the construction of an explicit solution for the contact value problem in the case, when the contact curve is a circle or an axis. In both cases the contact problems reduce to a system of Fredholm type equations, the solutions of which are vector-functions given on the border.

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STEADY OSCILLATION PROBLEMS IN THE LINEAR THEORY OF THERMOELASTICITY OF BINARY MIXTURES

The mathematical models of the mixtures of elastic solids are presented by Green A. E., Naghdi P. M., Steel T. R., Lempriere B. M., McNiven H. D., Tiersten H. F., Bowen R. M., Bedford A., Rushchitsky Y.

In this paper the boundary value problems of the binary mixtures of the elastic solids are investigated by means of the boundary integral method (BIM).

The fundamental solutions of the systems of the equations of static and steady oscillations for mathematical models of the binary mixtures of elastic solids are constructed in terms of elementary functions.

The uniqueness and existence theorems of interior and exterior boundary value problems of static and steady oscillations by means of BIM and multidimensional singular integral equations are proved. The Sommerfeld–Kupradze type radiation conditions are established.

The existence of eigenfrequencies of the interior homogeneous boundary value problems of steady oscillations is studied. The formulae of asymptotic distribution of eigenfrequencies and eigenfunctions are obtained.

The basic properties of plane waves are treated and existence theorems of eigenfrequencies of the interior homogeneous boundary value problems of steady oscillations are proved. The following connection between plane waves and eigenfrequencies is established.

Theorem 1. *If all plane waves propagating through a binary mixtures of elastic solids are damped, then the interior homogeneous steady oscillation problems have only the trivial solution, that is exist no oscillation eigenfrequency.*

Theorem 2. *If through a binary mixtures of elastic solids there propagates at least one plane wave with constant amplitude, then the existence of eigenfrequencies is possible in the interior homogeneous steady oscillation problems.*

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SOLUTION OF A TWO-DIMENSIONAL PROBLEM OF THE FILTRATION THEORY WITH THE PARTIALLY UNKNOWN BOUNDARY CONNECTED WITH UNDERGROUND WATER MOTION TOWARD THE EARTH EMBANKMENT SLOPE

A two-dimensional problem with the partially unknown boundary connected with steady-state motion of underground water toward the earth embankment slope is solved: the underground water motion domain is bounded from below by a horizontal impermeable foundation, while from above it is bounded by the unknown depression curve. In passing to the slope, the moving underground water forms a leakage interval, while a part of the moving liquid is contiguous to the water basin. Thus the boundary of the infinite singly-connected domain occupied by the moving liquid consists of the unknown curve and two segments – straight and semi-straight.

The porous medium is homogeneous, isotropic and nondifferentiable. Liquid motion in a porous mediun obeys the Darcy law. The liquid motion plane is brought into coincidence with a complex plane. The reduced complex potential is introduced, whose real and imaginary parts satisfy the Cauchy–Riemann conditions and two linearly independent boundary conditions.

Consideration is given to the complex planes of liquid motion, a complex potential, complex velocity and an auxiliary complex plane. On these complex planes, the domain occupied by the moving liquid is associated with singly-connected domains; for example, on the complex velocity plane, it is associated with a singly-connected circular pentagon one of whose vertices lies at the point at infinity. Characteristic properties are found at singular points and used to construct differential equations of the Fuchs class and its corresponding nonlinear Schwartz equation. Solutions of these equations are obtained.

To solve the filtration problem by means of solutions of the above-mentioned differential equations, three analytical functions are constructed, which conformally map the half-plane of the auxiliary complex plane on the domains of liquid motion, a complex potential and complex velocity. These functions satisfy the respective boundary conditions.

A system of higher transcendental equations is obtained for determining mechanical and geometrical characteristics of the problem and the abovementioned three domains. A solution algorithm is given for the obtained system of equations. A parametric equation of the depression curve is derived.

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SOME PROBLEMS OF POROELASTICITY

We propose to investigate the following problems and to develop efficient numerical processes for their solution:

1. Mathematical Problems of Poroelasticity (MPP): Modeling, Analysis.

1.1. The creation and justification of spatial nonlinear mathematical model for thermo-dynamic magneto-poroelastic media and analysis and comparisons with known theories.

1.2. The creation and justification of mathematical models for poroelastic plates and shallow shells having physical soundness. New physical layered effects explaining paradoxes peculiar to refined theories of plates in isotropic elastic case will be investigated.

1.3. The creation and justification nonlinear mathematical models for poroelastic beams with variable cross section. Construct and investigation of refined (in von Karman–Reissner sense) theories for elastic beams.

2. MPP: Investigations of some systems of nonlinear DE-s with averaged initial and boundary conditions corresponding to one and two dimensional (with respect to spatial coordinates) mathematical models for beams and plates.

2.1. The applications of generalized analytical functions nonlinear theory to two-dimensional mathematical models for poroelastic plates and shallow shells.

2.2. The development of nonlinear Volterra kind second type system and its application to some mathematical models for poroelastic beams.

3. MPP: Elaboration of numerical algorithms, creation of software and design of some practical objects.

In this part we consider only two-dimensional mathematical models of poroelasticity. Below D(x,y) denotes the connected domain:

3.1. If D represents a classical domain as circle, semicircle, ellipse, whole plane, first we prefer to use results of Analytical and Generalized Analytical Functions Theory and Potential Methods, constructing the nonlinear systems of integro-differential equations. Development of new numerical schemes.

3.2. If D represents a technical domain as curvilinear triangle, rectangle, trapezium, quarter of plane, then we prefer to use the schemes representing some modifications of our previously works.

The methods developing for investigations of corresponding problems represent nonlinear systems of integro-differential equations. Applications of BE or FE methods or Projective methods (using the apparatus of one or two variable orthogonal functions theory for rectangle or triangle regions) give also the new schemes for design.

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THE BASIC AND MIXED BOUNDARY VALUE PROBLEMS OF ELASTIC MIXTURES THEORY FOR INFINITE PLANE WITH CUTS

Suppose we have an infinite plane with curve cuts $S_1, S_2, \dots, S_n$. Let $S_J = p_j q_j$, $j = 1, 2, \dots, n$ be simple nonintersecting sufficiently smooth curves with positive direction from p_j to q_j. The infinite plane with cuts is denoted by D.

The following boundary value problems (BVP) are investigated for the homogeneous system of equations of elastostatics of the theory of mixtures.

Find a regular solution of the system in the domain D satisfying the one of the following boundary conditions:

(i) the displacement vector is given on both sides of the cuts (the first basic BVP);

(ii) the stress vector is given on both sides of the cuts (the second basic BVP);

(iii) the displacement vector is given on one side of the cuts and the stress vector is prescribed on the other side of cuts (the mixed BVP).

Applying the general representation formula of solution by means of complex valued potentials, the existence and uniqueness theorems are proved.

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ON SOME NONLINEAR PROBLEMS IN THE THEORY OF ELASTIC RODS

The problem of the bifurcation of equilibrated forms of the axis of the rod subjected to the joint action of axial compression force and distributed force of given intensity is investigated, taking into account tension-compression and displacement. The problems is described by a boundary value problem for a second order nonlinear ordinary differential equation depending on two real parameters (see [1], §8).

Questions on the existence of bifurcation points and continuous solution-branches of this boundary value problem are studied when axial compression force is proportional to transverse load. Functional are constructed to find bifurcation points which are critical values of forces. Asymptotic representations of small real solutions of this problem are given, which describe an approximate form of post-critical deformation of the rod.

In studying the problem of small bendings of rods of variable rigidity, the first eigenvalue (the critical value of load applied to the free end of the rod) of the linearized problem is expressed in terms of an energy functional (see, for instance, [2]). Of essential technical interest is the problem on a conditional maximum of the first eigenvalue with respect to rod rigidity. These questions led to the study of extreme properties of the first eigenvalue of the Sturm–Liouville operator with respect to the coefficients of a differential equation. The first eigenvalue is considered as the functional of these coefficients. The problem of a functional extremum is solved in the conditions of isoperimetric connections. Necessary and sufficient conditions are obtained for the first eigenvalue to reach its conditional extremum.

References

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SOME BODY DYNAMICS PROBLEMS OF ELASTICITY AND THERMOELASTICITY

Some body dynamics problems of the theory of elasticity and thermoelasticity are investigated for homogeneous and composite bodies with cavities when all physical factors produced by deformation are polynomially time-dependent.

Algorithms of the solution of the above-mentioned problems are constructed by reducing the latter to the definite three-dimensional problems of statics. Stress and displacement components, which define the state of the considered bodies, are represented as polynomials with respect to time whose coefficients are given by solutions of the three-dimensional problems of statics.

Some examples of bodies (solid and hollow balls) and oscillations (of a round membrane, rod and string) are considered. These examples illustrate the application of the problem solution algorithms.

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